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Bayesian inference to damping identification of fiber-reinforced composites from experimental modal data

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Abstract

For stochastic vibration analysis of the composite plate in-situ stochastic data of the elastic parameters and damping loss factors are essential. Deterministic values of the elastic and damping parameters of the composite plates can be identify using various optimization technique. In recent decades various stochastic inverse identification procedures are shown to evaluate the in-situ stochastic parameters of the elastic constants of the composite plate. However, identification of the damping loss factor of the composite plate has received less attention. A stochastic Bayesian inverse identification technique is used here to evaluate the in-situ uncertainty of the damping loss factors of the glass-epoxy composite plate. Experimentally evaluated first 4 modal damping ratio of the glass-epoxy composite plate are incorporated in Bayesian inverse inference formulation through likelihood function. Novelty of the study lies in the development Bayesian inverse formulation considering modal damping response to identify the stochastic damping loss factors of the glass-epoxy composite plate.

Keywords: Bayesian inverse inference, Loss factors, Composite plate.

1 INTRODUCTION

Inverse identification of the material parameters of a composite structure is popularly done by minimization of the error function between experimentally evaluated response and model response by using some optimization technique. This error function minimization technique can identify the material parameters of the system as a design variable. The identification of the material parameters using this approach can estimate the parameters deterministically without considering uncertainty involve in the modeling and material properties [1, 2, 3]. The randomness in the composite plate is arise due to variability of several factors such as fiber orientations, laminate sequences and various manufacturing uncertainty. The randomness of the material parameters should be considered while estimating the stochastic dynamic response of the composite plate. Generally an assumption of the variation of material properties are made prior to the stochastic dynamic analysis instead of considering in-situ variability of the material parameters. Therefore, a inverse framework for stochastic estimation of the in-situ uncertainty of the material properties of the composite plate is essential. In such cases stochastic representation of the experimentally evaluated modal response in terms of a probability distribution function (PDF) are necessary.

In recent years, stochastic inverse identifications of the material parameters such as elastic parameters and Poisson's ratio are done by generalized polynomial chaos (gPC) expansion method by Sephavand and Marburg [4, 5]. A collocation based gPC expansion method [8, 9, 6] is used for stochastic inverse identification of the parameters of the composite plate. The stochastic inverse identification of the parameters involve estimation of the unknown deterministic coefficients of the gPC expansion by optimization algorithm [7]. However, Bayesian inverse identification technique is also quite popular among the researchers [10, 11, 12] to identify the elastic properties of a dynamic system since 1979 [13]. Statistical properties such as mean and standard deviation (SD) of the identified parameters can be derived from the posterior distributions. The basic advantage of the Bayesian inverses technique is posterior PDF can be develop from a limited numbers of experimental observations. The evaluation of integral is most challenging task in multi-parameter Bayesian inverse inference. However, Markov Chain Monte Carlo (MCMC) [14, 15] became an efficient alternative to determine the posterior density without evaluating the integral. Various sampling based approaches such as, Metropolis-Hasting

(M-H) algorithm and Gibbs sampler [16] algorithm have been developed for improvement of the MCMC algorithm. Memory less M-H algorithm can efficiently derive the multi-parameter posterior PDF. More recently, Nagel [17] discussed Bayesian inverse problem with a direction to overcome the limitations of sampling based technique for determining the posterior probability density functions of the system parameters. Various advanced techniques and improvements of Bayesian inference technique are proposed in stochastic identification of the elastic constant [18, 19]. Gogu et al. [19] identified the elastic constants of the unidirectional laminate by tensile strength test. Of late, Rappela and Beex [20] identified material randomness of the discrete strut using Bayesian inverse updation and corresponding influences of the geometric randomness of the fiber. Rosić et al. [21] have proposed a linear Bayesian estimation of the unknown parameters in combination with the Karhunen-Loève and Polynomial Chaos expansion without using any sampling based technique such as, MCMC. This method can efficiently update non-Gaussian uncertainties. However, stochastic identification of the elastic parameters of multilayer bi-directional composite plate using Bayesian inverse inference technique from modal experimental data has not explored yet. Moreover, researchers have paid much attention to identify the elastic constants of the composite plate, while identification of the damping loss factors of the composite plate is indispensable requirement for the damped vibration analysis of the composite plate.

Yesilyurt and Habibe [22] have deterministically identified modal damping ratio of a composite beam by using short time Fourier transformation (STFT). Cherif et al. [23] have determined damping loss factors of a two dimensional orthotropic plate using inverse Wave method and compared with the classical methods. Li et al. [24] have identified loss factors of a carbon-epoxy composite plate using modal test data using complex modulus approach. However, stochastic identification of the damping loss factor has not been reported earlier. Here, loss factors of the multilayer bi-directional composite plate is identified using Bayesian inverse technique from experimentally evaluated modal damping ratio.

2 MATHEMATICAL FORMULATIONS

2.1 Finite element formulation of laminated composite plate

Classical thin plate theory is used for analysis of a laminated composite plate using first order shear deformation theory (FSDT). The stress $\boldsymbol{\sigma'}$ and strain $\boldsymbol{\varepsilon'}$ relationship for a thin unidirectional lamina is presented by the generalized Hook's law with reference to the principal material axes (1, 2, 3) as

$$\sigma' = \boldsymbol{C}\boldsymbol{\varepsilon'}. \tag{1}$$

Here, $\boldsymbol{C}$ is principal stress-strain relationship matrix [25] along the principal axes of the k^{th} lamina for the multilayer bi-directional laminated composite plate. The elements of the $\boldsymbol{C}$ matrix for the k^{th} orthotropic lamina are $C_{11} = E_{11}/(1-\nu_{12}\nu_{21})$, $C_{12} = \nu_{21}E_{11}/(1-\nu_{12}\nu_{21})$, $C_{22} = E_{22}/(1-\nu_{12}\nu_{21})$, $C_{33} = G_{12}$, $C_{44} = G_{23}$ and $C_{55} = G_{13}$. Here E_{ii}, G_{ij} and ν_{ij} are the set of elastic constants such as, Young's moduli, shear moduli and Poisson's ratio of the lamina, respectively. The stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\varepsilon}$ relationship of the lamina with reference to global laminate axes (x, y, z) is written as

$$\boldsymbol{\sigma} = \boldsymbol{Q}\boldsymbol{\varepsilon}, \tag{2}$$

in which,

$$\boldsymbol{Q} = \boldsymbol{\mathcal{T}}^{-1}\boldsymbol{C}\boldsymbol{\mathcal{T}}. \tag{3}$$

Here $\boldsymbol{\mathcal{T}}$ is transformation matrix [25] to relate the principal lamina axes and laminate axes. The element stiffness matrix is of the laminated is presented as

$$\boldsymbol{K_e} = \int_{A_e} \boldsymbol{B}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{B}\,\mathrm{d}A_e. \tag{4}$$

In which, $\boldsymbol{B}$ [25] is the strain-displacement matrix and written as

$$\bar{\boldsymbol{\varepsilon}} = \boldsymbol{B}\boldsymbol{\delta}. \tag{5}$$

where $\bar{\boldsymbol{\varepsilon}} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_{xy} \quad \kappa_x \quad \kappa_y \quad \kappa_{xy} \quad \varepsilon_{xz} \quad \varepsilon_{yz}]^{\mathrm{T}}$ is the strain and curvature vector and $\boldsymbol{\delta} = [u_{0j} \quad v_{0j} \quad w_{0j} \quad \theta_{xj} \quad \theta_{yj}]^{\mathrm{T}}$ is the nodal displacement vector of the composite plate. Eight-nodded element is used for finite element (FE) analysis of te composite plate.The mid-plane stress resultant $\bar{\boldsymbol{\sigma}}$ and strain $\bar{\boldsymbol{\varepsilon}}$ of the laminate are related by stiffness matrix $\boldsymbol{D}$ as [25]

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{D}\bar{\boldsymbol{\varepsilon}}. \tag{6}$$

In which $\bar{\boldsymbol{\sigma}} = [N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy} \quad Q_x \quad Q_y]^{\mathrm{T}}$ and $\boldsymbol{D}$ is given by

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix}, \tag{7}$$

where $A_{ij}, B_{ij}, D_{ij} = \sum_{k=1}^{N} \int_{z(k-1)}^{z_k} (Q_{ij})_k (1, z, z^2) \mathrm{d}z,\ i, j = 1, 2, 6$ and $A_{ij} = \sum_{k=1}^{N} \int_{z(k-1)}^{z_k} \kappa (Q_{ij})_k \mathrm{d}z,\ i, j = 4, 5,\ \kappa = 5/6$. The elemental mass matrix is written as

$$\boldsymbol{M_e} = \int_{A_e} \boldsymbol{N}^{\mathrm{T}} \boldsymbol{\rho} \boldsymbol{N} \mathrm{d}A_e, \tag{8}$$

where $\boldsymbol{\rho}$ is the inertia matrix. The global stiffness matrix $\boldsymbol{K}$ and the global mass matrix $\boldsymbol{M}$ are developed after assembling the elemental stiffness and mass matrices, $\boldsymbol{K_e}$ and $\boldsymbol{M_e}$, respectively. Therefore, Undamped modal analysis involves the solution of

$$[\lambda_i^2 \boldsymbol{M} + \boldsymbol{K}]\phi_i = 0, \quad i = 1, 2, ..., n \tag{9}$$

and extract the modal frequency λ_i and mode shape ϕ_i of the laminated composite plate with n numbers of degree of freedom (DOF) in FE model.

2.2 Modal damping formulation of laminated composite plate

During vibration, dissipation of the specific energy in the form of heat and acoustic radiation is responsible for the passive damping of the composite plate. Mathematical estimation of the damping of the composite plate is necessarily simple and yet accurately represent the damping of the composite plate. Viscoelastic damping model is efficiently represent the energy dissipation from the vibrating composite plate via complex modulus approach. It is assumed that stress and strain are harmonically time dependent for linear viscoelastic material and are presented as $\sigma = \sigma_0 e^{i\omega t}$ and $\varepsilon = \varepsilon_0 e^{i\omega t}$. Linear viscoelastic constitutive relationship is expressed as

$$\sigma(t) = \int_{-\infty}^{t} C^*(t - \tau) \mathrm{d}\varepsilon(\tau), \tag{10}$$

where complex modulus is expressed as a combination of a real and a imaginary component as $C^* = C' + iC''$. Real component C' and imaginary component C'' of the elastic moduli are termed as storage moduli and loss moduli, respectively. Complex elastic constants of the composite material in the principal lamina direction are evaluated as

$$E_{11} = E_{11}(1 + i\eta_{11}), \quad E_{22} = E_{22}(1 + i\eta_{22}), \quad G_{12} = E_{12}(1 + i\eta_{12}). \tag{11}$$

Here in η_{11}, η_{22} and η_{12} are the damping loss factors along the longitudinal, transverse and shear directions of the lamina, respectively. Accordingly, complex elastic constants are inserted in Eq. (2) and corresponding complex stiffness matrix is presented as

$$\boldsymbol{K}^* = \boldsymbol{K}_{\mathrm{R}} + i\boldsymbol{K}_{\mathrm{I}}. \tag{12}$$

Here $\boldsymbol{K}_{\mathrm{R}}$ storage stiffness matrix and $\boldsymbol{K}_{\mathrm{R}}$ is loss stiffness matrix. Hence, damped dynamic equation of the composite plate is written as

$$\boldsymbol{M}\ddot{x} + \boldsymbol{K}^{*}x = \boldsymbol{f}. \tag{13}$$

Assumed solution of the Eq. (13) is $x = \{\phi_i^*\}e^{i\lambda_i^* t}$, where ϕ_i^* and λ_i^* are n^{th} complex eigen mode and complex eigen frequency. Homogeneous solution of the Eq. (13) is written as

$$[\lambda_i^{*2}\boldsymbol{M} + \boldsymbol{K}]\phi_i^* = 0, \quad i = 1, 2, ..., n. \tag{14}$$

The complex eigen frequency is written as

$$\lambda_i^* = \lambda_{i\mathrm{R}}^*(1 + i\eta_i)^{1/2}. \tag{15}$$

Thus modal loss factor η_i of the composite plate is written as

$$\eta_i = \frac{\mathrm{Im}(\lambda_i^{*2})}{\mathrm{Re}(\lambda_i^{*2})}. \tag{16}$$

The ideal forward problem to estimate the modal loss factor η_i of the composite plate stated in the following form with reference to the Eqs. (14), (16)

$$\eta_i = G(\eta_{11},\ \eta_{22},\ \eta_{12},\ E_{11},\ E_{22},\ G_{12}) \quad i = 1, 2, ..., n. \tag{17}$$

Considering the fact that nominal values of the elastic parameters are known and distributions of the η_i are evaluated experimentally, then stochastic damping loss factors η_{11}, η_{22} and η_{12} along the principal directions of the lamina can be identified by inverse stochastic procedure. Corresponding forward model in close form is written as

$$\boldsymbol{d} = G(\boldsymbol{m}), \tag{18}$$

where set of simulated modal loss factors for ideal case is define by $\boldsymbol{d} = \{\eta_i\}^{\mathrm{T}}$ and $\boldsymbol{m} = \{\eta_{11}\ \ \eta_{22}\ \ \eta_{12}\}^{\mathrm{T}}$ is the vector of identifiable parameters of the inverse problem.

2.3 Bayesian inverse model

Considering the forward formulation stated in Eq. (18), classical inverse problem involves identification of model parameter $\boldsymbol{m}$ from the actual observation of data set $\tilde{\boldsymbol{d}}$. Thus, model lead to relation

$$\tilde{\boldsymbol{d}} = G(\boldsymbol{m}) + \boldsymbol{\varepsilon}, \tag{19}$$

where $\boldsymbol{\varepsilon}$ is modal additive error account for deviation between simulated value $\boldsymbol{d}$ and observed values $\tilde{\boldsymbol{d}}$. The components of $\boldsymbol{\varepsilon}$ are i.i.d. random variables. The Bayesian inverse problem concerned with estimating the parameters $\boldsymbol{m}$ given a set of observation data $\tilde{\boldsymbol{d}}$. The posterior probability density of the model parameters $\boldsymbol{m}$ takes the form

$$p(\boldsymbol{m}|\tilde{\boldsymbol{d}}) = \frac{p(\tilde{\boldsymbol{d}}|\boldsymbol{m})p(\boldsymbol{m})}{\int p(\tilde{\boldsymbol{d}}|m)p(\boldsymbol{m})\mathrm{d}\boldsymbol{m}}. \tag{20}$$

Herein $p(\boldsymbol{m})$ indicate the prior probability of the parameters $\boldsymbol{m}$. Likelihood $p(\tilde{\boldsymbol{d}}|\boldsymbol{m})$ is define as a function of $\boldsymbol{m}$, such as $\mathscr{L}(\boldsymbol{m}) = p(\tilde{\boldsymbol{d}}|\boldsymbol{m})$. A typical assumption for likelihood function for observed data $\tilde{d}_j$ are normally distributed with a SD σ and expected value $G(\boldsymbol{m})$ and can write as

$$p(\tilde{d}_j|\boldsymbol{m}) = \frac{1}{\sigma\sqrt{2\pi}}exp\left(-\frac{(\tilde{d}_j - G(\boldsymbol{m}))^2}{2\sigma^2}\right) \tag{21}$$

and likelihood is stated for n numbers of observational data as

$$\mathscr{L}(\boldsymbol{m}) = \prod_{j=1}^{n} p(\tilde{d}_j|\boldsymbol{m}) \tag{22}$$

$$\mathscr{L}(\boldsymbol{m}) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^n}\left[\exp\left(-\frac{(\tilde{d}_j - G(\boldsymbol{m}))^2}{2\sigma^2}\right)\right]^n. \tag{23}$$

The integral at the numerator of Eq. (20) is a normalizing constant and described by c. Inverse identification of parameters $\boldsymbol{m}$ from md set of modal data can be written using the theory of total probability as [26]

$$p(\boldsymbol{m}|\tilde{\boldsymbol{d}}) = \prod_{i=1}^{md} p(\boldsymbol{m}|\tilde{d}_i) \tag{24}$$

$$p(\boldsymbol{m}|\tilde{\boldsymbol{d}}) \propto p(\boldsymbol{m})^{md} \prod_{i=1}^{md} \mathscr{L}_i(\boldsymbol{m}) \tag{25}$$

$$p(\boldsymbol{m}|\tilde{\boldsymbol{d}}) \propto p(\boldsymbol{m})^{md} \frac{1}{\left((2\pi)^{md/2}\prod_{i=1}^{md}\sigma_i\right)^n} \exp\left[\sum_{i=1}^{md}(-0.5\boldsymbol{\mathcal{A}}_i^T \sigma_i^{-2}\boldsymbol{\mathcal{A}}_i)\right], \tag{26}$$

where $\boldsymbol{\mathcal{A}}_i$ is define as $\sum_{j=1}^{n}\left\{(\tilde{d}_{ij} - G(m_i))^2\right\}$. The strategy for numerical solution of the stochastic inverse problem to identify the parameters is proposed using Markov chain Monte Carlo (MCMC) method. The idea behind the MCMC is to construct posterior distribution without evaluating the normalizing constant c. The Metropolis-Hastings (M-H) algorithm construct a stationary Markov chain whose stationary distribution equal to the target distribution. M-H algorithm involve generating new sample point y of Markov chain from a proposal distribution $q(\cdot|x^{(k)})$ conditional to the current state $x^{(k)}$ and then accepts and rejects the new sample y with certain probability of acceptance. Generation of sufficient numbers of sample of a Markov chain with a stationary distribution is equivalent to the target posterior distribution $p_X''(x)$. The stationary distribution of the Markov chain is obtained by ignoring the initial burn-in period of the proposal distribution.

Here in $\boldsymbol{m}$ denotes vector of model parameters and $\boldsymbol{d}$ is set of simulated data for ideal case. The forward model operator G predicts the model output data set $\boldsymbol{d}$ in terms of eigen frequency as a function of model parameters m. In the present paper, model parameters are E_{ii} and G_{ij}, and the forward model yield the data output in the form of modal frequency λ_i.

3 NUMERICAL EXAMPLE

A spectral modal Bayesian inverse inferences technique is presented herein to identify the stochastic damping loss factors of the glass-epoxy composite plate. An experimental free vibration analysis is conducted on 12-layers glass-epoxy composite plate in free-free boundary conditions. 100 numbers of identical composite plate of dimension $250 \times 125 \times 2$ mm^3 are used for the free vibration analysis. The modal frequency and modal damping ratio of the composite plate is determine by impulse hammer technique. To identify stochastic damping loss factors, elastic parameters of the glass-epoxy composite plate are considered as deterministic and nominal values of the elastic parameters are identified in [27] using inverse gPC identification. The nominal vales of the elastic parameters and mass density are given in Table 1. These values were identified from the first 4 experimentally evaluated eigen frequency. In this paper the loss factors of the composite plate are evaluated from experimentally evaluated first 4 modal damping ratios using Bayesian inference technique. An innovative MCMC is used to construct the posterior distribution of the loss factors. The innovation lies on the construction of the likelihood function from the 4 modal data and simultaneously identified 3 loss factors by inverse operation.
Experimentally evaluated modal sampling data is used to update the damping loss factors from uninformative prior. Random walk of the M-H algorithm, is used to generate posterior samples of $\{\eta_{11}\ \ \eta_{22}\ \ \eta_{12}\}^{\mathrm{T}}$ using 50

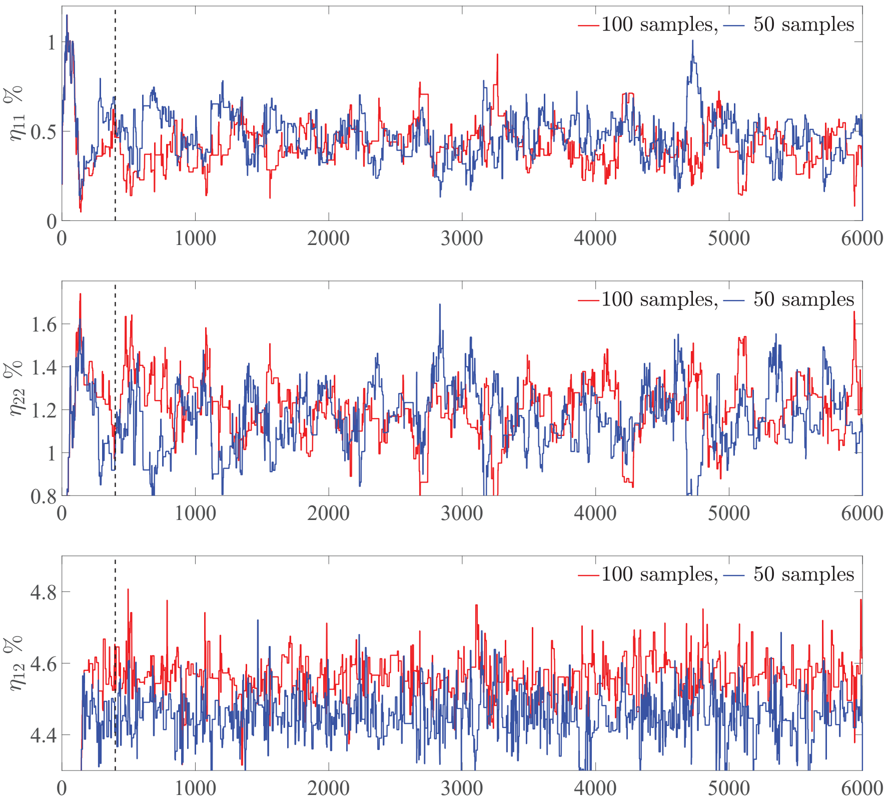

Figure 1. MCMC sample for posterior distribution of the loss factors due to proposal SD $\sigma_\eta = \{0.005\ 0.025\ 0.1\}^{\mathrm{T}}$.

and 100 experimentals sample data. The choice of proposal distribution is very important to specify the jump towards the target distribution. Generally normal distribution, centered about the current sample point, is used as a proposal distribution. However, asymmetric proposal distribution like log-normal distribution [28] is used here to avoid the generation of the sample points smaller than 0. The SD of the proposal PDF is proposed as $\sigma_\eta = \{0.005\ 0.025\ 0.1\}^{\mathrm{T}}$ with log-normal distribution. Same starting points of the identifiable parameters are considered here as $\{x^{(0)}_{\eta_{11}}\ x^{(0)}_{\eta_{22}}\ x^{(0)}_{\eta_{12}}\}^{\mathrm{T}} = \{0.2\ 0.2\ 0.2\}^{\mathrm{T}}$. However, in Fig. 1 it is observed that each loss factors are converged towards its target distributions after initial burn-in period for both cases from a uninformative prior. Sample statistics of the parameters are obtained after discarding initial 400 samples. The stochastic parameters of the identified loss factors are given in Table 2 discarding the initial burn-in period. Table 2 reveals that variation of the statistical parameters of the identified loss factors are remain within a limited range of variation for two types of sample data. Therefore, proposed modal Bayesian inverse inferences technique can efficiently identify the stochastic loss factors of the composite plate from the modal experimental data.

Table 1. Mean values of the elastic parameters and density of the investigated plate as identified in [27]

Elastic moduli [GPa]	$E_{11} = 68.714$	$E_{22} = 27.401$	$G_{12} = 6.122$
Poisson's ratio [-]	$\nu_{12} = 0.24$	$\nu_{21} = 0.24$	–
Density [gm/cm^3]	2.1143	–	–

Table 2. Identified stochastic loss factors of the glass-epoxy composite plate

Parameters	Prior		Posterior (50 samples)		Posterior (100 samples)	
	Mean	SD	Mean	SD	Mean	SD
η_{11} [%]	0.8	0.32	0.47	0.12	0.42	0.11
η_{22} [%]	3	1.20	1.16	0.15	1.20	0.13
η_{12} [%]	5	2	4.46	0.09	4.56	0.05

4 CONCLUSION

An stochastic Bayesian inverse identification technique is used to identify the loss factors of the glass-epoxy composite plate from the experimentally evaluated modal damping ratio. In Bayesian inverse formulation the likelihood function is developed from the sufficient numbers of modal data and the same likelihood function is used to identify the three loss factors of the composite plate simultaneously. 50 and 100 numbers of first 4 modal damping ratios are used to update the posterior distribution via likelihood functions. To avoid calculation of the integration in the denominator of the Bayesian equation MCMC algorithm has been used to determine the posterior distribution of the loss factors. Sufficient numbers of posterior sample points are generated using MCMC approach. The efficient learning of the inverse model is indicated in Fig. 1 by suitable convergence of the identifiable parameters with 50 and 100 numbers of sample. Random walk of MCMC is supported log-normal proposal distribution. Thus efficient stochastic identification of the set of loss factors of the glass-epoxy composite plate are shown in this paper using Bayesian inverse identification technique.

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