Curved Support Structures and Meshes with Spherical Vertex Stars

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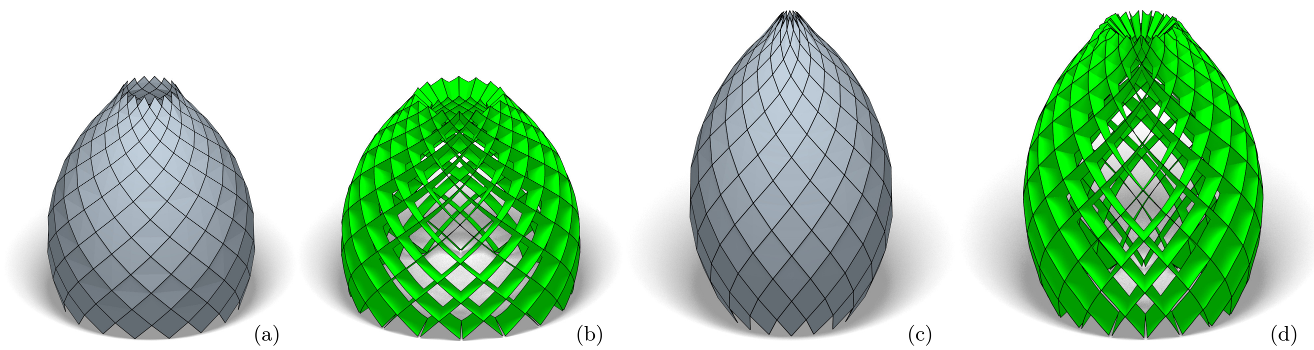

Figure 1: (a) A mesh with spherical vertex stars, computed from an unduloid surface. All vertex spheres are of the same radius. (b) The associated support structure consisting of developable strips with circular unrolling. (c) Deformed mesh, preserving sphere radii and edge lengths. (d) Associated support structure. Unrolled strips are identical to those shown in (b).

ABSTRACT

The computation and construction of curved beams along freeform skins pose many challenges. Controlling the curvature of design surfaces and beam networks, and using the elastic behavior of material to shape these grids, opens up new strategies for fabrication-aware design. We show how to use surfaces of constant mean curvature (CMC) to compute beam networks with beneficial properties, both aesthetically and from a fabrication perspective. To explore variations of such networks we introduce a new discretization of CMC surfaces as quadrilateral meshes with spherical vertex stars. The computed variations can be seen as a path in design space – exploring possible solutions in a neighborhood, or represent an actual erection sequence by elastic transformation.

CCS CONCEPTS

- **Mathematics of computing** → **Mathematical optimization**;
- **Computing methodologies** → **Shape analysis**;

KEYWORDS

CMC surfaces, support structures, elastic deformation

SIGGRAPH'18, August 2018, Vancouver, Canada

ACM ISBN 123-4567-24-567/08/06.
https://doi.org/10.475/123_4

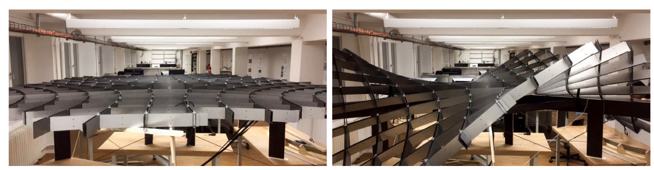

Figure 2: Elastic deformation of a grid into a beam network following asymptotic directions on a minimal surface.

ACM Reference Format:
Martin Kilian, Hui Wang, Eike Schling, Jonas Schikore, and Helmut Pottmann. 2018. Curved Support Structures and Meshes with Spherical Vertex Stars. In *Proceedings of ACM Siggraph conference (SIGGRAPH'18)*. ACM, New York, NY, USA, Article 4, 3 pages. https://doi.org/10.475/123_4

1 INTRODUCTION

Gridshells are highly efficient structures, because they carry loads through their curved shape with very little material. The main motivation for this work is research conducted at the Technical University of Munich, Fig. 2. A pavilion, realized as a gridshell, was built from steel lamella with straight unrolling. The initially planar lamella grillage can be deformed elastically, allowing an erection process without scaffolding. Geometrically, this is a family of grids with *planar vertex stars*. We generalize this idea to quadrilateral meshes with *spherical vertex stars*, allowing circular unrollings of strips.

2 APPROACH

We model curved support structures as networks of developable strips attached *orthogonally* to a design surface. Such a network is an idealized representation of the center planes of beams. To cover a design surface with beams of straight or circular unrolling, individual curves of the network need to follow curves of constant normal curvature. In addition to this geometric property one typically also has to satisfy aesthetic requirements related to the layout of curves like close to 90 degree intersection angles or as-square-as-possible faces of the network. On a generic design surface a network satisfying all those properties is hard to compute and may not even exist. Hence we propose a method that restricts the designer to a subset of the space of admissible shapes. Constant-mean-curvature surfaces serve as the starting shapes of our method as they can be covered by a network of curves with constant normal curvature, featuring close to right angle intersections and almost square faces. Such networks are close to meshes with *spherical vertex stars* and generalize the notion of meshes with planar vertex stars appearing in the study of asymptotic nets [1]. More precisely we discretize CMC surfaces as quad meshes such that: (i) a vertex and its four neighbors lie on a common sphere, (ii) all such sphere radii are equal and (iii) edge polylines intersect at right angles. Preserving the spherical vertex star property and the constant radius during deformation allows us to explore the shape of nearby support structures (which no longer represent CMC surfaces):

• Starting from a CMC surface, we compute an isothermic mesh M on top of it. For CMC surfaces such a mesh M always exists and is defined as a *principal mesh* with *as-square-as-possible* faces. This yields a highly aesthetic cell layout, plus, by following the diagonals of M, we can extract a support structure S along curves of constant normal curvature, allowing for beams with straight or circular unrolling.

• By construction, the quad-dominant mesh S is close to a mesh with spherical vertex stars. Mathematically, the sphere we are talking about is the so called *Meusnier sphere*. We optimize S to satisfy the spherical vertex star condition in the least squares sense. This optimization will also ensure that all vertex spheres have the same radius. The value of this radius approximates $1/H$, with H equal to the mean curvature of the design surface.

• In order to study elastic shape variations of S we preserve (i) the spherical vertex star condition, (ii) the radii of spheres and (iii) edge lengths of S during handle based shape editing. All optimization is done using the method presented in [2].

3 RESULTS

We used an unduloid surface to initialize the network shown in Fig. 1(a). The deformation was achieved by moving a ring of vertices at approximately half the height of the structure slowly inwards.

Our method can also be used to identify an elastic erection sequence similar to the process illustrated in Fig. 2. An

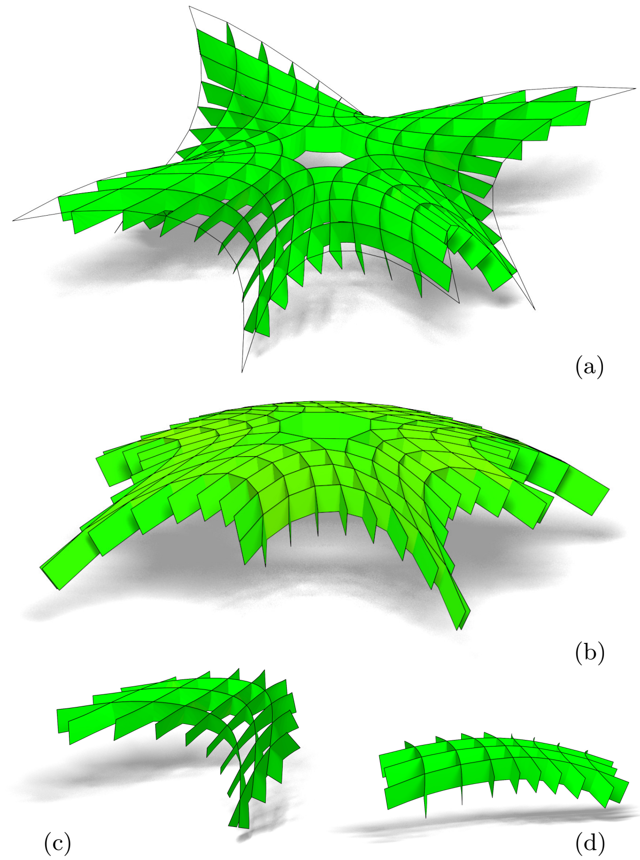

Figure 3: (a) Mesh with spherical vertex stars and associated support structure. (b) Global 'unrolling' onto a common sphere. (c) Lower left part of the structure and (c) its 'unrolling'. The common sphere can be thought of as scaffolding during construction on which the beams are assembled before the elastic erection process starts.

example is shown in Fig. 3. Note that even for a globally 'unrollable' structure it may be beneficial to consider smaller parts for several reason: the global structure may pass through singular states which are not physically viable; smaller substructures allow for easy assembly.

ACKNOWLEDGMENTS

This work was supported by SFB-Transregio program *Geometry and Discretization* (FWF grant no. I 2978) and by the project "Geometry and Computational Design for Architecture and Fabrication" at Vienna University of Technology.

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