Chapter 2 Principles of Geometric Modeling

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Abstract The three-dimensional geometry of a building is a vital prerequisite for Building Information Modeling. This chapter examines the principles involved in representing geometry with a computer. It details explicit and implicit approaches to describing volumetric models as well as the basic principles of parametric modeling for creating flexible, adaptable models. The chapter concludes with an examination of freeform curves and surfaces and their underlying mathematical description.

2.1 Geometric modeling in the context of BIM

A Building Information Model contains all the relevant information needed for the planning, construction and operation of a building. The three-dimensional description of the geometry of a building is one of the most important aspects without which many BIM applications would not be possible. The availability of a model in three dimensions offers significant advantages over conventionally drawn plans:

• The planning and construction of the building can be undertaken using a 3D model rather than separate plans and sections. Drawings are then generated from the 3D model, ensuring that the separate drawings always correspond and remain consistent with one another. This almost entirely eradicates a common source of errors, especially when alterations are made to the plans. But a three-dimensional geometric model on its own is not sufficient for generating plans that conform

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with current standards. Further semantic information also needs to be provided, for example denoting the construction type or material, as building plans are commonly represented in a symbolic or simplified form, which cannot be generated from the 3D geometry alone.

- With a 3D model, collision analyses can be conducted to determine whether parts
 of a model or building elements within a model overlap. In most cases this indicates a planning error or oversight. The detection of such collisions is especially important for coordinating the work of different trades, for example when
 planning wall openings and penetrations for plumbing, ducts or other technical
 installations (see Chap. 19).
- A 3D model facilitates easy quantity take-off as quantities can be calculated directly from the volume and surface area of the model elements. Further special rules are typically still required to conform to standards, e.g. simplified quantity approximations (see Chap. 24).
- The availability of the building geometry in 3D is essential for associated calculation and simulation methods (see Chaps. 20 and 21). The necessary mechanical or physical model can often be generated directly from the geometric model, obviating the need to laboriously re-enter geometric data in a parallel system and the associated risk of entry errors. Many simulation methods, however, require simplifications to the model or model transformations to function effectively. Structural analyses, for example, are often calculated using dimensionally-reduced models.
- 3D models make it possible to compute photo-realistic visualizations of building designs (renderings) including shadows and surface reflections (see Fig. 1.1). This is particularly relevant for communications with clients and helps architects assess the spatial qualities and lighting conditions of their designs. For photo-realistic visualization, information on the materials and their surface qualities is also required in addition to the 3D geometry.

The digital representation of the three-dimensional geometry of a building design is therefore one of the most fundamental aspects of Building Information Modeling. To properly understand the capabilities of modeling tools and exchange formats, one needs to know the basic principles of computer-aided geometric modeling, as described in this chapter. In addition, this chapter also introduces parametric modeling as a means of creating flexible geometries that can be easily adapted to meet new boundary conditions. The chapter concludes with an overview of modeling freeform curves and surfaces, which are gaining increasing relevance in building constructions

A key determining factor for the capabilities of a BIM modeling tool is the quality of the geometric modeling kernel used. This is a software component that provides support for elementary data structures and operations for representing and processing geometric information. The same geometric modeling kernels is often used for several related software packages, and sometimes even licensed for use by other software vendors. Two examples of commonly-used geometric modeling kernels include ACIS (Spatial, 2015) and ParaSolid (Siemens, 2015).



Fig. 2.1 A 3D model serves as the basis for a rendering to create a photo-realistic impression of a building design. © C. Preidel, reprinted with permission

2.2 Solid modeling

There are two fundamentally different approaches to modeling the geometry of three-dimensional bodies: *Explicit modeling*, which describes a volume in terms of its surface, and is therefore often also known as Boundary Representation (BRep). *Implicit modeling* by contrast employs a sequence of construction steps to describe a volumetric body, and is therefore commonly termed a procedural approach. Both methods are used in BIM software and in the corresponding data exchange formats, and both are part of the IFC specification (see **Chap. 6**). The following section describes each in turn.

2.2.1 Explicit modeling

2.2.1.1 Boundary representation methods

Boundary Representation is the most common and widespread method for describing three-dimensional bodies using a computer. The basic principle involves defining a hierarchy of boundary elements. Typically, this hierarchy comprises the elements *Body, Face, Edge* and *Vertex*. Each element is described by elements from the level beneath, i.e. the body is described by its faces, each face by its edges, each edge by a start and end vertex. This system of relationships defines the *topology* of the modeled body, and can be described with the help of a graph (see Fig. 1.2), which is known as the *vertex-edge-face graph*, or vef graph.

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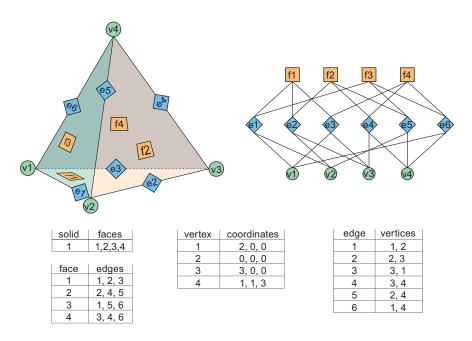


Fig. 2.2 A simple BRep data structure containing the information necessary to describe a pyramid. The Vertex-Edge-Face-Graph describes the relationship between the vertices, edges and faces, and therefore the topology of the body.

This topological information must then be augmented with geometric dimensions to fully describe the body. If a geometric body has only straight edges and flat surfaces, geometric information is only required for the nodes, i.e. the coordinates of the vertices. If the geometric kernel permits curved edges and surfaces, geometric information describing their shape or curvature is also required. This is described below in more detail in Sect. 1.4.

The data structure used to describe topological information usually takes the form of lists of variable length. The body refers to the faces that enclose it, the surfaces to the edges that bound it, and each edge to its start and end vertices.

This data structure is, however, only suitable for describing simple bodies without cut-outs or openings. To describe more complex volumes, the data model must be extended. Fig. 1.3 shows the object-oriented data model of the ACIS modeling kernel (Spatial, 2015), which is used by a variety of CAD and BIM software applications. With this data model, a *Body* can be comprised of several so-called *Lumps* that are not connected to one another. These *Lumps* are in turn described by several *Shells*, which make it possible for volumes with one or more openings or cut-outs. *Shells* can be comprised of any number of *Faces*, which in turn are described by one or more *Loops* that bound the faces. Because several loops are permitted per face, it is possible to define faces with holes, which are a prerequisite for modeling openings, recesses and holes.

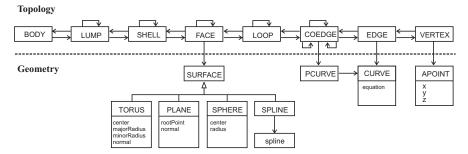


Fig. 2.3 The data model of the ACIS geometric modeling kernel

A further characteristic of this model is that loops do not refer directly to edges but to so-called *CoEdges* that have a consistent orientation to the respective face. These then refer to the actual *Edges*, which in turn are defined by their start and end Vertices. The bottom section of the figure shows the geometric information that can be associated with the faces, edges and vertices.

The resulting data model is extremely powerful, and can be used to describe almost any arbitrary body. It is implemented in the ACIS data exchange format, which is supported by several BIM systems, and is replicated in a slightly modified form in the IFC data model (see **Chap. 6**).

2.2.1.2 Triangulated surface modeling

A much-simplified variant of boundary representation is the description of the surface of a body as a triangle mesh. While curved surfaces cannot be described precisely, they can be approximated by choosing a finer mesh size to achieve the desired degree of accuracy. Triangulated surface modeling is often used in visualization software, for describing the surface of a terrain, or as input for numerical calculations and simulations. The description of curved surfaces as multiple faces requires much more storage capacity than analytical descriptions (see Sect. 1.4).

The underlying data structure commonly takes the form of a so-called *Indexed Face Set*. Here the coordinates of the vertices are stored as an ordered and numbered (indexed) list. The triangular faces are then defined by the indexes within the point list. This method avoids the repeated (redundant) storage of point coordinates and possible resulting geometry errors (gaps, overlaps) resulting from imprecisions.

The *Indexed Face Set* is a simple data structure and therefore robust and quick to process. It is used in several geometry data formats such as VRML, X3D and JT, as well as in the BIM IFC data structure (see **Chap. 6**). The commonly-used STL geometry format is likewise based on a triangulated description of bodies but, unlike the Indexed Face Set, stores the explicit coordinates of each individual triangle. This results in larger data sets and the lack of topological information in the STL format means that the derived geometry can contain errors, such as gaps between faces or overlapping sections of the individual triangles.

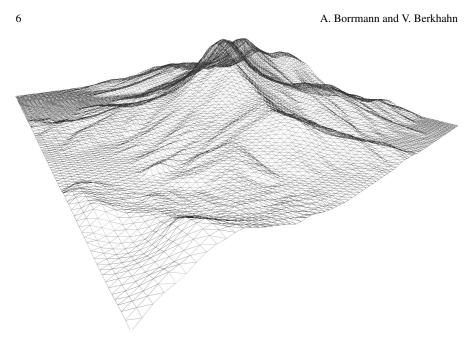


Fig. 2.4 Digital terrain models are usually modeled as triangulated surface meshes.

2.2.2 Implicit modeling

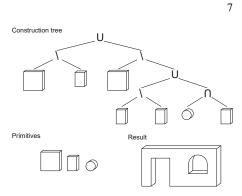
Implicit methods for modeling geometries store the history of the creation of a modeled 3D body. As such they are known as procedural methods. They represent an alternative approach to the explicit methods described above, which store just the result of a what may have been a long and complex modeling process.

In CAD and BIM systems, a hybrid approach is often used in which the individual modeling steps of the construction history are recorded for the user while the system makes snapshots of the resulting explicit description of the geometry to reduce computational load and improve display times.

2.2.2.1 Constructive Solid Geometry

A classical approach to the procedural description of 3D geometries is the *Constructive Solid Geometry* (CSG) method, which employs predefined basic objects – so-called primitives –, such as cubes, cylinders or pyramids and combines them using Boolean operators such as union, intersection or difference to create more complex objects. This process of combination results in a construction tree that describes the generation of the 3D body (see Fig. 1.5). The dimensions of the basic bodies are usually parametrized so that they can be easily adapted to the respective application.

Fig. 2.5 The CSG method is based on the combination of solids using the Boolean operators union, intersection and difference.



While a relatively large spectrum of bodies can be constructed using CSG, the use of a small number of simple objects is often too limiting. As such, the pure CSG method is only rarely used, although it is supported by the IFC data model and other systems for data exchange purposes.

Many 3D CAD and BIM systems have adopted the principle of Boolean operators and extended their functionality significantly by making it possible to apply them to any previously modeled 3D object. This offers a powerful means of intuitively modeling complex three-dimensional objects. In the field of BIM, the definition of subtraction solids plays an important role in the modeling of openings and penetrations.

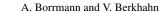
2.2.2.2 Extrusion and rotation methods

Many CAD and BIM systems provide the ability to generate 3D geometries by extrusion or rotation (Fig. 1.6). With these methods, a 2D geometry (typically a closed surface) is moved along a path or 3D curve defined by the user to create a 3D solid.

When the path along which the shape is drawn is straight, the results is an *Extrusion*, when curved a *Sweep*. Using a dedicated setting, the user can define whether the 2D profile remains parallel to its original plane or whether it turns to remain perpendicular to the path over the length of the path. Extrusion methods are used in building construction to generate beams with a constant or variable profile. A rotation volume is similar to an extrusion except that the 2D surface is rotated around an axis defined by the user.

Lofting is a variant of the above in which several cross-sections are defined and positioned one behind the other in space. The cross-sections can differ in size and shape from one another. The CAD or BIM system generates a body out of these cross-sections, interpolating the sections between them.

Extrusion and rotation functionality for generating 3D bodies is provided in many BIM tools, and is included in the IFC data format.



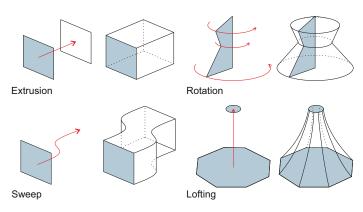


Fig. 2.6 Extrusion and rotation methods for creating solid bodies

2.2.3 A comparison of explicit and implicit methods

With respect to data exchange, implicit methods have several advantages over explicit representations, most notably the ability to trace the modeling steps, the ability to easily modify the transferred geometry by editing the construction steps and a much smaller quantity of data to transfer. A major proviso in the data exchange of implicit model descriptions is, however, that the target system must support and be able to precisely reproduce all the operations used to generate the model geometry in the source system. This makes the implementation of a data exchange interface considerably more complex for the software producer.

The ability to edit the construction steps in implicitly modeled geometries requires the automatic reconstruction of the building element. Although this rarely needs any manual interaction from the user, it can be computationally intensive for complex elements. In addition, editing a construction step can prevent later construction steps from being executed properly so that these may also need editing.

In the case of explicitly modeled geometries, only direct editing is possible. One can manipulate specific control points to ensure the continuity of surfaces or to adapt the shape of surfaces to match the respective requirements.

2.3 Parametric modeling

An exceptionally important trend in the building sector is parametric modeling (Pottman et al., 2015), with which it is possible to define a model using dependencies and constraints. The result is a flexible model that can be quickly and easily adapted to meet new or changing conditions.

Parameters can be as simple as geometric dimensions, for example the height, width, length, position and orientation of a cuboid. Relationships between param-

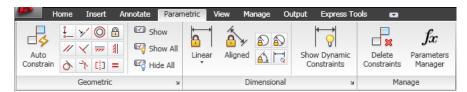


Fig. 2.7 User interface for defining parametric geometries in Autodesk AutoCAD

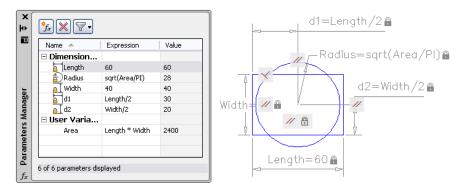


Fig. 2.8 Right: a parametric sketch with defined geometric and dimensional constraints. Left: Using a parameter manager, it is possible to specify equations that define the relationship of parameters to one another. In the example shown, the parameter constraints ensure that the square and circle always bound the same area.

eters, so-called dependencies, can be defined with user-definable equations. This can be used, for example, to ensure that all walls in a story have the same height as the story-height. If the height of the story is changed, all wall heights change accordingly.

The concept of parametric CAD systems originated from the field of mechanical engineering, where it has been used since the 1990s. These systems used an approach based on parametrized sketches. The user would create an 2D drawing (the sketch) comprising all the desired geometric elements in proportions that roughly corresponded to the final object. These geometric elements would then be assigned constraints in the form of geometric constraints or dimensional constraints (Fig. 1.7). Geometric constraints can, for example, define that two lines must meet at their ends, that two lines are perpendicular to one another or parallel to one another. Dimensional constraints, on the other hand, define only dimensional values such as length, distance or angle. Equations can be defined to define relationships between different parameters (Fig. 1.8). This parametrized sketch then serves in the next step as a basis for an extrusion or rotation operation that generates the final parametrized three-dimensional body. Such bodies can then be combined with one another using CSG operations. So-called features can also be added to the final bodies, for example the application of a chamfer or the boring of holes. These features comprise a series of geometric operations, each controllable via their own parameters.

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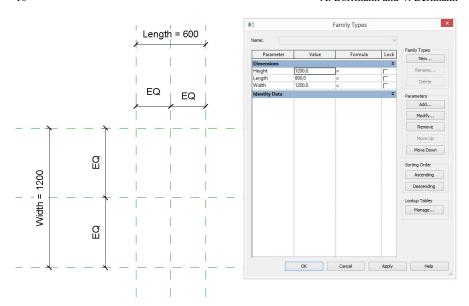


Fig. 2.9 Definition of a construction element family in Revit, showing how dimensions are linked to parameters.

The combination of parametrized sketches and procedural geometric descriptions is an extremely powerful mechanism for defining flexible 3D models that affords users a high degree of freedom as well as precise control of the generated model.

This form of parametric modeling is not currently supported by BIM products. At present, only pure 3D modeling tools such as SolidWorks, CATIA and Siemens NX provide this functionality, but without support for semantic modeling. One exception is Digital Project by Gehry Technologies that comprises a fully parametric modeling kernel augmented with a catalog of building-related construction elements that detail their semantic structure.

At present, BIM tools implement the concept of parametric modeling with a limited degree of flexibility. Parametric definitions are applied at two different levels: the level of the creation of parametrized building element types and the level of the orientation and positioning of building elements within a specific building model.

To create parametrized object types (typically called "families"), reference planes and/or axes are first defined and their position specified with the help of distance parameters. Here too, the relationship between parameters can be defined with the help of equations. The resulting bodies can then be generated with their edges or faces aligned with respect to the reference plane.

When creating the building model itself, the user cannot generate new parameters but only specify values already defined in the families or for the respective project. It is, however, possible to define the following constraints when aligning construction elements:

- Orientation: Construction elements must be arranged either horizontally or vertically to one another or to a reference plane.
- Orthogonality: Construction elements remain perpendicular to one another.
- Parallelism: Construction elements remain parallel to one another.
- Connection: Two construction elements are always connected.
- Distance: The distance between two construction elements remains constant.
- Same-size dimensions: Two dimensions specified by the user must be the same size.

While the implementation of parametric systems is more limited in comparison with defining the building geometry, it can still provide a sufficiently high degree of flexibility while keeping the model dependencies manageable.

BIM products that support this kind of parametric modeling include Autodesk Revit (Fig. 1.9), Nemetschek Allplan, Graphisoft ArchiCAD and Tekla Structure.

2.4 Freeform curves and surfaces

Bodies with straight edges and surfaces are easily represented using boundary representation (BRep method). The conceptual design of more sophisticated and complex architectural designs can, however, also require the modeling of arbitrarily curved edges and surfaces. These curved geometries are known as freeform curves and surfaces. Freeform geometries are described with the help of parametric representations which, compared to approximations (e.g. polygon triangulation), make it possible to model a curve or surface with absolute precision. The data volume required for the parametric description of freeform geometries is also much less than that needed for approximated methods.

The following section outlines the principle methods for describing curved surfaces, and how these surfaces are represented.

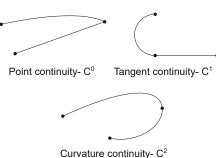
2.4.1 Freeform curves

Freeform curves are also known as splines. These are curves that are comprised of a series of polynomials. To ensure the overall curve is smooth, the joins between the segments of the curve must satisfy given continuity conditions. There are three different stages of continuity which are termed C^0 -, C^1 - and C^2 – continuity (see Fig. 1.10).

- C^0 *continuity* stands for point continuity and means that two curves are connected without a break between them.
- C^1 continuity stands for tangent continuity and means that two curves are connected at a point and share a common tangent direction at the join point.



Fig. 2.10 Continuity conditions at the join between two curves



• C^2 – continuity stands for curvature continuity and means that two curves are connected at a point, share a common tangent direction and a common curvature at the join point.

Freeform curves are described mathematically as parametric curves. The term "parametric" derives from the fact that the three coordinates in space are the function of common parameters (commonly termed u). These parameters span a given value range (typically 0 to 1) and the evaluation of the three functions produces the path of the curve in the space.

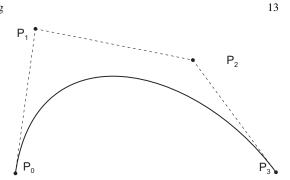
The most common types of freeform curves are Bézier curves, B-splines and NURBS. All three types are defined by a series of control points: the first and last of these lie on the curve, while those in-between are only approximated by the curve. Moving a control point changes the arc of the curve, making it possible to adjust curves intuitively in the computer interface. The control points form a characteristic polygon, the first and last segments of which determine the tangents for the start and end points of the curve.

Mathematically all three curve types are the sum of the multiplication of the control points with the basis function. These basis functions are different for each of the three curve types. As such they are fundamental to determining the shape of the different curves and are described as follows:

Bézier curves. The basis functions for Bézier curves consist of Bernstein polynomials. The degree p of the resulting curves is determined by the number of control points n where p = n - 1. This means, however, that curves with a large number of control points result in a very high degree polynomial. In addition, control points are not isolated from each other: changing the position of one, therefore has global impact on the entire course of the curve.

B-splines. B-splines were developed to overcome the limitations of Bézier curves. The primary advantage is that the degree of the curve can be defined largely independently of the number of control points. It needs only remain beneath the number of control points (p < n). As such, it is possible to combine the smoothness of low degree polynomials (typically p = 3) with a higher number of control points. To achieve this, the B-spline is comprised of piecewise sections polynomials of a cho-

Fig. 2.11 A Bézier curve described by four control points.



sen degree, whereby the continuity c = p - 1 at the join point. The basis for this is a hierarchical basis function that is recursively defined.

NURBS. Non-Uniform Rational B-splines (NURBS) are based on B-splines but additionally make it possible to assign a weighting to each control point (Piegl and Tiller, 1997). This makes it possible to further influence the course of the curve, which is necessary to precisely represent regular conical sections (circles, ellipses, hyperboles). Consequently, NURBS are the standard means for describing curves and are implemented by many BIM systems and geometric modeling kernels.

2.4.2 Freeform surfaces

Freeform surfaces add an additional dimension to the description of freeform curves. For this a second parameter is introduced, typically given as v, that also spans a predefined value range. The combination of all specified values of u and all specified values of v produces the desired freeform surface.

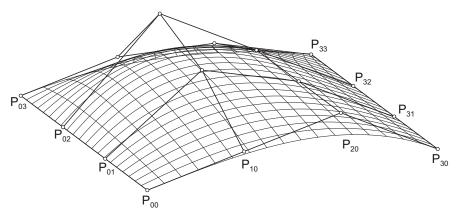


Fig. 2.12 NURBS patch with a field of 4x4 control points

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As with the description of curves, one also differentiates between Bézier surfaces, B-spline surfaces and NURBS surfaces. The respective advantages and disadvantages of these curve types apply equally to the corresponding surfaces. As such, NURBS surfaces are by far the most flexible type of freeform surfaces, and can be used to precisely model spherical and cylindrical surfaces. Fig. 1.12 shows a NURBS surface and its corresponding network of control points.

Larger surfaces are generally assembled out of a series of individual "patches" with a set mathematical description. Where the patches adjoin one another, continuity conditions need to be satisfied. The most common continuity condition is $C^2 - continuity$, i.e. that the patch surfaces meet without changing the curvature of the surface.

2.5 Further reading

The field of geometric modeling is extensive and complex and this chapter only presents a basic overview. Readers interested in more detailed aspects of geometric modeling can find out more from the following literature:

Pottman et al. (2007) provide a good overview of the different forms of geometric modeling with a discussion of their relevance for and impact on architectural design. Mortenson (2006) has become a standard work on computer-aided geometric modeling, now available in its third edition. Shah and Mantyl (1995) have also authored a standard work that focuses on parametric modeling with an in-depth discussion of the underlying mathematics and data structures. With respect to the mathematical description of freeform surfaces, the NURBS book by Piegl and Tiller (1997) is highly recommended.

2.6 Summary

Geometric modeling is an important basis for digitally modeling buildings. The representation of a building as a 3D volumetric model makes it possible to derive consistent plans and sections, to determine possible collisions between construction elements, to automate quantity take-off and to pass data on to calculation and simulation systems.

There are two principle approaches to geometric modeling. The explicit description of the model surfaces known as Boundary Representation, which is modeled by a hierarchy of boundary relationships between body, face, edge and vertex. A special variant of this is the triangulated description of model surfaces. The implicit method, by contrast, is a procedural approach that describes the history of the creation of the modeled body. Typical methods include Constructive Solid Geometry and extrusion and rotation methods. As both explicit and implicit geometric description methods have specific advantages and disadvantages, many BIM systems employ a hybrid

approach in which the user models a body using the implicit procedural method and the system internally takes a snapshot of the resulting explicit description at each point in the history of its description. Both approaches are also used for BIM data exchange formats.

Parametric modeling makes it possible to assign parameters, dependencies and constraints to geometric models. This results in flexible models that can be quickly and easily adapted to meet changing boundary conditions. Parametric approaches are always based on implicit methods of describing geometry.

Freeform curves are mathematically described as parametric curves. Three coordinates in spaces are defined as a function of common parameters that are defined within a predefined value range. The computation of the three functions produces the path of the curve. Control points can be used to intuitively control the shape of the freeform curve. Depending on the definition of the underlying basis functions, one differentiates between Bézier, B-spline and NURBS curves. This same differentiation also extends to freeform surfaces, resulting in Bézier, B-spline and NURBS surfaces. Complex surfaces can be created by assembling a series of so-called patches making sure that they satisfy given continuity conditions at their joins.

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