Optimal prioritization of inspections in structural systems considering component interactions and interdependence

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Abstract: Prediction of deterioration in structural systems is associated with large uncertainties. Inspections can reduce these uncertainties and support the planning of measures to ensure the integrity of structural assets, but inspections are costly and should be optimized. Past research and applications of risk-based inspection planning have treated sequentially the questions “when to inspect?” and “where to inspect?” to limit the computational cost of optimizing inspection plans. To optimize inspections in larger structural systems, we develop a methodology that accounts for component interactions and interdependence such as stochastic dependence in deterioration processes at different locations, structural interactions and progressive damage evolution. The methodology involves a hierarchical dynamic Bayesian network to compute the updated system reliability with component inspection results. The optimization utilizes a heuristic for defining inspection strategies at the system level. In particular, component inspections are prioritized based on their value of information (VoI). We investigate heuristics that combine component characteristics which are closely linked to the VoI. We define a Prioritization Index and study its effect for different combinations of component structural importance, uncertainty and correlation. For numerical investigations, the methodology is applied to an idealized steel structure subject to fatigue deterioration.

1 Introduction

Civil and structural assets deteriorate over time due to processes such as fatigue, wear or corrosion, and can eventually become non-operational or even fail. Their state of deterioration can be described and predicted only with uncertainty due to the stochastic nature of these processes [3, 13]. To reduce this uncertainty, the asset operator can devise inspection and maintenance plans (I&M). Through inspections, the operator obtains new information on the state of the structure and reduces the uncertainty on the system condition. However, the inspection and maintenance actions can represent a significant part of the total life cycle cost of a structure. This paper addresses the optimization of these actions, so that the total expected I&M cost over the design life of the structure is as low as possible without affecting the operability of the system.
Multiple studies have developed I&M planning methods and tools combining Bayesian methods and stochastic deterioration models [7, 8, 13, 18, 20]. An I&M plan can be modelled as a sequential decision problem, as illustrated by the decision tree in Figure 1. This type of problem has been studied extensively and several general methods have been proposed to obtain a global optimal strategy [2, 11]. These methods include the Partially Observable Markov Decision Process (POMDP), which has been utilized to solve single and multi-components I&M

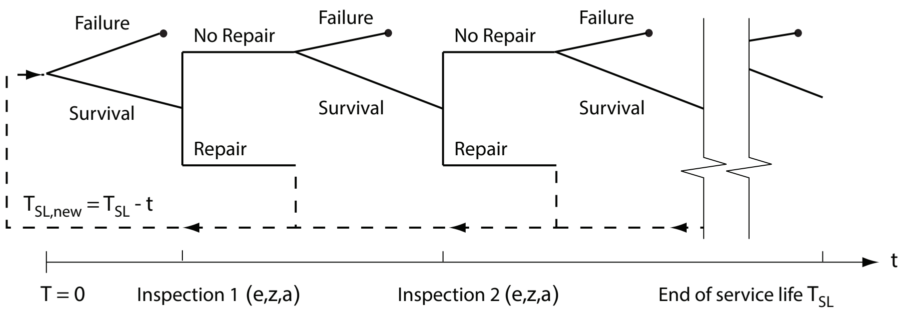

Figure 1: Simple decision tree in risk based inspection planning [13].

optimization problems [6]. However, the exact solution of the POMDP for multi-component systems is an intractable computational challenge [2, 9, 6]. Approximate solutions have been investigated, but they do not consider components interdependence and their joint effect on the system condition [6]. For the same reasons, most risk-based planning optimization approaches have simplified the system and focused on the optimization at the component level [8, 18]. However, these simplified models consider separately and sequentially the conditions of the components and do not include the strong correlation among the states of multiple components. The method proposed in [4] models the component interaction by adopting a hierarchical dynamic Bayesian network (DBN) to calculate the system probability of failure at any point in time t, given inspection observation and repair decisions of the system components up to that time t. This offline method is based on state-of-the art deterioration models, is applicable to large multi-components systems, and takes into account component interdependence and their joint effect on the state of the system. The DBN is utilized in a methodology to evaluate the cost of any strategy for an I&M plan [5]. The methodology utilizes a heuristic to reduce the optimization space of possible strategies at the system level and a Monte Carlo simulation to obtain the expected cost of an adopted inspection and maintenance strategy. The heuristics approach plays the double role of eliminating sub-optimal and impractical strategies for the asset operator, and pre-selecting subsets of strategies. The strategy that minimizes the expected cost within the reduced space is not a global optimum but may be a good compromise and can be compared to any other strategy being put forward as an alternative.

In [5], component prioritization for inspection is considered only in a simplified manner, through the value of the component probability of failure. This paper extends the methodology in [5] and combines component importance within the system and component probability of failure to propose a new ranking index and a parameter η for the heuristic prioritization of components for inspection. This extended methodology and proposed component prioritization is applied to a steel structure.

2 The optimization problem

2.1 Terminology and parameters

The terminology, parameters of interest and basic assumptions in this study are outlined below.

- *Finite horizon model*: we evaluate the optimal decision plan for a deterministic service life time T in years. This assumption is reasonable since the durability of materials or technological advances are likely to make the structure obsolete after a certain time.

- *Time step t*: a discrete-time model is adopted, where a time step corresponds to one year. This is typical for most structures, but the methodology and formulae in this paper can be adjusted for different time step definitions. For the fatigue model presented in the numerical investigation, one time step corresponds to Δn fatigue stress cycles.

- *Policy* π_t: the set of rules adopted at time t guiding the decision process based on the information available at that time [1]. The policy gives the answer to the questions "Inspect?" {Yes/No}, "Where?" {component i, j...}, "Repair?" {Yes/No} in function of the inspection, repair and failure history.

- *Strategy* $\mathcal{S} = \{\pi_1, ..., \pi_t, ...\pi_T\}$: the set of policies for all decision steps. The universe $\mathscr{S}$ of strategies increases exponentially with the number of components and the number of states describing the condition of a component.

- *Inspection outcomes* $\mathbf{Z} = \{\mathbf{Z}_1, ..., \mathbf{Z}_T\}$: the $(\mathbf{Z}_t)$ are random variables containing the possible inspection history during the service life of the structure. The vector $\mathbf{Z}_t$ stores the information for components $1 \leq i \leq N$, if inspected at time t, and the inspection results.

- *Costs*: the utility of a strategy is expressed in monetary terms, in function of the different costs incurred during the service life [14].
  - c_C: cost incurred when an inspection campaign is launched. It includes the cost of installing inspection devices or transporting inspection operators to site, and the cost of impairing the operation of the system.
  - c_I: cost of inspection per component. It includes the cost of the time spent for each component inspection during an inspection campaign.
  - c_R: cost of repairing one component. It also includes the associated downtime.
  - c_F: cost incurred if the structure fails.

  We assume that the inspection and maintenance actions have a fixed price over the service life.

- *Discount rate* r: we consider a discounting factor for the costs incurred in the future. For the life cycle cost calculation, all costs are discounted to their present value by the discounting function $\gamma(t) = 1/(1+r)^t$.

2.2 Formulation of the optimization problem

The decision-maker searches for the optimal inspection-repair strategy $\mathcal{S}_{opt}$, which minimizes the expected cost over the service life of the structure:

$$\mathcal{S}_{opt} = \underset{\mathcal{S} \in \mathscr{S}}{argmin}(\mathbf{E}[C_T|\mathcal{S}]). \tag{1}$$

The expected cost $\mathbf{E}[C_T|\mathcal{S}]$ of a strategy $\mathcal{S}$ can be derived as follows [5]:

1. The total lifetime risk associated to a strategy $\mathcal{S}$ and an outcome $\mathbf{Z}$ is the sum of the discounted risk, namely the cost of failure multiplied by the probability of failure of the system $F_s(t)$, conditional on the inspection outcomes and repairs as prescribed by the strategy:
$$R_F(\mathcal{S}, \mathbf{Z}) = \sum_{t=1}^{T} c_F \cdot \gamma(t) \cdot Pr(F_s(t)|\mathcal{S}, \mathbf{Z}_{0:t-1}). \tag{2}$$

2. The lifetime risk is added to the lifetime inspection and repair costs, which are determined by the inspection and repair history:
$$C_T(\mathcal{S}, \mathbf{Z}) = C_C(\mathcal{S}, \mathbf{Z}) + C_I(\mathcal{S}, \mathbf{Z}) + C_R(\mathcal{S}, \mathbf{Z}) + R_F(\mathcal{S}, \mathbf{Z}). \tag{3}$$

3. Finally, the expected total life-time cost and risk, for a strategy $\mathcal{S}$, is given by

$$\mathbf{E}[C_T|\mathcal{S}] = \mathbf{E}_{\mathbf{Z}}[C_T(\mathcal{S},\mathbf{Z})] = \int_{\Omega_{\mathbf{Z}}(\mathcal{S})} C_T(\mathcal{S},\mathbf{z}) f_{\mathbf{Z}}(\mathbf{z})\mathrm{d}\mathbf{z}. \tag{4}$$

The solution of Eq. (1) is already non trivial for a single-component system and has been studied extensively [8, 13]. The complexity of the calculation increases for a multi-component system, due to the large number of possible strategies and inspection outcomes. The methodology summarized in section 2.3 addresses the calculation of the probability of failure in Eq. (2) at each time step, given an inspection and repair history, with a dynamic Bayesian Network with time and state-space discretization. The computation of the integral in Eq. (4) is performed with a Monte Carlo simulation. A solution to the optimization problem of Eq. (1) is given in section 3.

2.3 Dynamic Bayesian Network model

2.3.1 Deterioration model

Fatigue crack growth is modeled by Paris' law where the evolution of the crack depth D is described by:

$$\frac{dD}{dt} = C[\Delta S_e^M \pi^{\frac{M}{2}}] \cdot D(t)^{\frac{M}{2}}, \tag{5}$$

where C and M are empirical material parameters. C can be expressed as a function of M, i.e. $C = C(M)$ [13].
The fatigue stress range ΔS is described by a Weibull distribution with scale and shape parameters K and λ. The distribution of the equivalent fatigue stress range $\Delta S_e = (\mathbf{E}[\Delta S^M])^{\frac{1}{M}}$ is defined by Eq. (6) [13], as

$$\Delta S_e = K \cdot \Gamma\left(1+\frac{M}{\lambda}\right)^{\frac{1}{M}}. \tag{6}$$

One can integrate Eq. (5) between $t-1$ and t (one time step corresponding to Δn fatigue cycles), with initial condition D_{t-1}:

$$D_t = \left[\left(1-\frac{M}{2}\right) C \cdot (K\sqrt{\pi})^M \cdot \Gamma\left(1+\frac{M}{\lambda}\right)\Delta n + D_{t-1}^{(1-\frac{M}{2})}\right]^{(1-\frac{M}{2})^{-1}}. \tag{7}$$

Failure of a component is defined by the fatigue crack size exceeding a critical depth d_c.

2.3.2 Inspection model

The observation outcome Z_t is a random variable defined conditionally on D_t. The possible states of Z_t are defined on $\mathbb{R}^+$ by the discrete state {*"no crack detected"*}, i.e.$\{Z_t = 0\}$, and the continuous state {*"crack detected and measured as z"*}, i.e. $\{Z_t = z\}_{z>0}$. The corresponding hybrid distribution is defined in Eq. (8), where $\phi(.)$ is the standard normal probability density function, and $\Phi(.)$ is the standard normal cumulative distribution function:

$$\begin{cases} \Pr(Z_t = z|D_t = d) = \exp\left(-\frac{d}{\xi}\right) & \text{if } z = 0 \\ f_{Z_t|D_t=d}(Z_t = z) = (1-\Pr(Z_t = 0|D_t = d)) \cdot \frac{1}{1-\Phi\left(\frac{-d}{\sigma_\varepsilon}\right)} \cdot \phi\left(\frac{z-d}{\sigma_\varepsilon}\right) & \text{if } z > 0. \end{cases} \tag{8}$$

2.3.3 System DBN

The DBN models the deterioration process at the component level and at the system level as per Figure 2. K, M, C and λ are modelled as constants. For each component, D_t, M_t and K_t are random variables describing the state of the component at time t, with $M_t = M_{t-1}$ and

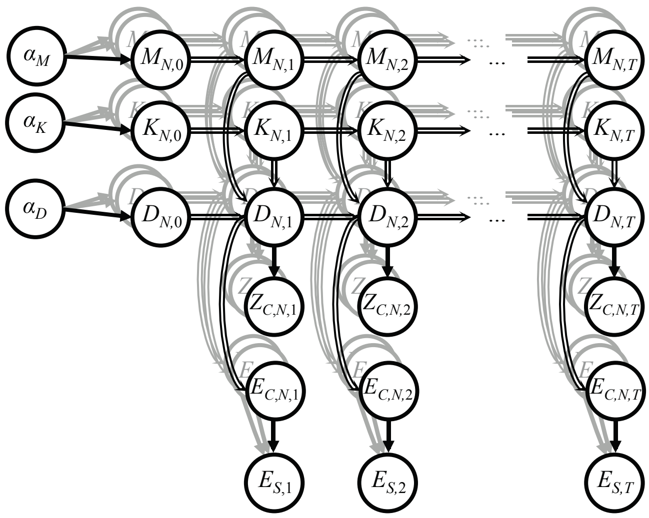

Figure 2: Hierarchical dynamic Bayesian network [4].

$K_t = K_{t-1}$. The effect of components' interdependence and correlation at the system level is modelled through the hyper-parameters α. The random variables are discretized and the conditional probability tables are pre-computated according to Eqs. (7 - 8). Exact Bayesian inference at each time step at the component level, and at the system level through the hyper-parameters, enables the calculation of the probability of failure for each component given the observation from all components. Finally, the updated probability of system failure F_s is calculated. The inference is performed following the algorithm described in [4].

2.4 Monte Carlo simulation

Monte Carlo simulation approximates the integral in Eq. (4) by:

$$\mathbf{E}[C_T|\mathcal{S}] \simeq \frac{1}{n_s}\sum_{j=1}^{n_s} C_T(\mathcal{S},\mathbf{z}_j), \tag{9}$$

where n_s is the number of samples and $(\mathbf{z}_j)_{1\leq j\leq n_s}$ are samples of the inspection history generated from sampled deterioration histories of the structure [5]. The number of samples determines the accuracy of the approximation. For the optimization problem, the required accuracy should allow two strategies to be compared with a small uncertainty. In our study, we find that 200 samples are sufficient to identify the optimum strategy.

3 Value of information and component prioritization

3.1 Heuristics

A heuristic is a simple method to generate I&M strategies and reduce the search space of strategies to a discrete subset. This approach is common in risk-based inspection planning and has been demonstrated to approximate the optimal strategy efficiently for individual structural elements [8]. The choice of the heuristics can be guided by operational constraints on the minimum inspection interval and the system reliability threshold. We characterize the heuristics at the system level with the following parameters [5].

1. Inspection campaigns are performed at fixed intervals ΔT.
2. The numbers of inspected components in each campaign is n_I.
3. If a threshold on the system reliability p_{th} is exceeded, an additional component is inspected. If no inspection campaigned was planned at that time, an additional inspection campaign is carried out.

4. Repairs of components are carried out if damage measurements exceed a repair criterion d_R.
5. Components are prioritized for inspection following a selection criterion described in section 3.2.

These five parameters define the strategies now considered for the optimization problem. The heuristic can yield a sub-optimal solution but reduces the complexity of the decision problem.

3.2 Prioritization Index $PI(\eta)$

We investigate several component prioritization criteria and their effect on the cost of the strategies. The value of information (VoI) is a good candidate for such a criterion and can be defined as the expected utility of inspecting component i at time t, i.e. the net gain of inspecting one component rather than doing nothing [11]. However, the calculation of the VoI demands significant computational efforts for multi-component systems [15, 19]. In the application of the DBN-MC method to a Daniels system, the components' probability of failure was used as a proxy for the VoI [5].
In this study, we aim to find an improved proxy that also accounts for component importance. Fundamentally, the information gained by inspecting a component is contained a) in the reduction of the uncertainty on the condition of this component and the corresponding effect on the system reliability; and b) in the reduction of the uncertainty on the condition of other components, through the components' interdependence. Indeed, more information is gained on the deterioration of other components from inspecting a component with a higher probability of failure [18].
To approximate the relationship between the probability of system failure $\Pr(F_s)$ and the probability of component failure, we introduce the Single Element Importance measure for component i (SEI_i), defined as the difference between the probability of failure of the intact system and the probability of failure of the system when only component i has failed [16]. Due to the conditional independence properties of the Bayesian network, the SEIs are independent of any observation and are constant in time. With F_i denoting the event "failure of component i", it is

$$SEI_i = \Pr(F_s|\overline{F_1},...,\overline{F_{i-1}},F_i,\overline{F_{i+1}},...,\overline{F_N}) - \Pr(F_s|\overline{F_1},...,\overline{F_N}). \tag{10}$$

Using the total probability theorem and inserting the definition of the SEI from Eq. (10), one can approximate the probability of system failure at a time t:

$$\Pr(F_s) \simeq \overbrace{Pr(F_s|\overline{F_1},...,\overline{F_N})}^{constant} + SEI_1 \cdot \Pr(F_1) + ... + SEI_N \cdot \Pr(F_N) + b, \tag{11}$$

where b represents the contribution of simultaneous component failure. Here, the conditioning event $\mathbf{Z}_{0..t-1,1..N}$ is omitted. Note that Eq. (11) is not actually used to calculate $Pr(F_s)$, but shows that the probability of system failure is a function of the terms $SEI_i \cdot \Pr(F_i)$ defined for each component, hence by a), the VoI for component i is a function of $SEI_i \cdot \Pr(F_i)$. Furthermore, as stated in b), the amount of information learnt on other components is related to the probability of failure $\Pr(F_i)$, through the components' interdependence.
The numerical study investigates the relative weight of the quantities SEI_i and $\Pr(F_i)$, as per a) and b), to find an improved proxy for the VoI. For this purpose, we introduce a Prioritization Index (PI) with an adjustable exponent η:

$$PI(\eta) = SEI_i^{\eta} \cdot \Pr(F_i), \text{ with } 0 \leq \eta \leq 1. \tag{12}$$

Other factors should be considered for the VoI, such as the effect of varying component correlations, the inspection quality, and the cost of inspection. For instance, an underwater part of the structure is more difficult and costly to inspect with the same accuracy as a part of the superstructure. We limit the study to equi-correlated components, with equal inspection quality and cost.

4 Numerical investigation

4.1 Zayas frame description

We apply the methodology to the Zayas frame [10]. This two-dimensional welded frame shown in Figure 3 is widely used for studying offshore structures. The structure is subjected to gravity and is loaded laterally at the top left node by a quasi-static point load L. The details of the loading and the DBN setup can be found in [4] and [12]. The components are defined as the 22 fatigue hotspots. Two hotspots belonging to the same element have the same SEI. The system failure is assessed through a pushover analysis [12], conditional on components states.

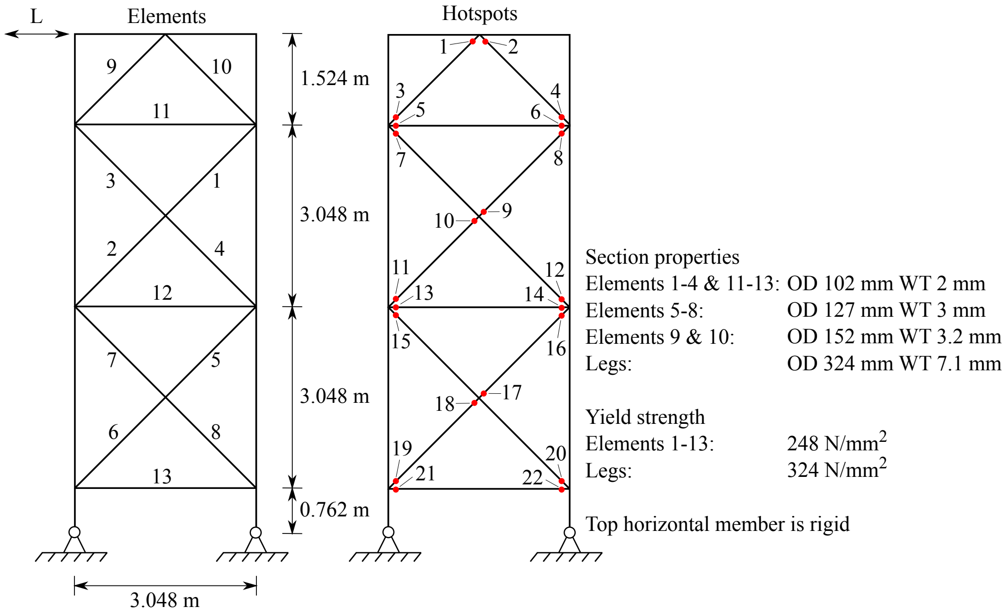

Figure 3: The Zayas frame [12, 10].

4.2 Fatigue model and DBN Parameters

It is here assumed that deterioration parameters for all components have the same marginal distributions and are equi-correlated, but the method is applicable for varying probabilistic models and correlation coefficients. The parameters of the DBN and the discretization process of the variables are summarized in [4]. The crack depth variable D is discretized in 100 intervals, as is the observation variable Z. The cost model assumed for the optimization is presented in Table 1, with the same inspection and repair costs for all components. To reduce the number of required computations, we assume that a repaired component performs like a new one [17].

Table 1: Cost model.

Inspection campaign, c_C	1
Component inspection, c_I	0.1
Component repair, c_R	0.3
System failure, c_F	10^4
Discount rate, r	0.02

4.3 Optimization

The optimization is performed exhaustively on the discrete set of parameters values as per Table 2. We use 200 samples for the MC simulation, which gives an acceptable variability for the approximation of the expected cost of one strategy and is sufficient to conclude on the strategy with the minimum cost.

Table 2: Parameters for the heuristic strategies.

Minimum time between inspection campaigns, ΔT [year]	{5, 10}
PoF threshold, p_{th}	$\{7 \cdot 10^{-4}, 1 \cdot 10^{-3}, 2 \cdot 10^{-3}\}$
Minimum number of inspected hotspots, n_I	{1,2,3,...,22}
Repair criterion, d_R	{0}
Prioritization parameter, η	{0, 0.2, 0.5, 0.7, 1}

4.4 Results

The results for the approximated expected cost of strategies with $\eta = 0$ are shown in Figure 4. From this graph, the best strategy with $\eta = 0$ is defined by $\Delta T = 10$, $p_{th} = 1e^{-3}$ and $n_I = 4$. The

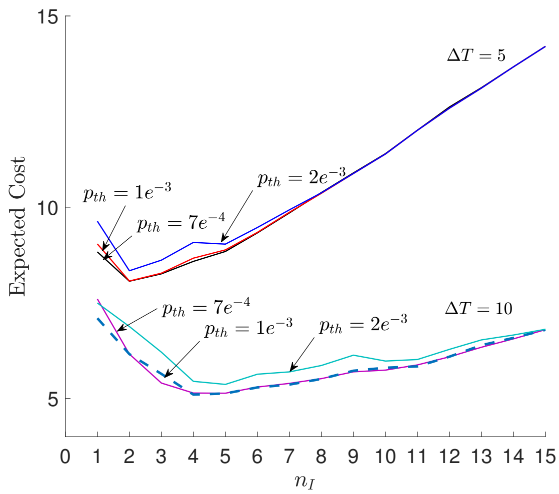

Figure 4: Strategy optimization for $\eta = 0$.

constraint of a minimum inspection interval along a probability threshold might not be optimal. In practice, it is however challenging to agree a strategy with the operator without minimum (regular) inspection intervals.

The influence of parameter η on the expected cost of strategies is illustrated in Figure 5. Varying η from 0 to 1 decreases the cost of the strategy in the order of 10%. The η factor also has a smoothing effect on the expected cost curve for the same number of simulation samples, which is due to a lower variance in the total life cycle cost.

Figure 6 shows that the greatest reduction in cost between $\eta = 0$ and $\eta = 1$ is in the risk of failure. All other costs are also reduced but to a lesser degree.

5 Conclusions

This paper extends the methodology for estimating the optimal inspection and maintenance strategy of large multi-component structural systems, described by Luque and Straub in [5],

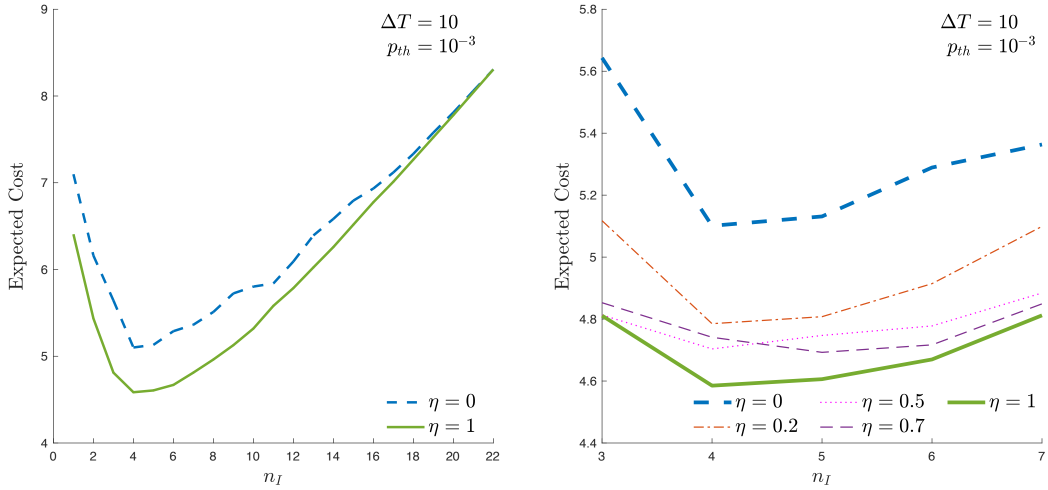

Figure 5: Influence of η on the expected cost of strategies with $\Delta T = 10$ and $p_{th} = 10^{-3}$.

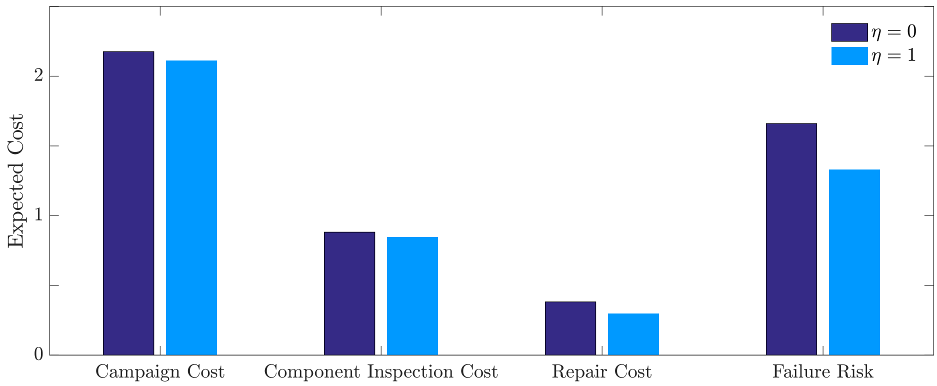

Figure 6: Decomposition of costs for the strategy $\Delta T = 10$, $p_{th} = 10^{-3}$ and $n_I = 4$.

within the space of strategies defined by heuristic parameters. The heuristic of component inspection was investigated with a Prioritization Index acting as a proxy for the value of information of inspecting a component. This Prioritization Index factors in the information gained at the system level through the component importance in the structural system, as well as the information learned about other components through the probability of failure of the component. The application of the methodology to a frame structure confirmed the computational efficiency of the dynamic Bayesian network model combined with Monte Carlo simulation for a more complex multi-component system. The proposed prioritization heuristic proved to be an effective measure to reduce the expected cost of a chosen strategy.

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