Reliability Analysis and Updating of Inspected Ship Structures subject to Spatially Variable Corrosion

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Abstract: We address the effect of spatial variability of corrosion on the reliability of ship structures conditional on inspection results. We apply a hierarchical spatial model to model the spatial variability of corrosion. To tackle the high-dimensional reliability and Bayesian updating problem, which arises from the spatial corrosion model, computations are performed by the BUS approach in combination with subset simulation. To reduce computation time in the evaluation of the time-dependent reliability, we propose to perform subset simulation in reverse chronological order. A series of numerical investigations is performed to investigate the effect of different spatial corrosion models, as well as the inspection coverage, on the resulting reliability.

1 Introduction

The integrity of ship structures is subject to significant uncertainty associated with deterioration processes, among which corrosion loss is the most common and often the most critical one [6]. Corrosion phenomena relevant to ship structures are uniform (general) corrosion, pitting corrosion, crevice corrosion and galvanic corrosion [12]. Uniform corrosion, which is the focus of this contribution, can reduce the structural capacity by a widespread reduction of plate thickness, leading to a loss in cross section. It is influenced by a variety of factors, such as age, type, cargo type, area of operation. These factors vary among ships and among position and type of elements within a ship structure. For example, the bottom shell exposed to sea water or a tank with corrosive cargo are more vulnerable to corrosion than other elements in the ship. Empirical models are available to describe the corrosion progress [5, 8, 16, 17]. Considering the significant uncertainty associated with the many influencing factors and their effect, such models should be formulated probabilistically.

To manage corrosion and the associated uncertainties, inspections are carried out regularly on ship structures. Inspection results reduce uncertainty and facilitate efficient repair and replacement of corroded elements. Because inspection results are necessarily incomplete and subject to uncertainty, they should be described in a probabilistic format. These can be achieved in a Bayesian framework, in which the prior corrosion model is combined with information from the inspections [4].

In this study, we apply a hierarchical spatial model following [13] to describe corrosion in a mid-ship section. Failure of a mid-ship section is taken as the relevant limit state. The reliability is computed conditional on inspections of the structure through plate thickness measurements. We thereby investigate the effect of the spatial dependence on the updated reliability. Computations are performed by means of the BUS approach with Subset Simulation [21], which can handle Bayesian analysis and reliability updating with a large number of random variables, as encountered when modeling spatially variable properties. To enhance computational efficiency, we propose to calculate conditional probabilities in reverse chronological order, which allows reusing samples generated in the previous time steps. We furthermore apply spectral decomposition to reduce the dimensionality of the problem.

In a numerical example, the framework is applied to calculating the probability of flexural failure of the mid-ship section of a tanker. We compare the results obtained with varying spatial corrosion models, to investigate the effect of modeling assumptions. To demonstrate the effectiveness of the framework, we compute results for varying number of inspected elements. Besides enabling a better use of inspection results, the proposed approach can potentially also be used for improved planning of inspections, in which the spatial dependence is accounted for.

2 Modelling Spatial Variability of Corrosion in Ship Structures

2.1 Remaining Thickness of Plate Under Corrosion

We express the remaining thickness w of a plate at time t as $w(t) = w_0 + M - D(t)$, where w_0 is the design plate thickness, M is the thickness margin, and D is the corrosion depth. The plate is the basic element, and we do not further consider the variability of uniform corrosion within the plate.

The corrosion depth $D(t)$ is calculated with a bi-linear corrosion model, in which the corrosion loss is zero if t is smaller than the coating life C_t and increases at a constant rate afterwards [19]. The corrosion rate, the coating life and the thickness margin M are modeled as lognormal random variables [13].

2.2 Hierarchical Representation of Corrosion in a Ship Cross-section

Spatial dependence of corrosion in ship structures has previously been addressed by grouping structural elements depending on location and type (e.g. bottom shell, side shell, deck plating), and representing each group by separate parameters [1, 3, 12]. In [13], a hierarchical Bayesian model was proposed for predicting corrosion progress, which hierarchical levels being e.g. fleet, frame, compartment and structural element (Figure 1). The correlation between two plate elements is a function of the level of hierarchies that is shared by the two plates. At the lowest hierarchical level, a random field describes the dependence between two plates within the same compartment. This model is capable of representing spatial dependence depending on location, type and mutual distance of elements.

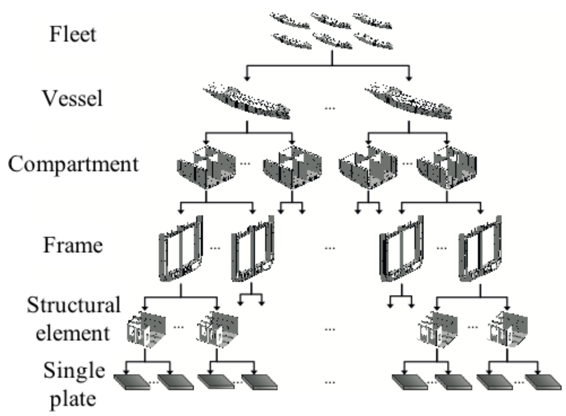

Figure 1: Hierarchical structure of the spatial corrosion model taken from [13].

In this study, we describe the spatial dependence of corrosion process in a mid-ship section following the hierarchical model as implemented in [11]. Four hierarchical levels are defined: frame, cell, structural element, and single plate element (e.g. stiffener, web, plating). For simplicity, the same hierarchical model represents spatial dependence of the corrosion rate R, the coating life C_t and the thickness margin M.
Figure 2 illustrates the effect of spatial dependence through random realizations of corrosion loss in a mid-ship section, where the variation of line color indicates how much the simulated corrosion loss deviates from the average corrosion loss. Figure 2(a) and (b) show realizations obtained with the hierarchical model, with different degrees of dependence (details in section 5.1). For comparison, a random realization of the case with independence among corrosion in the individual elements is included, which is clearly unrealistic. Finally, Figure 2(d) shows a random realization following the classical approach of considering spatial variability of corrosion in ship structures [3], wherein the cross section is divided into groups of elements (e.g. 8). Each group is characterized by a single random variable, implying full dependence within the group. Different groups are modeled as independent.
To reduce the dimension of the random variable vector, we employ a spectral decomposition of the correlation matrixes describing the spatially distributed corrosion rate, coating life and thickness margin. The number of eigenvalues is selected to ensure that more than 99% of total variability is captured by the approximation.

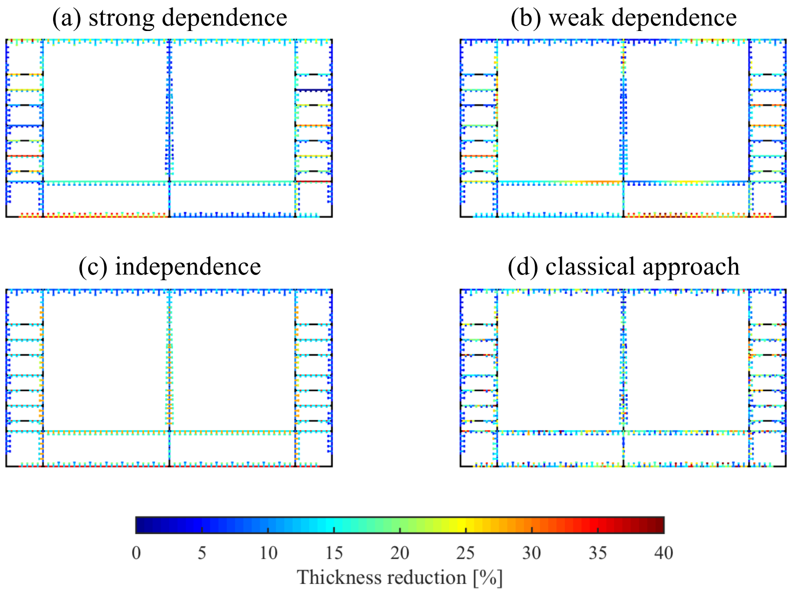

Figure 2: Realizations of thickness reduction due to corrosion with different spatial dependence.

3 Load and Resistance Models

3.1 Ultimate Bending Moment Capacity

Loss of steel by corrosion leads to a reduction of moment capacity. We calculate the ultimate moment capacity M_u of the hull girder section based on the incremental curvature method described in the IACS guideline [10]. We combine this method with the optimization scheme of [14] to reduce computational cost. Uncertainties in corrosion depth, Young's modulus, and yield stress are considered when computing the probability distribution of the ultimate moment capacity.

3.2 Vertical Bending Moment

The vertical bending moment acting on the hull girder section is calculated as the sum of stillwater moment and wave-induced moment, which are functions of ship size, operational condition, cargo history, sea states and additional parameters. The stillwater bending moment M_{sw} is here represented by a normal distribution [3, 7, 9, 15, 24]. Based on [7], mean value and standard deviation are defined as $\mu_{sw} = 0.70 M_{sw,d}$ and $\sigma_{sw} = 0.20 M_{sw,d}$, where $M_{sw,d}$ is the design stillwater moment calculated with IACS guidelines [10]. The extreme wave-induced moment M_{wv} is modeled by the Gumbel distribution with scale parameter a_{wv} and location parameter b_{wv} determined following [7]. Details on the calculation of the bending moment can be found in [11].

3.3 Limit State

In this study, flexural failure of a cross-section due to bending moment is taken to define the limit state, which is defined as follows:

$$g(\mathbf{X};t) = X_u M_u(\mathbf{X}_\mathbf{s}, t) - X_{sw} M_{sw} - X_{wv} M_{wv} \tag{1}$$

$\mathbf{X}_s$ is the vector of random variables affecting the ultimate moment strength; X_u represents the model uncertainties associated with ultimate moment strength calculation; X_{sw} and X_{wv} are the model uncertainties related to the stillwater and wave-induced moment load calculations. Model uncertainties are described by normal distributions following [15]. All random variables are combined in the vector $\mathbf{X} = [\mathbf{X}_\mathbf{s}, X_u, X_{sw}, X_{wv}, M_{sw}, M_{wv}]^\mathrm{T}$. A failure domain Ω_F is defined by the failure limit state function as $\Omega_F = \{g(\mathbf{X};t) \leq 0\}$.

4 Bayesian Updating

4.1 BUS Approach with Subset Simulation

In Bayesian analysis, the measurements are expressed by a likelihood function. In this study, the observation event is a set of plate thickness measurements w_m. We assume additive measurement error ϵ, so that the measurement at element i and time t is related to the true plate thickness $w_i(t)$ by $w_{m,i}(t) = w_i(t) + \epsilon_i(t)$. Measurement errors are modelled as independent normal random variables with zero mean and standard deviation σ_ϵ, i.e. $\epsilon_i(t) \sim N(0, \sigma_\epsilon)$. The resulting likelihood function is:

$$L(\mathbf{X}_\mathbf{s}) = \prod_t \prod_{i=1}^{N} L_i(\mathbf{X}_\mathbf{s}, t) = \prod_t \prod_{i=1}^{N} \exp\left[-\frac{1}{2}\left(\frac{w_{a,i}(\mathbf{X}_\mathbf{s}, t) - w_{m,i}(t)}{\sigma_\epsilon}\right)^2\right] \tag{2}$$

where N is the total number of inspected elements, w_a is the remaining plate thickness predicted with the implemented models. Following the BUS approach (Bayesian Updating with Structural reliability methods) proposed in [20 - 22], the likelihood function L describing Z can be cast into a structural reliability framework, by defining the observation limit state function:

$$h(\mathbf{X}_\mathbf{s}, U_0) = U_0 - \Phi^{-1}[cL(\mathbf{X}_\mathbf{s})] \tag{3}$$

U_0 is a standard normal random variable, Φ^{-1} is the inverse standard normal cumulative distribution function, and c is a positive constant that can be chosen following [21]. It should be noted that only the random variables $\mathbf{X}_\mathbf{s}$ which affect a remaining plate thickness are included in the observation limit state function. The observation limit state function defines a corresponding observation domain $\Omega_Z = \{h(\mathbf{X_s}, U_0) \leq 0\}$. The updated failure probability conditional on the observation event Z can be calculated as:

$$\Pr(F|Z) = \frac{\Pr(F \cap Z)}{\Pr(Z)} = \frac{\int_{\mathbf{X},u_0 \in \{\Omega_F \cap \Omega_Z\}} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} du}{\int_{\mathbf{X},u_0 \in \Omega_Z} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X} du} \tag{4}$$

To solve Eq. (4), we employ subset simulation [2]. This method expresses the failure event as the intersection of nested intermediate events and the probability of failure is evaluated through as product of (larger) conditional probabilities of these intermediate events. Because of the conditioning on Z, the classical subset simulation is modified and the conditional probability of failure is expressed as:

$$\Pr(F|Z) = \frac{\Pr(\cap_{i=0}^{M} F_i^*)}{\Pr(Z)} = \prod_{i=1}^{M} \Pr(F_i^*|F_{i-1}^*) = p_0^{M-1} p_M \tag{5}$$

F_i^* is the intersection of the intermediate failure event F_i as used in classical subset simulation and the observation event, $F_i^* = F_i \cap Z$. Therefore, the subset simulation starts with samples conditional on Z, since $F_0^* = Z$. The parameter $p_0 = \Pr(F_i^*|F_{i-1}^*)$ is taken as 0.1; $p_M = \Pr(F_M^*|F_{M-1}^*)$ is the conditional probability of failure computed in the last subset M. Details on the algorithm can be found in [22]. The adaptive MCMC algorithm proposed in [18] is used to improve the accuracy and efficiency of subset simulation.

4.2 Computing Reliability in Function of Time

The conditional probability of failure is computed in function of time *t*. The structural capacity of the ship reduces with time because of corrosion, which affects the probability of failure. To reduce computational effort, we propose to compute the reliability in different time steps sequentially, starting from the last time step.

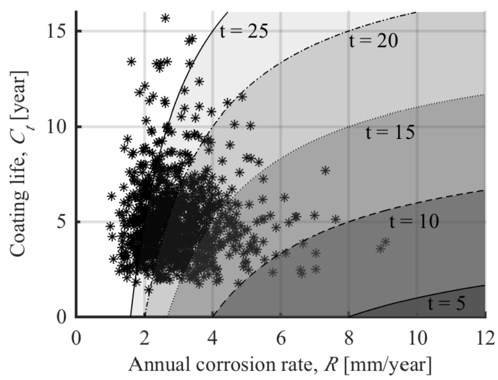

Figure 3: Limit state surfaces describing failure at different times t, where failure occurs when corrosion exceeds the plate thickness $w_0 = 40$mm. The limit state functions are $g(r, c_T) = 40mm - r(t - c_T)$, which potentially corresponds to Eq. (1). The black dots show Monte Carlo samples of R and C_T.

To illustrate the idea, Figure 3 shows failure domains for a simple deterioration limit state at different times. Clearly, the failure domain at time *t*–1 is a subset of that at time *t*. This property can be exploited in subset simulation, by computing the failure probabilities in reverse time order. The probability of failure at time step t can be written as $\Pr(F_t) = \Pr(F_t|F_{t+1}) \Pr(F_{t+1})$.

Hence, it is sufficient to compute $\Pr(F_t|F_{t+1})$, and the samples conditional on failure in time step $t+1$ are available as seeds for this computation.

5 Numerical Investigation

5.1 Problem Definition

A mid-ship section of a tanker described in [11] is investigated. The main parameters of the tanker are: length L=255m, breadth B=57m, height H=31.1m, block coefficient C_B=0.842, mean value of the Young's modulus E =207,000MPa, mean value of the yield stress σ_y=353MPa. The considered cross-section consists of 400 stiffened plate elements, and each of them consists of plating, stiffener and web, leading to a total of 1,200 basic elements.
Corrosion parameters are assigned in function of 8 distinct groups, in function of the location within the cross section. The corrosion rate R is modeled through lognormal random variables whose mean values are in the range 0.06 – 0.34 [mm/year], and whose C.o.V.s are 0.1 or 0.5. The coating life C_t is modeled by lognormal random variables with mean value equal to 5 years and C.o.V. 0.40. Details on the probabilistic modeling of the corrosion is provided in [11].
The hierarchical spatial model is defined with three levels: cross-section (level 1), cells (level 2) and structural element types (level 3). All basic elements are assigned to the hierarchical levels based on their location and type. For illustration purposes, we define a weak and a strong spatial dependence model, whose properties are summarized in Table 1. These models are compared to the classical approach, in which each corrosion parameter is represented by 8 random variables assigned to each of the 8 corrosion groups. In addition, we consider a model with all elements treated as independent.

Table 1: Correlation at each hierarchical level defined for two spatial dependence cases

Hierarchical level	Weak dependence	Strong dependence
Level 1: Frame	0.04	0.10
Level 2: Cell	0.19	0.32
Level 3: Struct. elem.	0.53	0.70

5.2 Reliability Updating with Thickness Measurements

A set of hypothetical thickness measurement data is generated through a Monte Carlo simulation based on the prior probabilistic models and the strong dependence case (Table 1). Measurement uncertainty is represented by a normal distribution with σ_ϵ=1mm. Inspections are performed in years 5, 15 and 25, at selected locations indicated in Figure 4.
Subset simulation is carried out with 1,000 samples per subset level. Calculations are repeated 10 times to estimate the sampling variability. Through the spectral decomposition, the 1,200 random variables representing each of the corrosion parameters are compressed to 50 random variables in the case of the weak spatial dependence model and to 33 random variables in the case of the strong dependence model.

5.2.1 Effect of Spatial Dependence Model

We first perform an analysis considering inspection of 6 elements (Table 3). The resulting failure probabilities are shown in Figure 5. One can clearly observe the effect of the inspection in later years in the case of the weak and the strong spatial dependence model. The classical approach leads to smaller changes in the probability of failure with inspection outcomes. As expected, when considering plates as independent, no change in the probability of failure is observed, because the inspections provide information only about the 6 inspected basic elements (out of 1,200).

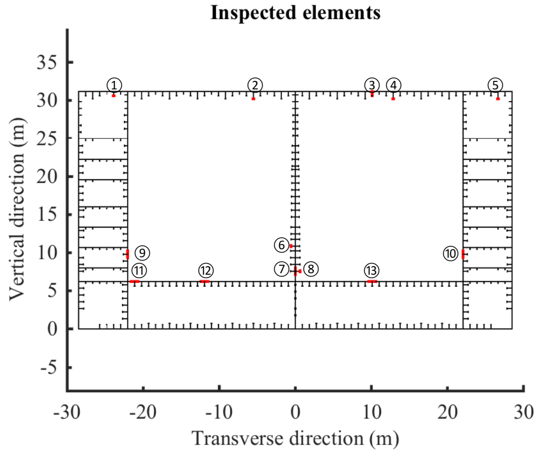

Figure 4: Configuration of inspected elements among cross-section

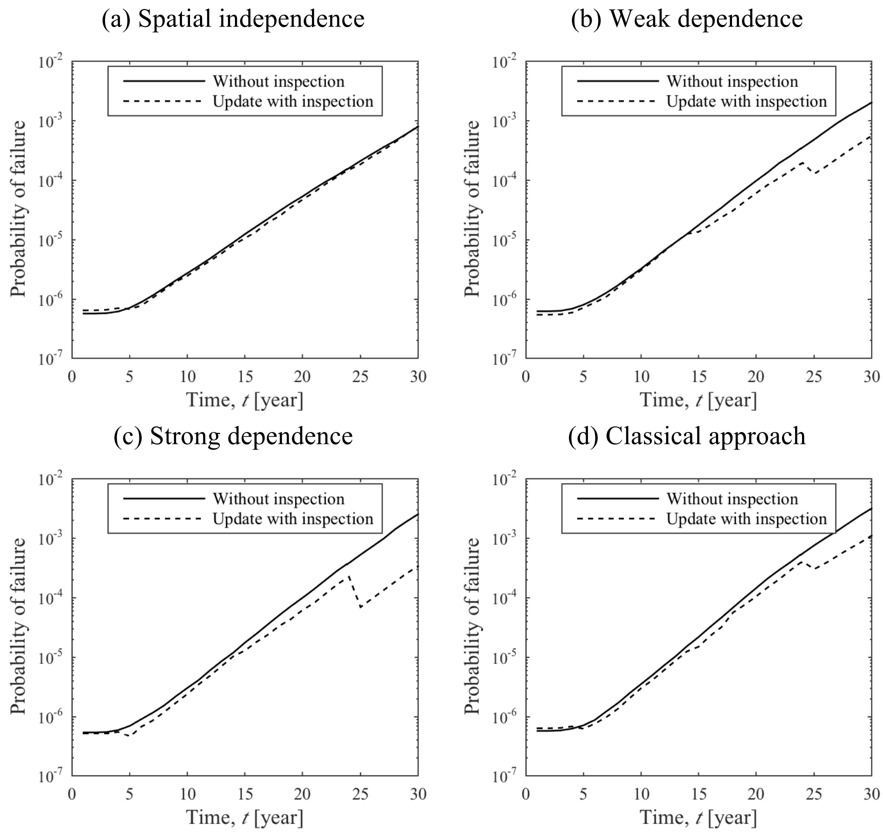

Figure 5: PoF conditional on the thickness measurements under varying spatial dependence assumptions

The sampling variability of the estimated probability of failure (PoF) is shown in Figure 6 and Table 2. The C.o.V. of the PoF estimate increases with decreasing probability of failure and with increasing number of inspections. The latter effect occurs because additional data leads to a smaller $\Pr(Z)$ and therefore to an increased number of subsets.

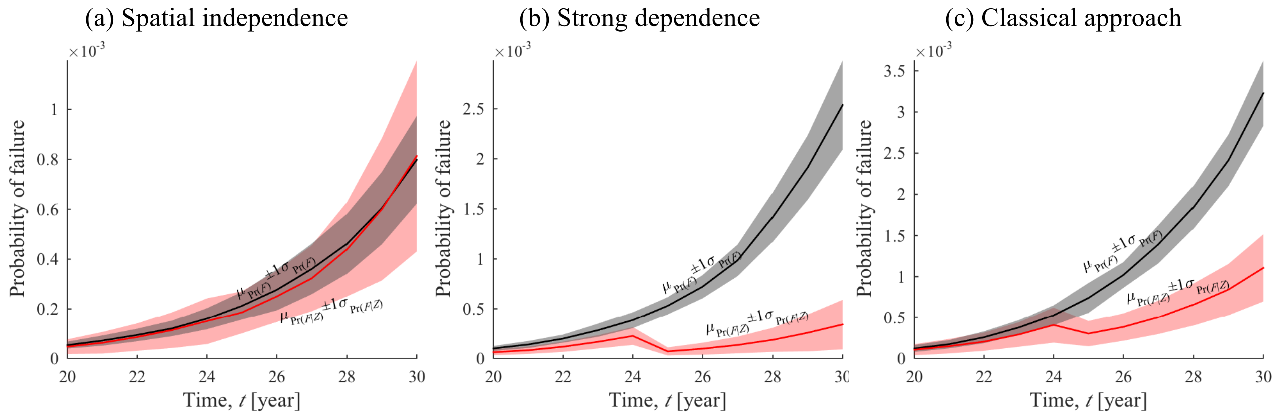

Figure 6 Sampling variability in the PoF estimates for different spatial dependence models. The solid line shows the mean estimate, and the shaded areas indicate ± one standard deviation.

Table 2: C.o.V of PoF at selected years

Time [yr]	Independence		Weak dep.		Strong dep.		Classical	
	P(F)	P(F|Z)	P(F)	P(F|Z)	P(F)	P(F|Z)	P(F)	P(F|Z)
24	0.27	0.62	0.27	0.78	0.21	0.40	0.23	0.53
25	0.26	0.46	0.24	0.79	0.16	0.58	0.25	0.51
30	0.22	0.47	0.21	0.62	0.15	0.73	0.12	0.37

5.2.2 Effect of Number of Inspected Elements

We analyse the effect of number of inspected elements on the conditional failure probabilities. We consider three different sets of inspected elements following Table 3 and Figure 4. The results are presented in Figure 7. Inspecting more elements provides more information, leads to a stronger reduction in uncertainty and on average results in reduced conditional failure probabilities. However, this must not necessarily hold for specific outcomes. For unfavourable inspection outcomes, the PoF can even increase compared to the prior probability of failure.

Table 3: Elements chosen for inspection for three case studies

Inspection variable	# of inspected elements
3 elements	7, 12, 13
6 elements	1, 3, 7, 9, 12, 13
13 elements	1-13

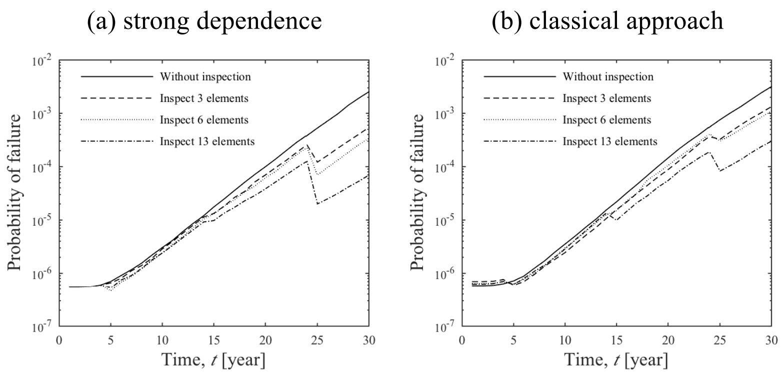

Figure 7: PoF with different number of inspected elements.

6 Summary and Conclusions

We investigate the effect of explicitly modeling the spatial variability of corrosion processes in ship structures in the context of reliability analysis and reliability updating. We apply the BUS approach with subset simulation for reliability updating, which can handle efficiently the large number of random variables arising from the modeling of spatial variability. Even after performing a spectral decomposition of the corrosion parameters, the number of random variables is in the order of 100–200. To further enhance the computational efficiency, we implemented a chronologically reverse computation of the probability of failure, exploiting the fact that the failure domain at time t is a subset of the one at time $t + 1$.

To understand the effect of different assumptions in the spatial modeling of corrosion, we compare the results obtained with different dependence models. Neglecting spatial dependence (either by assuming full correlation or independence) leads to unrealistic assumptions. Furthermore, there are clear differences among the results obtained with the different dependence models, but further numerical investigations are necessary to make general conclusions. It is also necessary to extend the current investigation to consider all critical cross sections in the ship jointly. Such investigations are a straightforward extension of the study presented here.

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