Stochastic Vibro-acoustic Analysis in the Uncertain Thermal Environment

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Introduction

Noise inside the cabin like structure of the car compartment, fuselage of aircraft is an important aspect of vibro-acoustic analysis considering the strict environmental regulations. The vehicular structure can be ideally represented by an enclosed rectangular cabin like structure with one or more vibrating wall and thus forming an acoustic cavity inside the structural domain. Composite materials are increasingly used in automobile and aerospace industries due to high specific stiffness, low specific weight and thermal stability. The acoustic responses inside the cabin are significantly modify with the application of the composite structure as a flexible wall in comparison to thick and acoustically rigid wall. The dynamic behavior of composite structure is interacted acoustically to modify the noise inside the enclosed cavity. Moreover, during the service life the structures are subjected to the random thermal environment due to high speed of the vehicles, aerodynamic friction and hight variations. The elastic moduli and damping of composites are varied significantly with the variation of the temperature. Therefore, study on stochastic vibro-acoustic response of the enclosed cavity due to random temperature increment is essential to design the quite ambiance inside the acoustic enclosure.

The studies on coupled structural-acoustic analysis have been carried out extensively by researchers since several decades. Interior acoustic domain can be modeled by the finite element (FE) method [1] or boundary element (BE) method [2, 3]. The coupled structural-acoustic analysis can be carried out by modeling both domain viz. structural and acoustic, using FE method [4, 5]. On the other hand, it is convenient to model the interior acoustic domain by BE discretization and continuity condition of the particle velocity and pressure are used for structural-acoustic coupling [6, 7]. Niyogi et al. [8] presented a FE-BE based coupled structural-acoustic formulation for an enclosed cavity with single or multiple composite walls. More recently, Sarigül and Karagözlü [9] have presented modal structural-acoustic coupling analysis of a composite plate to determine the coupled natural frequency of the system. Moreover, elastic properties of the composite structure degrade while exposed to the thermal environment. Sai ram and Sinha [10] reported that with increment of the temperature the eigen frequencies of the composite plate are decreased for various boundary conditions. In fact, the variation of the temperature is random in nature rather than deterministic. Therefore, elastic properties of the composite structure are varied with the random variation of the temperature, thus resulting eigen frequencies and mode shapes of the composite structure are varied accordingly. The acoustic responses inside the enclosed cavity become random due to the interaction of the vibrating composite structure which is subjected to the random thermal environment. The enriched deterministic studies in the field of interior structural-acoustic using FE-FE and FE-BE coupled formulation with the application of the composite structure is available. Different researchers also explored the deterministic variation of the frequency response of the composite plate with the increment of the temperature, but they did not consider the uncertainty of the thermal environment. However, the uncertainty of the frequency responses of the composite plate in thermal environment due to random elastic properties has reported by Kumar [11] using perturbation method. The application of the perturbation has an inherent limitation of the applicability with small coefficient of variation. In recent past, Sephavand et al. [12] and Sepahvand [13] have explored the stochastic acoustic response due to uncertain material properties of the composite plate using generalized Polynomial chaos (gPC) expansion.

An overview of literatures reflect that deterministic interior coupled structural-acoustic analysis already reported without considering the thermal effect on the composite plate. Moreover, uncertainty in acoustic response of the flexible cavity due to random temperature increment is never been reported earlier. Therefore, the present paper aims to report the uncertainty of the sound pressure level (SPL) inside the flexible cavity due to random temperature increment.

Mathematical formulations

Mathematical modeling of the multiple walls structure in thermal environment is done by FE method considering proper folded plate transformation. The interior acoustic domain of the enclosure is modeled by BE method. The continuity of the pressure and velocity is used to couple the structure and acoustic domains. Temperature increment is considered as an uncertain parameter and is represented by non-sampling based gPC expansion method. The FE-BE coupled formulation is used as a deterministic solver for the collocation based gPC expansion method.

Stochastic FE formulation

A laminated composite plate of uniform thickness h with n numbers of arbitrarily oriented thin lamina is used for the analysis. The displacement fields, element geometry, and shape functions, as presented by Sai ram and Sinha [10], are taken for FE modeling. The generalized displacement field in terms of mid-plane displacement is described by $\{d\} = \{u \quad v \quad w \quad \theta_{\mathrm{x}} \quad \theta_{\mathrm{y}}\}^{\mathrm{T}}$. The constitutive equation of the composite plate subjected to the thermal environment is stated as

$$\{F^r\} = [D]\{\varepsilon^*\} - \{F^N\}, \tag{1}$$

in which D is the stress-strain relationship matrix, ε^* is generalized mid-plane strain vector and thermal resultant force and moment vector is denoted by F^N. The uncertainty of the temperature increment is modeled by the truncated gPC expansion, and is expressed as [14]

$$T(\xi) = \sum_{i=0}^{N} a_i \Psi_i(\xi). \tag{2}$$

The elastic properties of the composite became uncertain due to the uncertain thermal environment. The stochastic dynamic system properties of the composite plate using FE formulation are written as a function of random temperature increment

$$\begin{aligned} K_e(T(\xi)) &= \int_{A_e} B^T D(T(\xi)) B \mathrm{d}A_e \\ K_{Ge}(T(\xi)) &= \int_{A_e} G^T S_r(T(\xi)) G \mathrm{d}A_e. \end{aligned} \tag{3}$$

Here, $K_e(T(\xi))$ and $K_{Ge}(T(\xi))$ are stochastic elemental stiffness and geometric stiffness matrices, respectively. The elemental mass matrix M_e does not depend on the random temperature increment and presented as

$$M_e = \int_{A_e} N^T \bar{m} N \mathrm{d}A_e, \tag{4}$$

in which $\bar{m}$ is the distributed inertia matrix. An orthogonal transformation matrix $\mathscr{T}$ [8] is proposed to relate the global displacement d' of the composite plate with the local displacement d of the composite plate as $d = \mathscr{T} d'$. The local elemental matrices of the composite structure are transform into global coordinate system as

$$\begin{aligned} K'_e(T(\xi)) &= \mathscr{T}^T K_e(T(\xi)) \mathscr{T} \\ K'_{Ge}(T(\xi)) &= \mathscr{T}^T K_{Ge}(T(\xi)) \mathscr{T} \\ M'_e &= \mathscr{T}^T M_e \mathscr{T}, \end{aligned} \tag{5}$$

and assembled thereafter to obtain the global matrices $\boldsymbol{K}(T(\xi))$, $\boldsymbol{K_G}(T(\xi))$ and $\boldsymbol{M}$ of the composite structure. The equation of stochastic eigen value problem due to thermal uncertainty take the following form

$$\big(\boldsymbol{K}(T(\xi)) + \boldsymbol{K_G}(T(\xi))\big)\phi(T(\xi)) = \omega(T(\xi))^2 \boldsymbol{M} \phi(T(\xi)). \tag{6}$$

Here, uncertain eigen frequencis $\omega(T(\xi))$ and uncertain mode shapes $\phi(T(\xi))$ are represented by the truncated gPC expansion.

The stochastic impedance relationship for the damped multi-degree freedom system subjected to the harmonic loading can be stated as

$$\begin{aligned} \{\boldsymbol{v}(\xi)\} = \Big[&\Omega[\phi(T(\xi))] \\ &\mathrm{diag}\Big(\frac{2\Omega\omega_k(T(\xi))\zeta_k + i(\omega(T(\xi))^2 - \Omega^2)}{(\omega_k(T(\xi))^2 - \Omega^2)^2 + 4(\Omega\zeta_k\omega_k(T(\xi)))^2}\Big) \\ &[\phi(T(\xi))]^T\Big]\{f\}e^{(i\Omega t)}. \end{aligned} \tag{7}$$

Here, $\{\boldsymbol{v}(T(\xi))\}$ denotes nodal structural velocity vector, Ω is the forcing frequency and nodal force f_k is multiplied by initial phases in the form of, $f_k = f_{ok} e^{i\Omega t}$ for k^{th} global degree of freedom system. The modal stochastic eigen frequency and eigen mode shape are expressed as $\omega_k(T(\xi))$ and $\phi(T(\xi))$ respectively, and ζ_k denote the modal damping ratio of the vibrating structure. The velocity and force components normal to the interacting boundary are used while coupling with the acoustic system equation.

BE formulation

The interior acoustic domain is discretized by 2 - dimensional eight-noded BE mesh. The Helmholtz equation for the time harmonic acoustic problem is written as

$$\bigtriangledown^2 p + k^2 p = 0 \tag{8}$$

in which, p is the acoustic pressure and k is the wave number of the acoustic medium. For interior acoustic domain linear system equation is written as

$$[H]\{p\} = [G]\{v\}. \tag{9}$$

Stochastic FE-BE coupling

Selecting only the normal velocity and the pressure components of the flexible surfaces, deterministic mobility relation is derived from Eq. (7) by converting nodal forces to nodal pressure term and written in the form of

$$\{v\} = [\mathscr{Q}]\{p\}. \tag{10}$$

Here, $[\mathscr{Q}]$ is the mobility matrix relating normal nodal velocity component with the nodal pressure term of the vibrating surface. Eq. (9) is solved to determine the sound pressure level at the boundary by inserting the proper boundary conditions. The elemental normal nodal velocities on the rigid boundary surface are specified in prior. However, elemental normal nodal velocity and the nodal pressure are unknown on the interacting surface. The nodal velocity on the interacting boundary is replaced by using the Eq. (10) in terms of p. After proper manipulation the Eq. (9) transforms into the following form

$$\{p\} = [\mathscr{A}]\{v\} \tag{11}$$

and solves for nodal boundary pressure p. The stochastic mobility relation in Eq. (7) has developed with uncertain eigen frequency and uncertain mode shape. Thus Eq. (11) is shifted to the stochastic system equation

$$\{p(\xi)\} = [\mathscr{A}(\xi)]\{v(\xi)\}. \tag{12}$$

The stochastic pressure $p(\xi)$ at each forcing frequency Ω can be expressed by truncated gPC expansion as

$$p(\xi) = \sum_{i=0}^{N} b_i \Psi_i(\xi). \tag{13}$$

Here, $\Psi_i(\xi)$ is the polynomial basis function and b_i is the unknown deterministic coefficients. The deterministic unknown coefficients are determine by stochastic

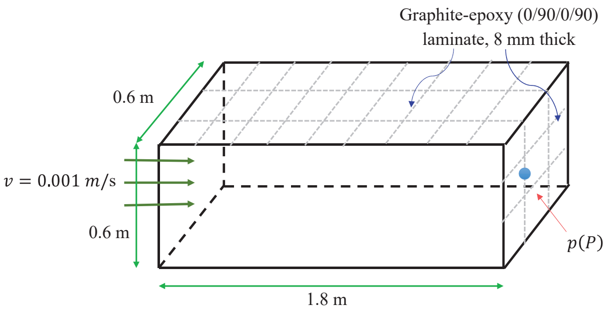

Figure 1: Geometry of the acoustic cavity

collocation method. A set of deterministic solution at n numbers of temperature at the predefine collocation points, are generated. The Eq. (13) is solved to determine the unknown deterministic coefficients and corresponding statistical properties of the SPL at a given forcing frequency.

Numerical example

A coupled interior acoustic analysis is presented here for a rectangular box type structure in uncertain thermal environment. A 8 mm thick graphite-epoxy composite plate is used in the top and right walls which are subjected to the uncertain thermal environment. The left wall acting as a rigid piston and the remaining walls are considered as a rigid, see Figure 1. The temperature dependent elastic properties of the graphite-epoxy plate is presented in Table 1. The mean μ of the random temperature increment are 25 K and 50 K, considering 300 K as a reference temperature. The corresponding standard deviations of the random temperature increment are 5 K and 10 K. The uncertainty of the SPL at for various random mean temperatures are are estimated at the P point on the right wall.

Table 1: Elastic moduli of graphite-epoxy lamina at different temperatures, cf. [10], $G_{13} = G_{12}$, $G_{23} = 0.5G_{12}$.

Elastic moduli (GPa)	Temperature, T (K)			
	300	325	350	375
E_{11}	130	130	130	130
E_{22}	9.5	8.5	8.0	7.5
G_{12}	6.0	6.0	5.5	5.0

Uncertainty of SPL due to random temperature increment

The uncertainty of the SPL at the P point on the right flexible wall has been studied due to random temperature increment at the damping ratio $\zeta = 0.5\%$. Coupled interior acoustic analysis at random temperature of 325 K and 350 K are presented in Figure 2 and 3, respectively. The peak of the sound pressure level has observed at acoustic mode of the box. The first five eigen frequencies of the folded composite plate at the mean random temperature of 325 K and 350 K are (855.8, 928.7, 1114.8, 1332.8, 1522.7 rad/s) and (811.6, 878.1, 1056.7, 1264.0, 1460.4 rad/s), respectively. The uncertainty of the SPL is observed near the mean structural eigen frequencies. Moreover, the uncertainty band near the interaction zone increases with the increment of random mean temperature. The distributions of the SPL near the first two eigen frequencies (812, 878 rad/s) at the random mean temperature of 350 K are shown in Figure 4. It is observed from Figure 4 the distributions are non-Gaussian in nature and corresponding mean and deterministic SPLs are significantly apart from each other.

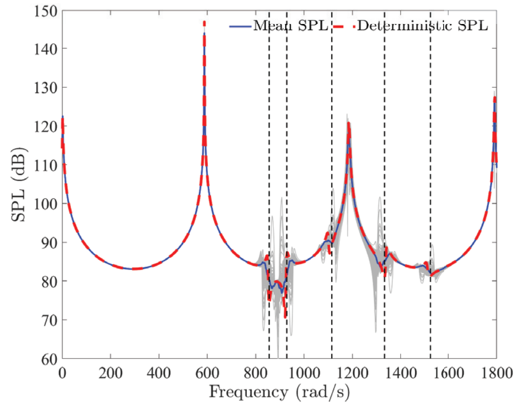

Figure 2: SPL at boundary at random mean temperature of 325 K and $\zeta = 0.5\%$

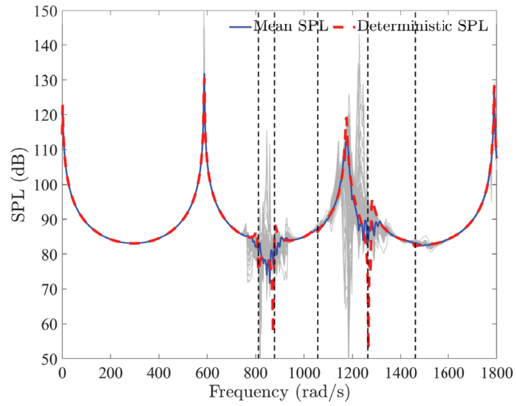

Figure 3: SPL at boundary at random mean temperature of 350 K and $\zeta = 0.5\%$

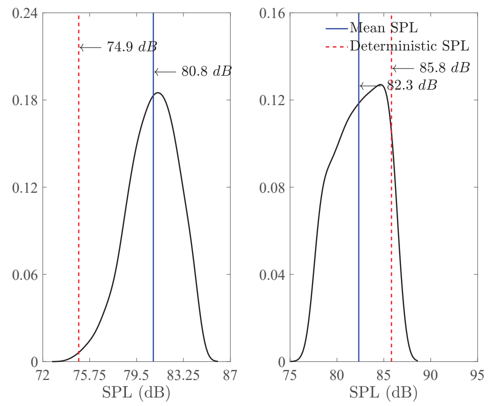

Figure 4: PDFs of SPL at 812 rad/s and 878 rad/s, respectively at random mean temperature of 350 K and $\zeta = 0.5\%$

Uncertainty of SPL for various damping ratio

The uncertainty of the SPL at random mean temperature of 350 K for damping ratio $\zeta = 2.0\%$ is plotted in Figure 5. It is observed from Figure 3 and 5, the dispersion of the SPL near the interaction zone is reduced due to application of the higher damping ratio. Moreover, Figure 6 represents that mean and deterministic SPLs remain close to each other with the application of the higher damping ratio. The uncertainty distribution of the SPL at the higher random temperature are also reduced with the application of the higher damping ratio.

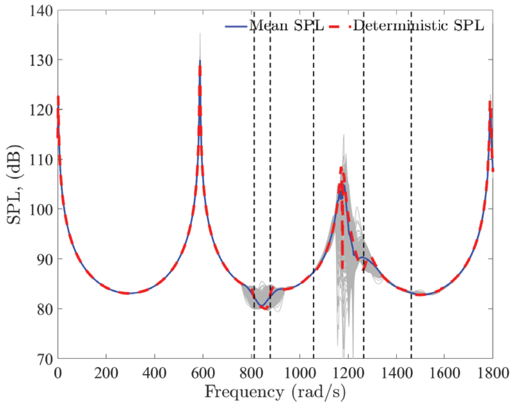

Figure 5: SPL at boundary at random mean temperature of 350 K and $\zeta = 2.0\%$

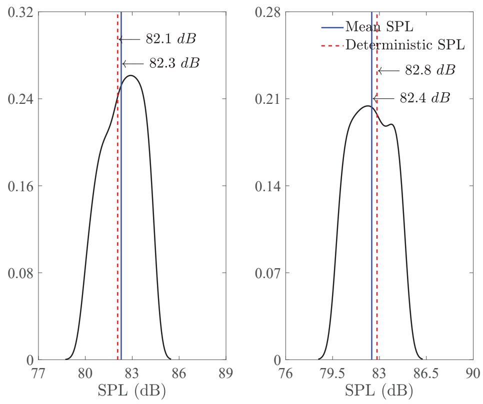

Figure 6: PDFs of SPL at 812 rad/s and 878 rad/s, respectively at random mean temperature of 350 K and $\zeta = 2.0\%$

Conclusion

A stochastic based gPC expansion method in accordance with coupled interior acoustic analysis is presented here to study the uncertainty of the SPL due to random temperature increment with the application of various damping ratio. The level of uncertainty increases with the increment random mean temperature near the interaction zone. At the lower damping ratio the deterministic SPL remains away from the maximum probability density. However, with the application of the higher damping ratio deterministic SPL lies near the maximum probability density even at the higher random temperature.

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