Optical Communication Using the Nonlinear Fourier Transform

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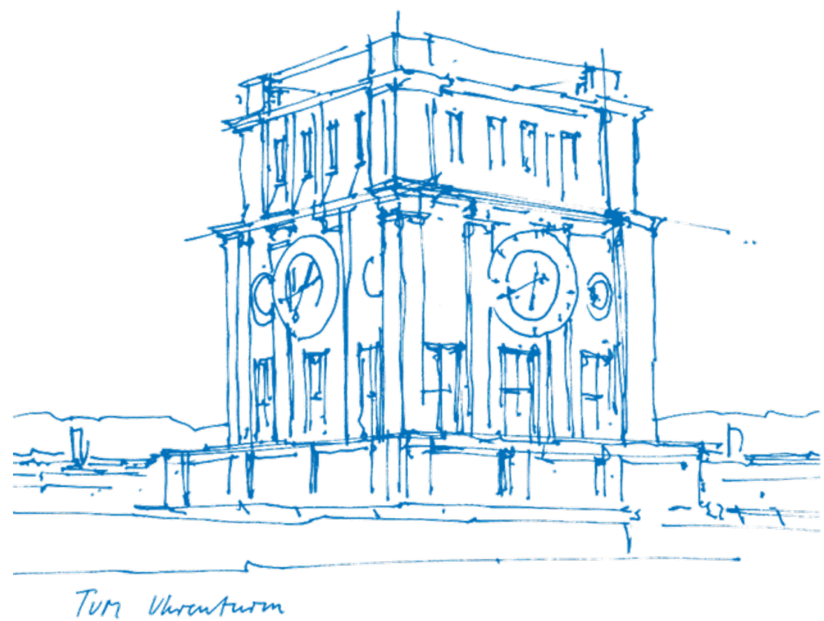

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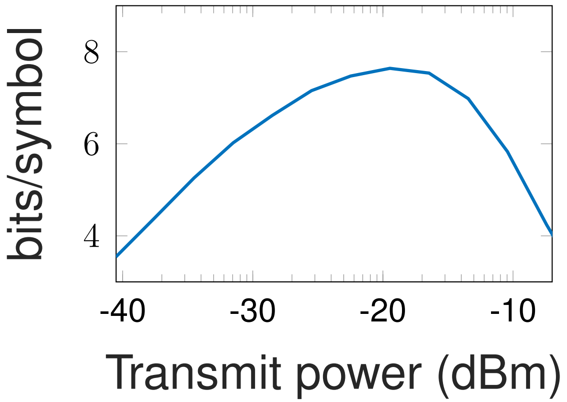
Motivation

- The nonlinearity of the optical fiber channel imposes a capacity peak on linear transmission systems



5 WDM channels @ $20\ \mathrm{GHz}$
Guardband: $5\ \mathrm{GHz}$
Distance: $2000\ \mathrm{km}$
RRC pulses, multi-ring modulation, 64 rings, 128 phases

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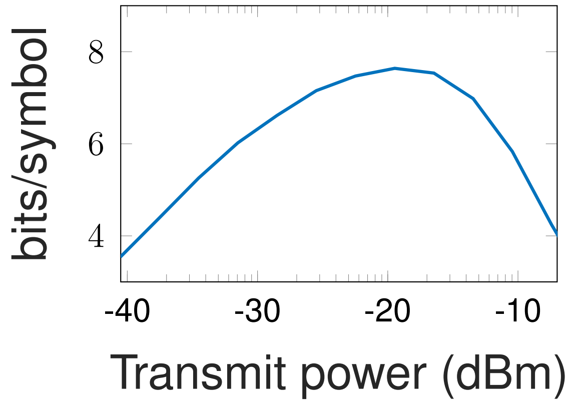

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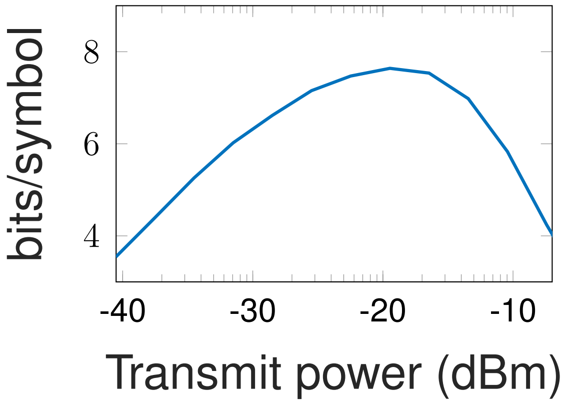

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- The Nonlinear Fourier Transform (NFT) provides a domain in which the noise-free channel is multiplicative
- Challenges: modeling noise, spectral efficiency...

Outline

The optical channel and the NFT

Information transmission using the NFT

Effect of noise

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The Nonlinear Schrödinger Equation (NLSE)

$$\frac{\partial Q(Z,T)}{\partial Z} = -j\frac{\beta_2}{2}\frac{\partial^2 Q(Z,T)}{\partial T^2} + j\gamma \left|Q(Z,T)\right|^2 Q(Z,T) + N(Z,T)$$

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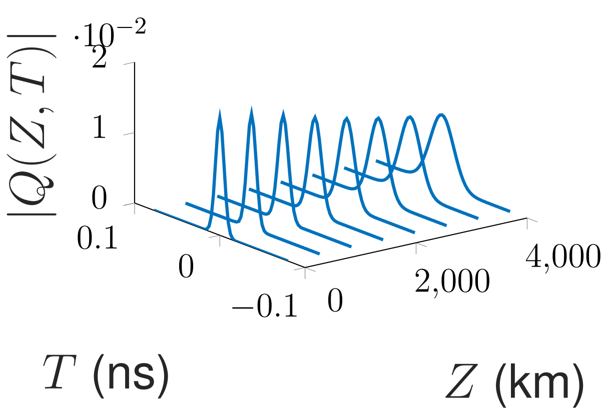
Dispersion

- Linear term
- Causes temporal broadening



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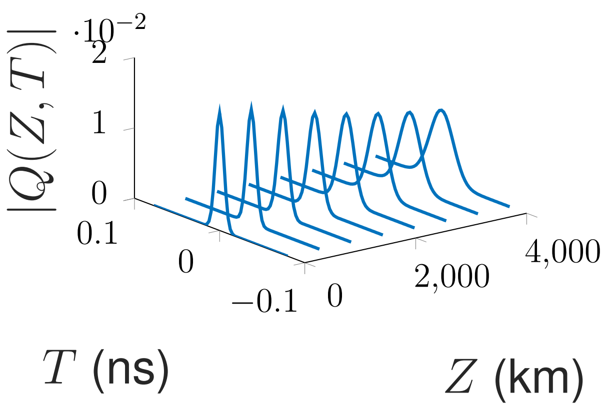
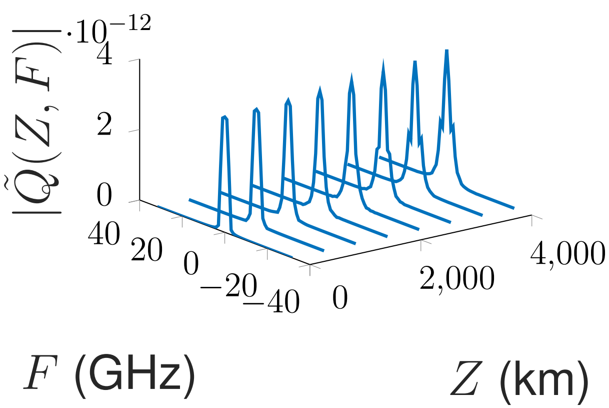
Nonlinearity

- Causes frequency mixing (spectral broadening, SPM, XPM, FWM)





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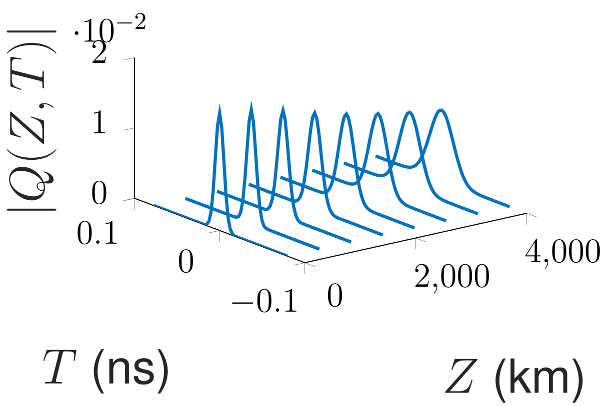
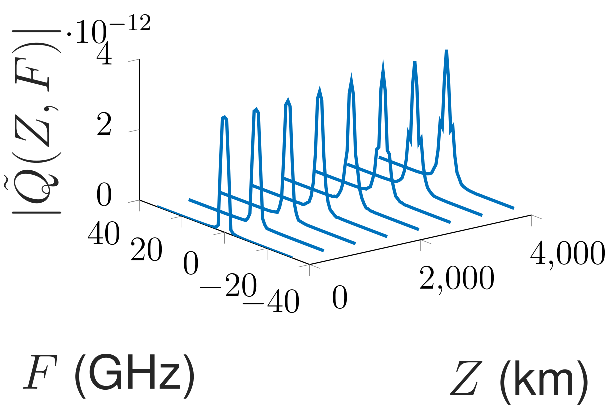
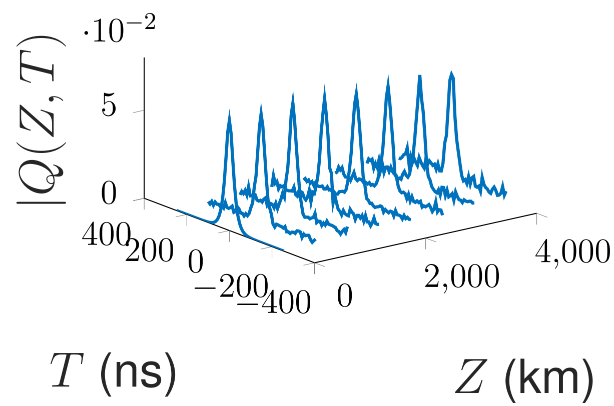
Noise

- Distributed along the fiber
- Mixes nonlinearly with signal!







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Normalized Nonlinear Schrödinger Equation (NLSE)

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$$T = T_0 \cdot t$$

$$Z = 2\frac{{T_0}^2}{|\beta_2|} \cdot z$$

$$Q(Z,T) = \frac{1}{T_0}\sqrt{\frac{|\beta_2|}{\gamma}} \cdot q(z,t)$$

$$\mathbb{E}\left[N(Z,T)N^*(Z',T')\right] = \frac{\beta_2^2}{2\gamma {T_0}^4} \cdot \mathbb{E}\left[n(z,t)n^*(z',t')\right]$$

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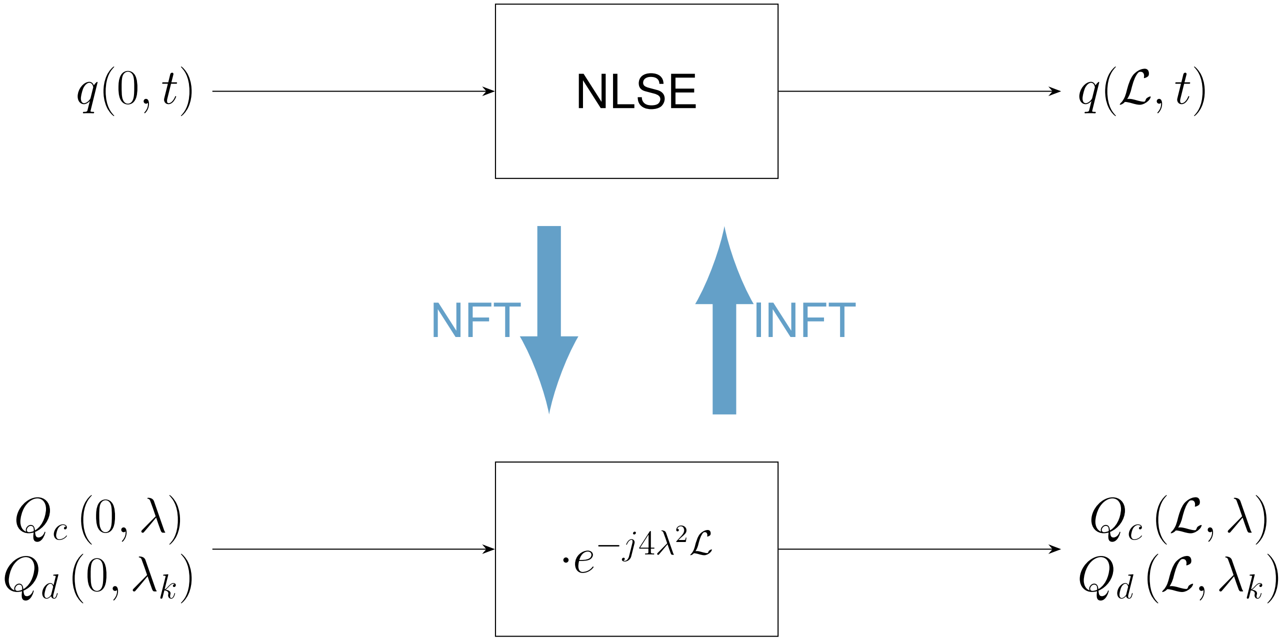
T_0 is a **free parameter**. Can be used to jointly set *power*, *duration* and *bandwidth*.

The Nonlinear Fourier Transform (NFT)

- Motivation: find a domain in which the noise-free NLSE channel is multiplicative (similar to FT in LTIs):



The Nonlinear Fourier Transform (NFT)

- Lax pair: two operators L and M

$$L = j \begin{pmatrix} \frac{\partial}{\partial t} & q(z,t) \\ -q^*(z,t) & \frac{\partial}{\partial t} \end{pmatrix}, \quad M = \begin{pmatrix} 2j\lambda^2 - j|q(z,t)|^2 & -2\lambda q(z,t) - jq_t(z,t) \\ 2\lambda q^*(z,t) - jq_t^*(z,t) & -2j\lambda^2 + j|q(z,t)|^2 \end{pmatrix}$$

such that the condition:

$$L_z = ML - LM$$

implies the NLSE.

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  such that the condition:

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  implies the NLSE.
- Main idea: the eigenvalues λ of L are invariant under propagation along z

The Nonlinear Fourier Transform (NFT)

- Step 1: solve the Zakharov-Shabat system:

$$Lv(t,\lambda) = \lambda v(t,\lambda); \quad v(t,\lambda) \underset{t \to -\infty}{\longrightarrow} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-j\lambda t}$$

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  - **Continuous spectrum:** $Q_c(\lambda) = \frac{b(\lambda)}{a(\lambda)}$, $\lambda \in \mathbb{R}$
  - **Discrete spectrum:** $Q_d(\lambda_k) = \frac{b(\lambda_k)}{a_\lambda(\lambda_k)}$, $\lambda_k \in \mathbb{C}^+, a(\lambda_k) = 0$

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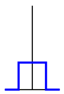
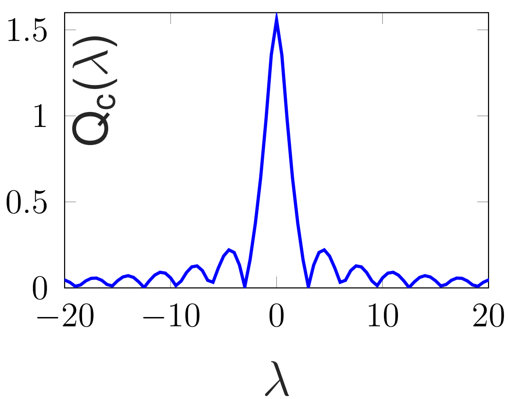
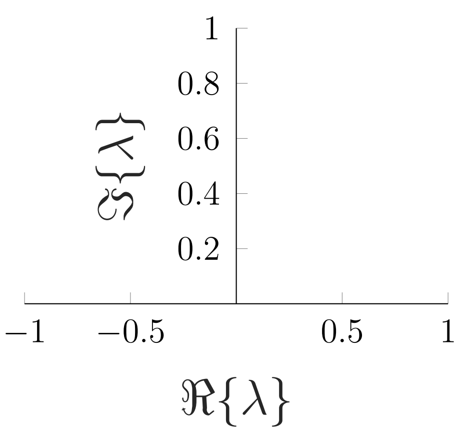
- Equivalent of Parseval's identity:

$$\int_{-\infty}^{\infty} |q(t)|^2 \, \mathrm{d}t = \frac{1}{\pi} \int_{-\infty}^{\infty} \log\left(1 + |Q_c(\lambda)|^2\right) \, \mathrm{d}\lambda + 4 \sum_{k=1}^{K} \Im\{\lambda_k\}$$

Example: Rectangular pulse with varying amplitude

- $A = 1$: 





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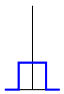
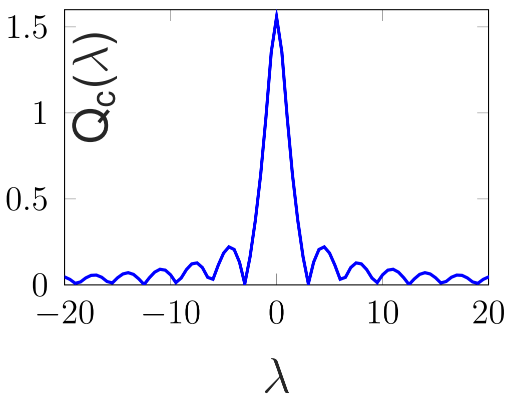
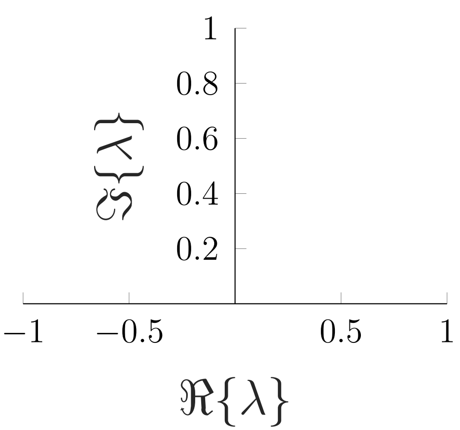
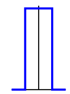
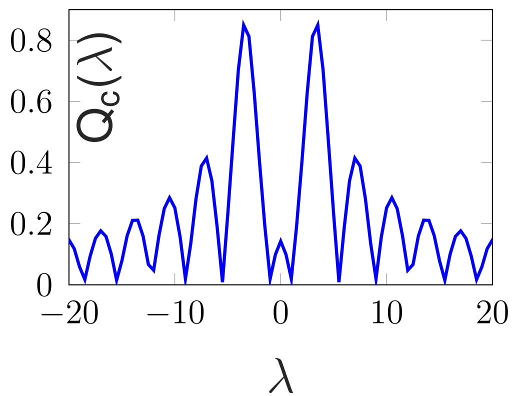
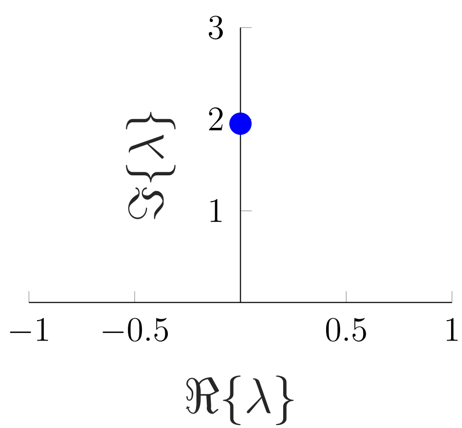
- $A = 1$: 





- $A = 3$: 





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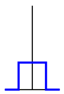
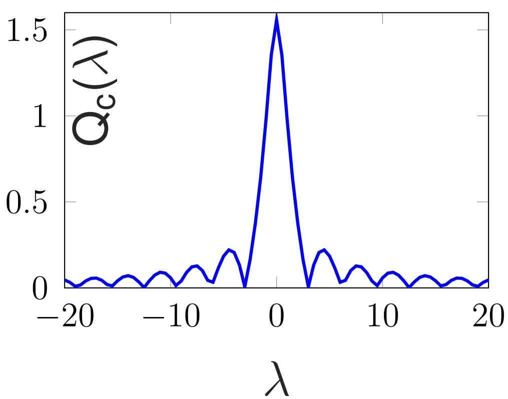
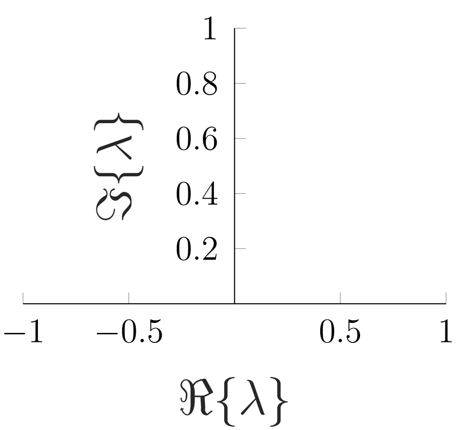
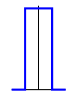
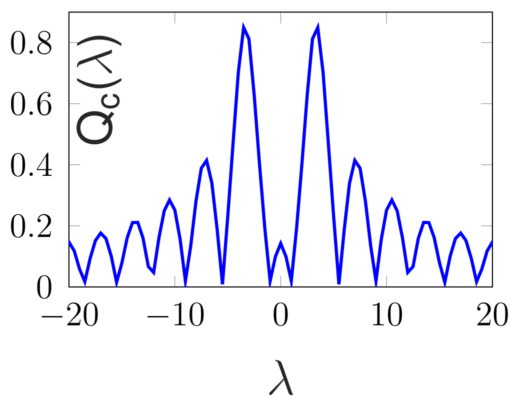
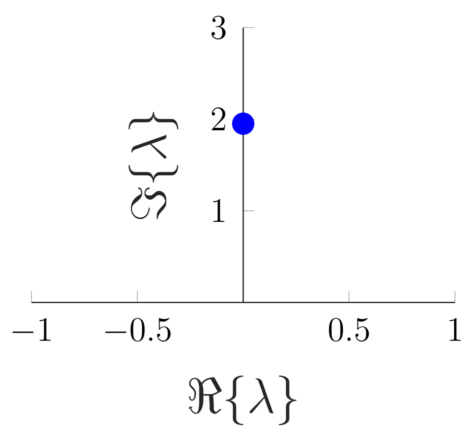
- $A = 1$: 

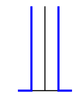


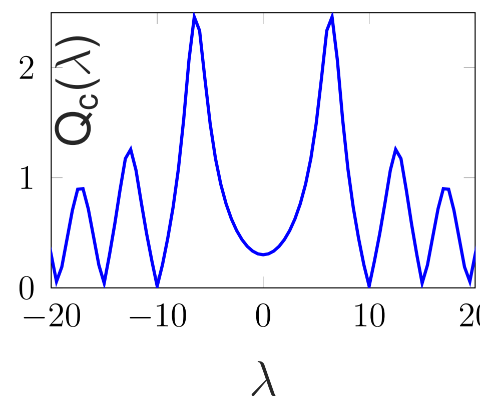


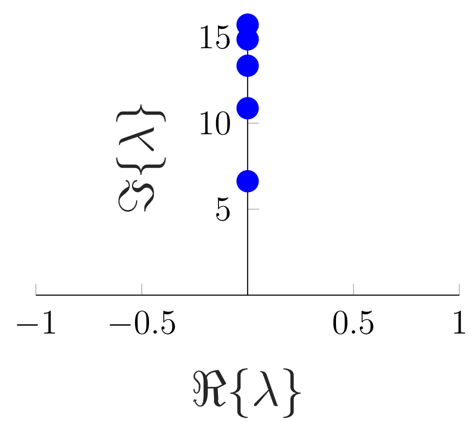
- $A = 3$: 





- $A = 16$: 





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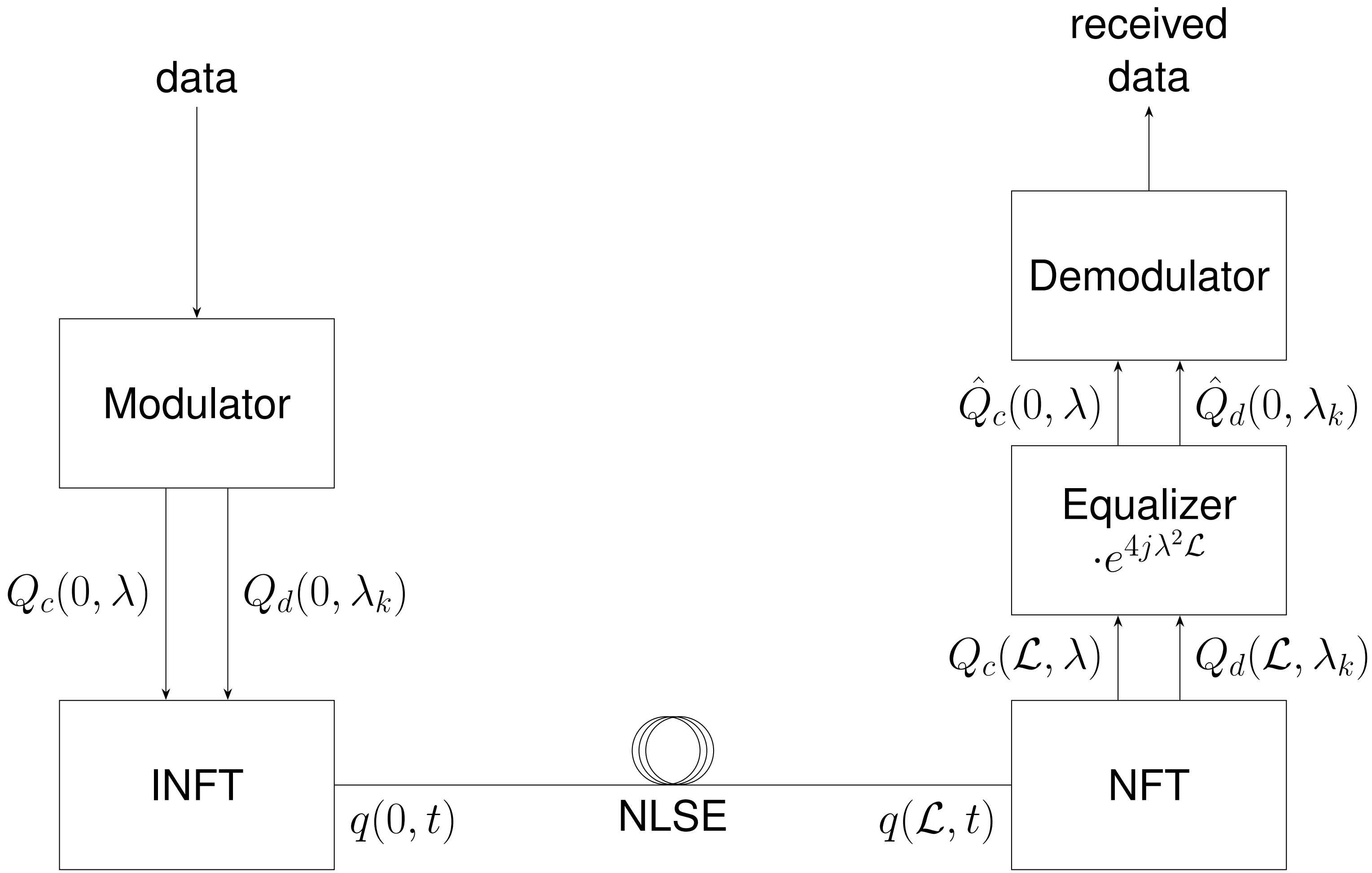

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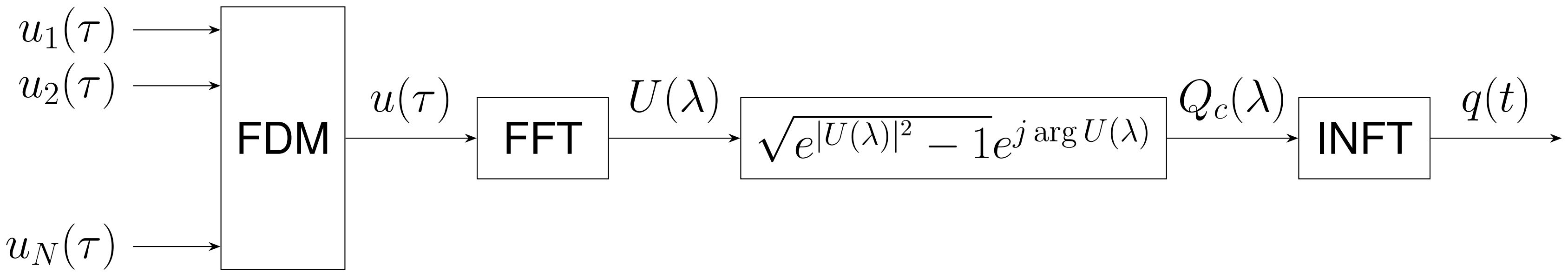
Modulation of the continuous spectrum



- From Parseval, the signal

$$U(\lambda) = \log\left(1 + |Q_c(\lambda)|^2\right) e^{j \arg Q_c(\lambda)}$$

has energy $E/2$, where E is the energy of $q(z,t)$

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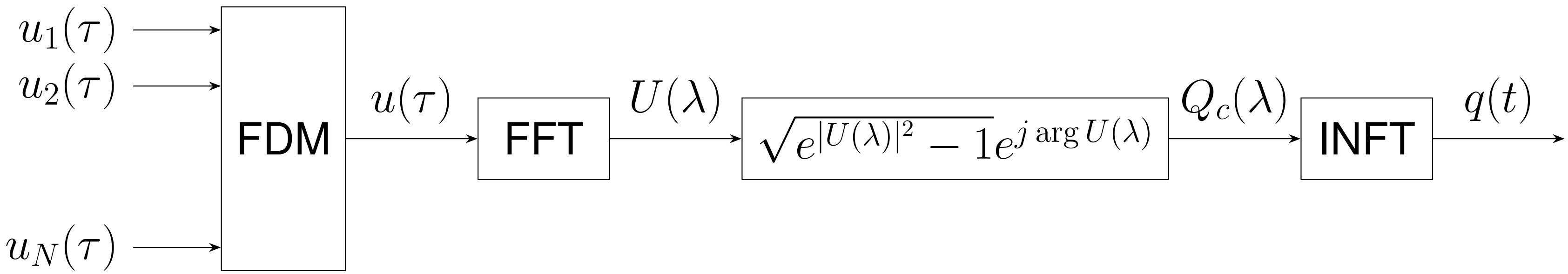

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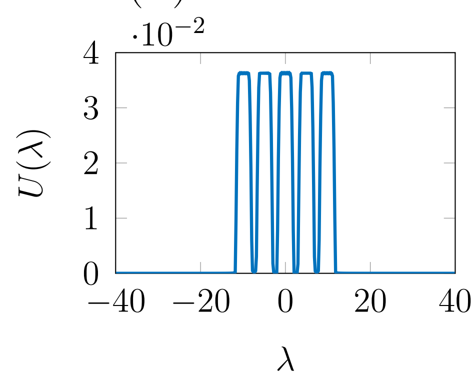
- User channels are multiplexed in $U(\lambda)$



Continuous spectrum: simulation parameters

- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$

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- 5 FDM channels, Root Raised Cosine pulses with roll-off $\beta = 0.25$
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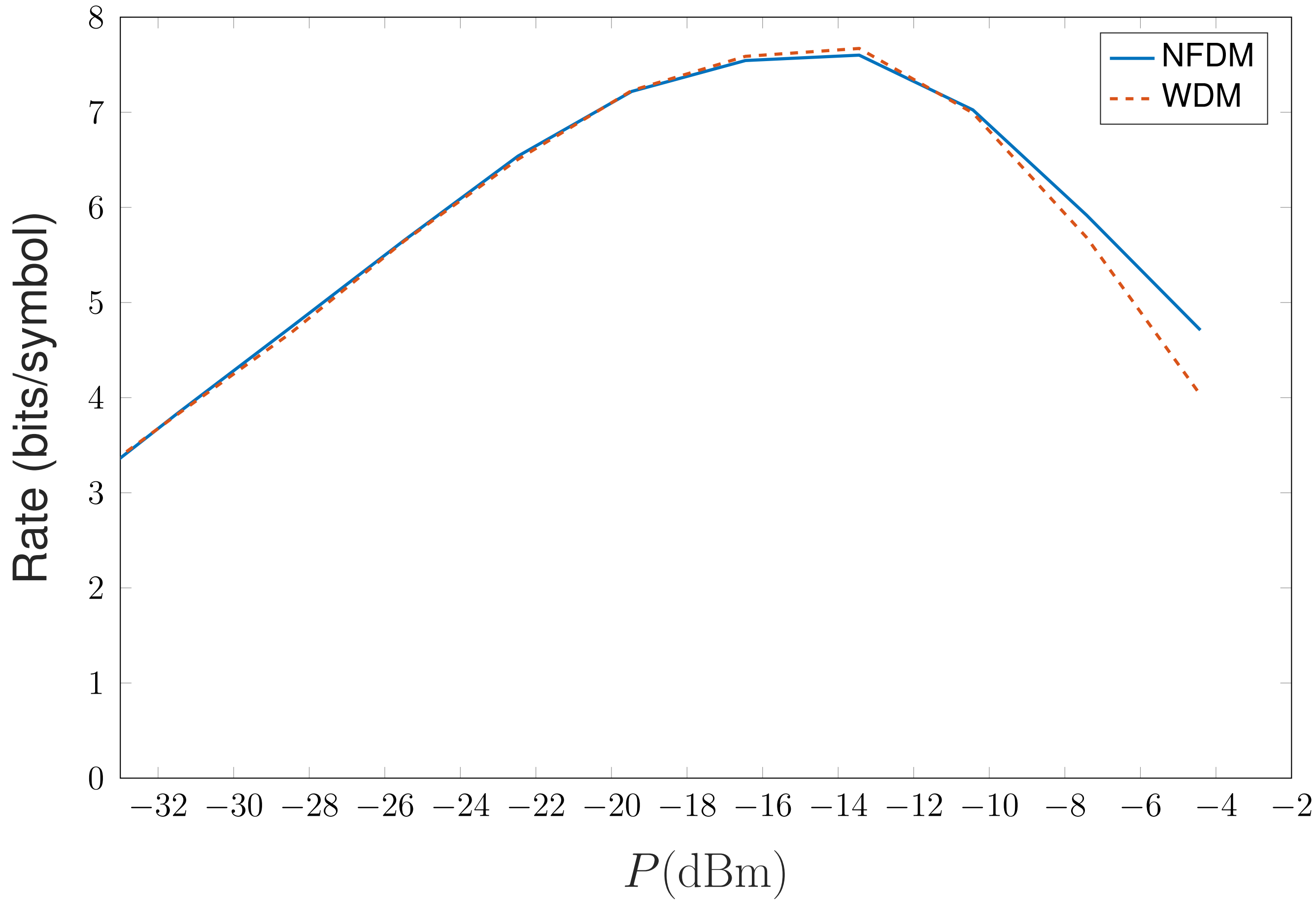
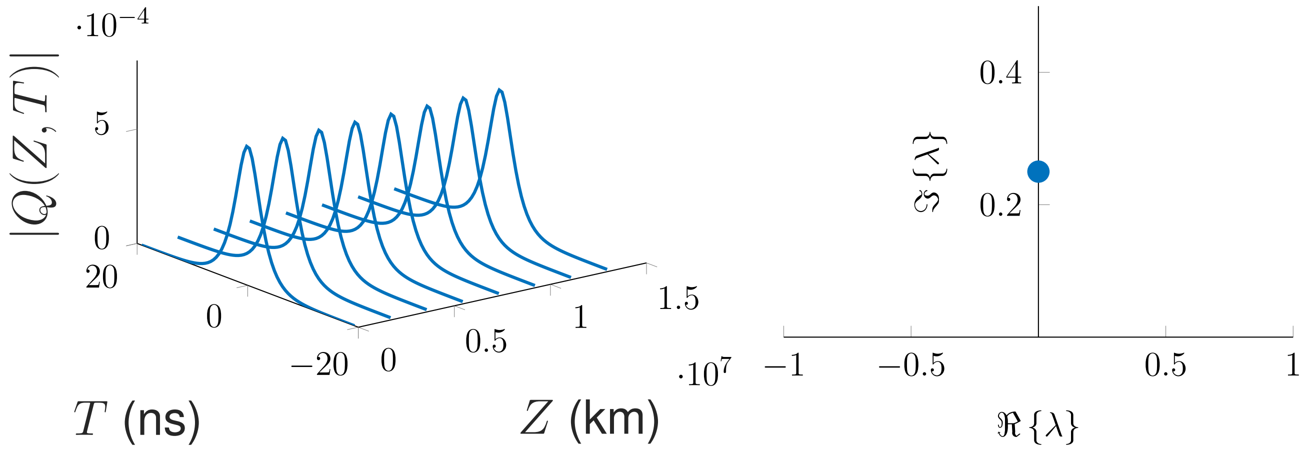
Parameter	Symbol	Value
Dispersion coefficient	β_2	$-21.667\ \mathrm{ps}^2/\mathrm{km}$
Nonlinearity parameter	γ	$1.2578\ \mathrm{W}^{-1}\mathrm{km}^{-1}$
Fiber length	$\mathcal{L}$	$1000\ \mathrm{km}$
Channel bandwidth	B	$10\ \mathrm{GHz}$
Guard band	B_{guard}	$2.5\ \mathrm{GHz}$
Noise spectral density	N_{ASE}	$6.4893 \cdot 10^{-19}\ \mathrm{W} \cdot \mathrm{s}$

Continuous spectrum: simulation results



Modulation of the discrete spectrum: solitons



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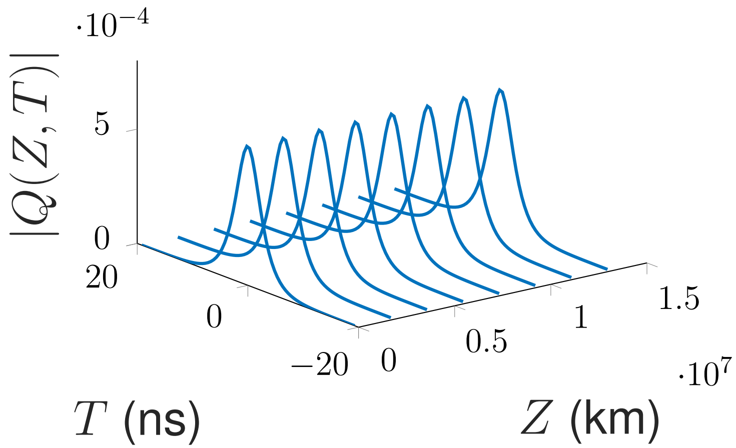

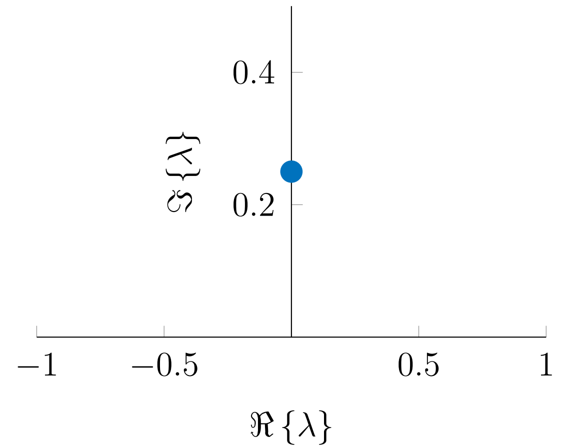

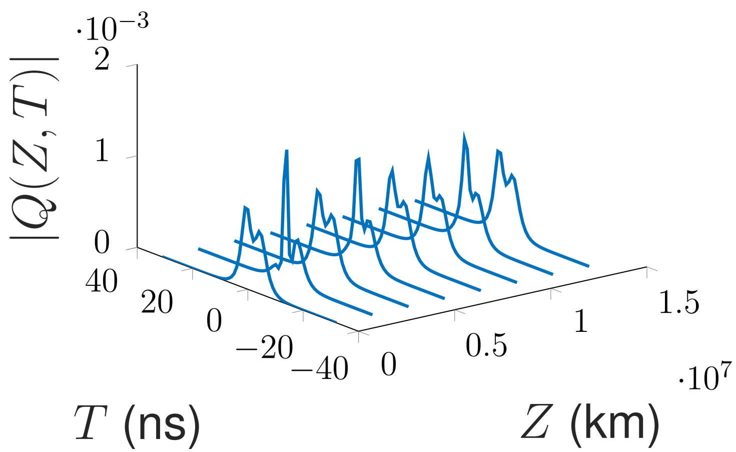

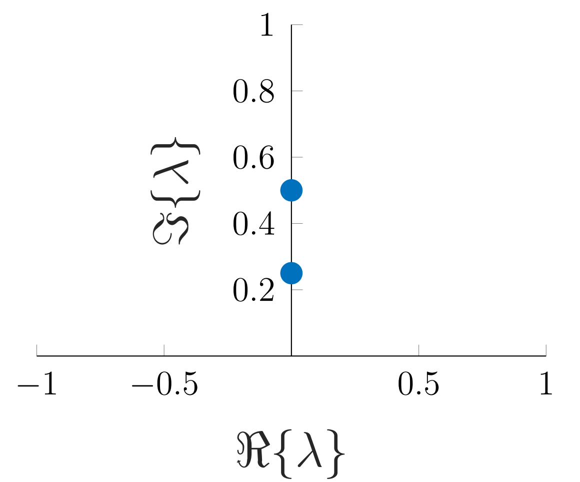

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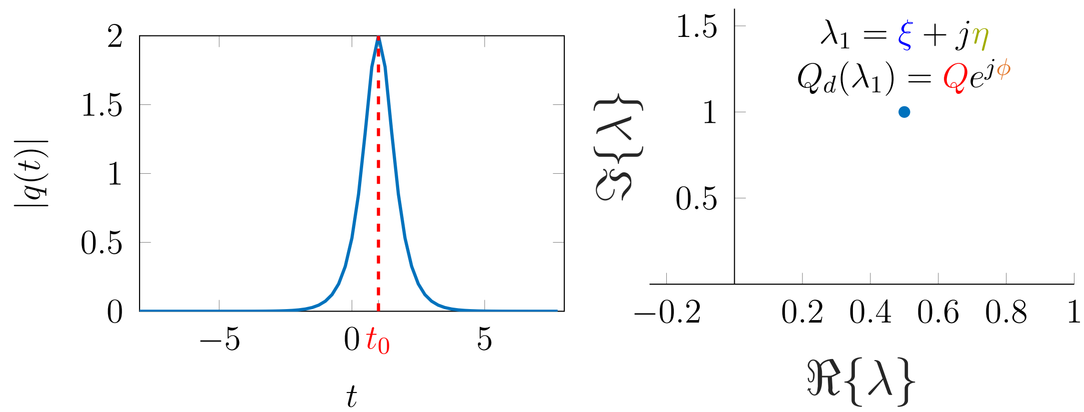
Parameters of a 1-soliton



$$q(z,t) = -je^{-j\phi}e^{-4j\left(\xi^2-\eta^2\right)z}e^{-2j\xi t}2\eta\,\mathrm{sech}\left(2\eta t + 8\xi\eta z - \ln\frac{Q}{2\eta}\right)$$

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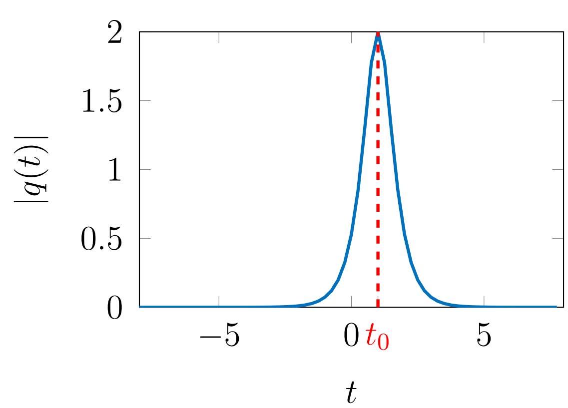

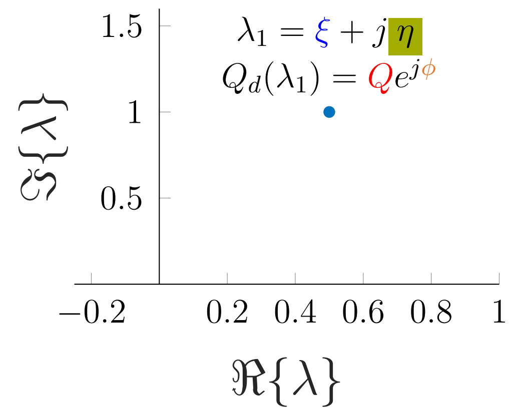

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- Bandwidth: $B = 1.0726\eta$

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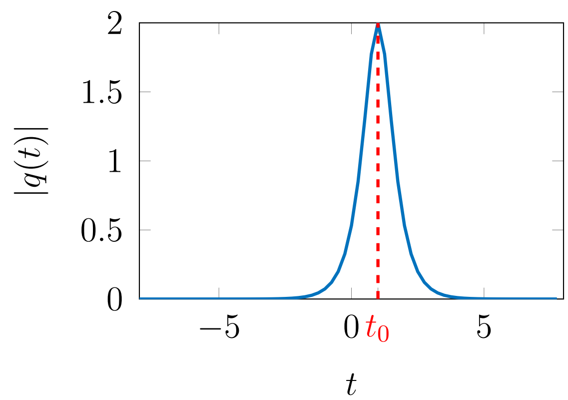

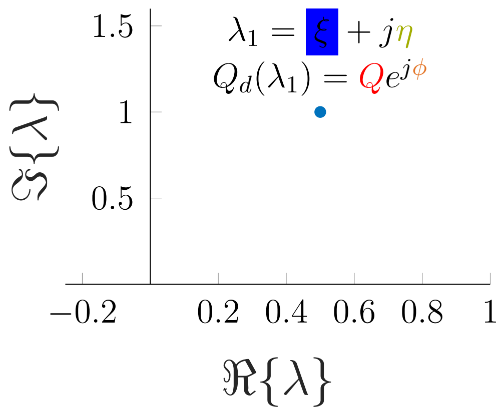

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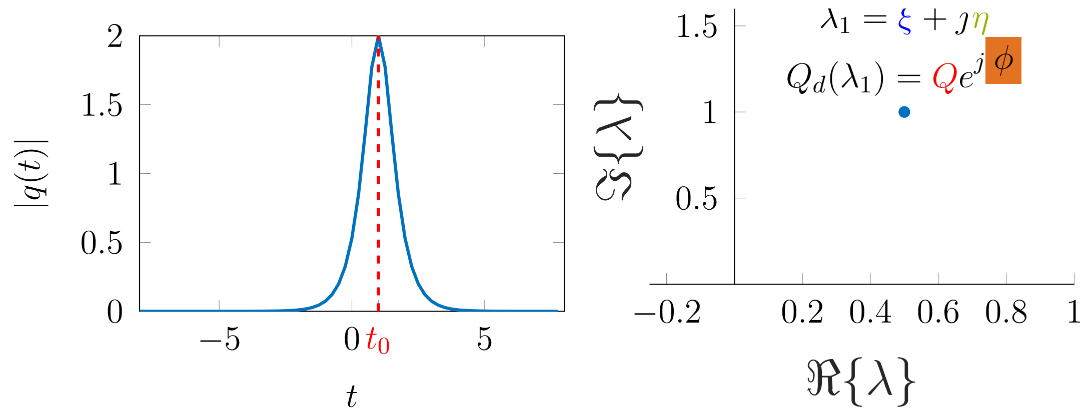

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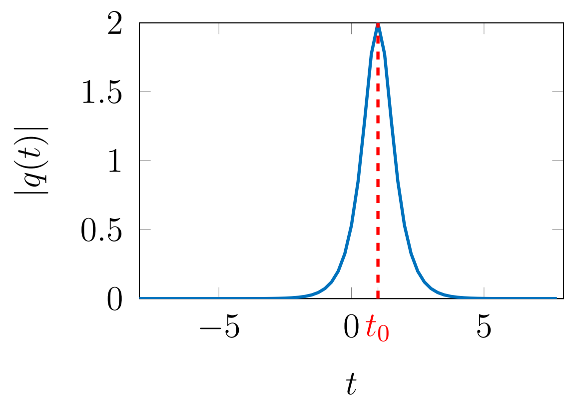

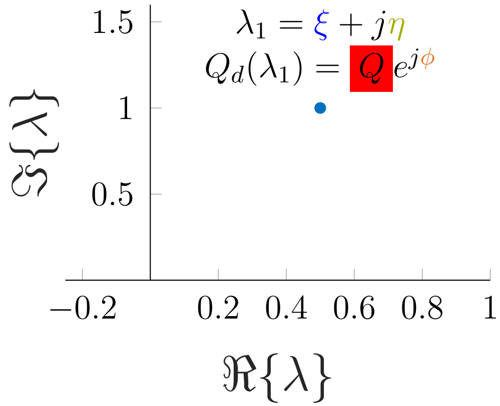

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Perturbation analysis of a 1-soliton[1]

$$\frac{\partial}{\partial z}q(z,t) = j\frac{\partial^2}{\partial t^2}q(z,t) + 2j\left|q(z,t)\right|^2 q(z,t) + \epsilon n(z,t)$$

where $\epsilon \ll 1$.

[1] J. Yang, "Soliton Perturbation Theories and Applications," *Nonlinear Waves in Integrable and Nonintegrable Systems*, ch. 4, pp. 119–162, 2010

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Multi-scale perturbation analysis:

$$q(z,t) = q_0(z,t) + \epsilon q_1(z,t) + \epsilon^2 q_2(z,t) + \cdots$$

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where the four parameters depend on the **slow distance** $Z = \epsilon z$:

$$\eta = \eta(Z) \quad \xi = \xi(Z) \quad \phi = \phi(Z) \quad t_0 = t_0(Z)$$

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Perturbation analysis of a 1-soliton

Substituting $q_0(z,t)$ into the $\mathcal{O}(\epsilon)$ equation yields:

$$\frac{\mathrm{d}\eta}{\mathrm{d}Z} \sim \mathcal{N}_{\mathbb{R}}\left(0, \eta/2\right) \qquad \frac{\mathrm{d}\xi}{\mathrm{d}Z} \sim \mathcal{N}_{\mathbb{R}}\left(0, \eta/6\right)$$

$$\frac{\mathrm{d}t_0}{\mathrm{d}Z} \sim \mathcal{N}_{\mathbb{R}}\left(0, \frac{\pi^2}{96\eta^3}\right) \qquad \frac{\mathrm{d}\phi}{\mathrm{d}Z} \sim \mathcal{N}_{\mathbb{R}}\left(0, \frac{1}{72\eta}\left(12+\pi^2\right) + \frac{\pi^2\xi^2}{24\eta^3}\right)$$

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Assuming η and ξ do not change much along propagation:

$$\eta(\mathcal{L}) \sim \mathcal{N}_{\mathbb{R}}\left(\eta(0), \frac{\eta(0)}{2} N_{\mathrm{ASE}}\mathcal{L}\right)$$

$$\xi(\mathcal{L}) \sim \mathcal{N}_{\mathbb{R}}\left(\xi(0), \frac{\eta(0)}{6} N_{\mathrm{ASE}}\mathcal{L}\right)$$

$$t_0(\mathcal{L}) \sim \mathcal{N}_{\mathbb{R}}\left(t_0(0), \frac{\pi^2}{96\eta(0)^3} N_{\mathrm{ASE}}\mathcal{L}\right)$$

$$\phi(\mathcal{L}) \sim \mathcal{N}_{\mathbb{R}}\left(\phi(0), \left[\frac{1}{72\eta(0)}\left(12+\pi^2\right) + \frac{\pi^2\xi(0)^2}{24\eta(0)^3}\right] N_{\mathrm{ASE}}\mathcal{L}\right)$$

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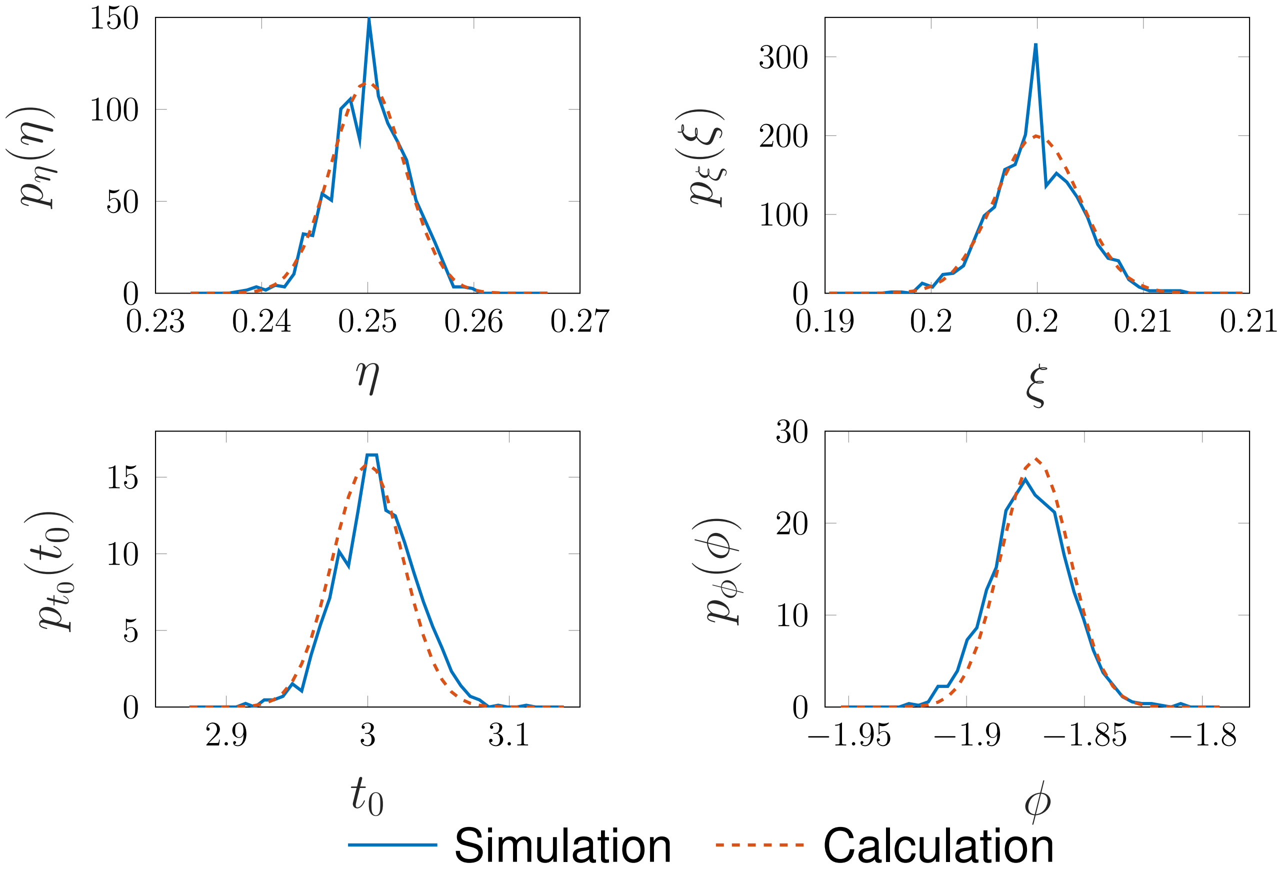
Perturbation analysis of a 1-soliton ($z = 0.9578$)



Perturbation of eigenvalues of a multi-soliton

- Joint work with Vahid Aref (Nokia Bell Labs Stuttgart)

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Perturbation of eigenvalues of a multi-soliton

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- Take the DFT of the Zakharov-Shabat system (c_n is the DFT of the pulse):

$$\left(\begin{array}{c|c} -\frac{2\pi}{M}\mathrm{diag}\left(-N,\ldots,N\right) & -j\begin{pmatrix} c_0 & \cdots & c_{-N} & \cdots & 0 \\ \vdots & c_0 & \ddots & \ddots & \vdots \\ c_N & \ddots & c_0 & \ddots & c_{-N} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & c_N & \cdots & c_0 \end{pmatrix} \\ \hline -j\begin{pmatrix} c_0^* & \cdots & c_N^* & \cdots & 0 \\ \vdots & c_0 & \ddots & \ddots & \vdots \\ c_{-N}^* & \ddots & c_0^* & \ddots & c_N^* \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & c_{-N}^* & \cdots & c_0^* \end{pmatrix} & \frac{2\pi}{M}\mathrm{diag}\left(-N,\ldots,N\right) \end{array}\right) \mathbf{a}_k = \lambda_k \mathbf{a}_k$$

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- Use **matrix perturbation theory** [1] to obtain the statistics of λ_k

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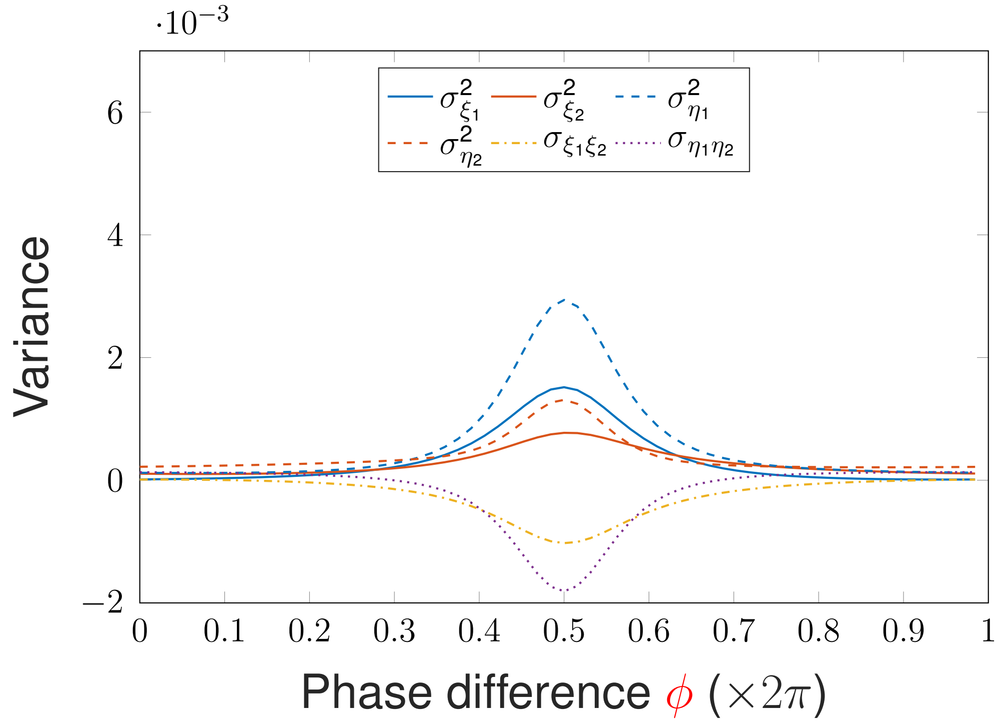
Perturbation of a 2-soliton

- 2-soliton with parameters ($\lambda_i = \xi_i + j\eta_i$)

$$\lambda_1 = 0.3j \quad Q_d(\lambda_1) = 1.8$$
$$\lambda_2 = 0.6j \quad Q_d(\lambda_2) = 3.6e^{j\phi}$$



Numerical demonstration of information transmission

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System	$\{\lambda_k\}$	Modulation of $\{q_{k\ell}\}$
1-soliton	$\lambda_1 = 2.5j$	q_{10}: 32 rings, 128 phases
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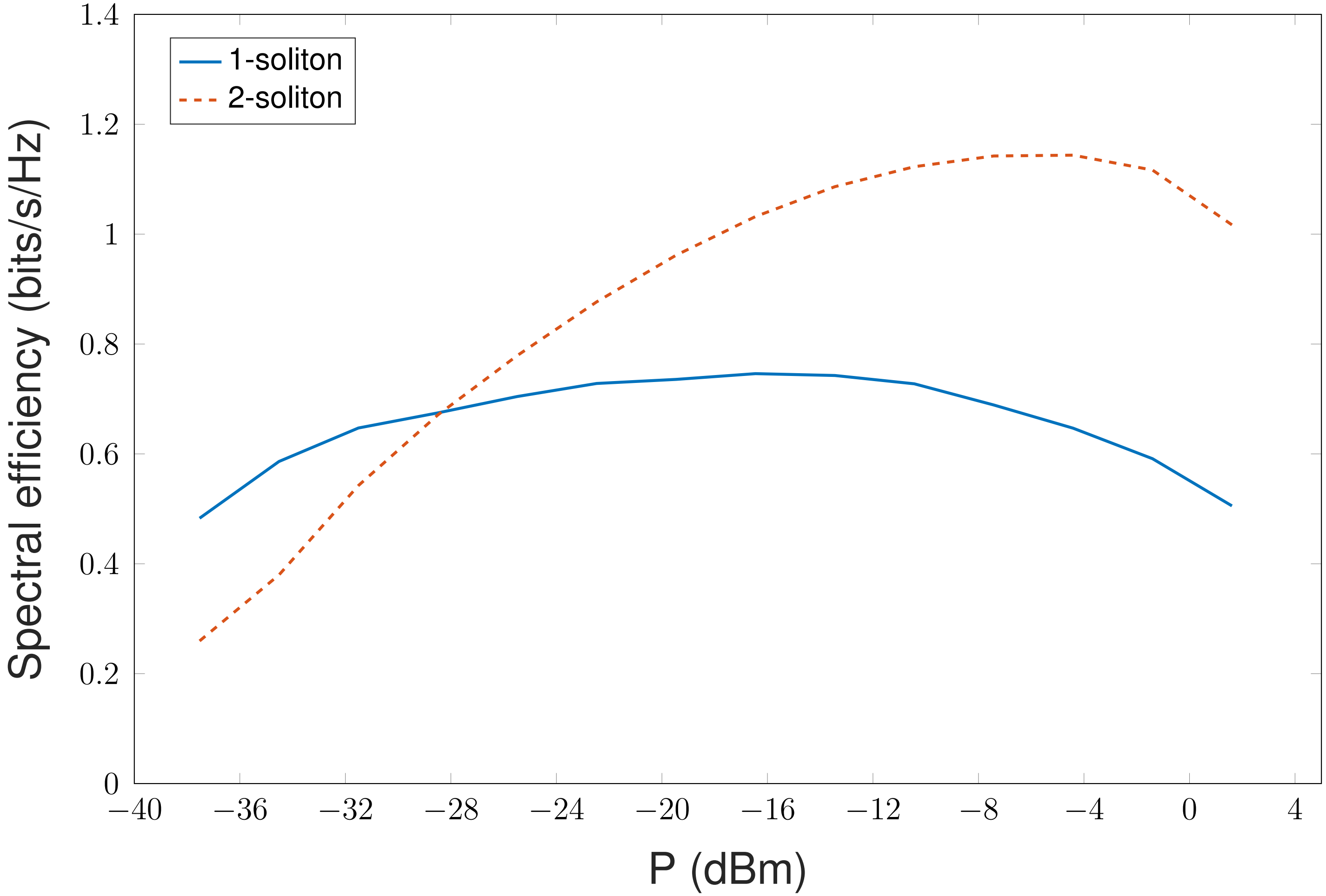
Parameter	**Symbol**	**Value**
Dispersion coefficient	β_2	$-21.667\ \mathrm{ps}^2/\mathrm{km}$
Nonlinear coefficient	γ	$1.2578\ \mathrm{W}^{-1}\mathrm{km}^{-1}$
Fiber length	z	$4000\ \mathrm{km}$
Noise spectral density	N_{ase}	$6.4893 \cdot 10^{-24}\mathrm{Ws/m}$

Numerical demonstration of information transmission



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- Challenges: spectral efficiency, effect of noise on norming constants