1 Multilevel Adaptive^2 Sparse Grid Stochastic Collocation

Parametrized models are often anisotropic in nature. Methodologies based on adaptivity can leverage this anisotropy in the sense that they can implicitly detect the spaces which have significant impact on the solution. The aforementioned phenomenon is valid in Uncertainty Quantification as well, where some stochastic parameters influence the solution more than others. Thus, it is recommended to also apply adaptive methods in this field. Here, problem classes involve differential models having stochastic parameters where the anisotropy might for example be due to a parametrization using a Karhuenen-Loeve expansion or varying sensitivities of the uncertain parameters. A common method of choice to exploit anisotropy is dimension-adaptive sparse grid stochastic collocation first introduced by [Gerstner, Griebel, 2003].

We build on this method by introducing a multilevel approach where we add an additional adaptive layer. Thus, we define two different spaces in which we perform adaptivity: on the upper layer, we construct an abstract dimensional-adaptive sparse grid operation over the parametric space. Here, problem-dependent dimensions might be the discretization in time and/or space, the number of uncertain parameters and the tolerance of the adaptive sparse grid stochastic collocation approach. In each subspace we compute a dimension-adaptive sparse grid stochastic collocation solution over the stochastic space for a prescribed disrectization level and tolerance defined by the upper layer. Finally the solutions are combined using the combination technique on the upper level similar to a telescopic sum.

We apply the proposed approach to several problems –parametric partial differential equations, ordinary differential equations with random parameters and random ordinary differential equations. This deterministic method offers solutions with the same accuracy as a single high resolution adaptive sparse grid, however with much lower computational cost by leveraging nestedness and anisotropy of the underlying problem. Due to its adaptive nature it also competes with classic multilevel or multiindex approaches.