Consistent estimation of station coordinates, Earth orientation parameters and selected low degree Earth's gravity field coefficients from SLR measurements

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Introduction

- Precise geodetic parameters such as positions of crust-fixed geodetic tracking stations (TRF), Earth orientation parameters (EOP) and Stokes coefficients of the Earth’s gravity field model are the basis for positioning, navigation and monitoring of dynamic processes on the Earth and in space.
- Up to now, these parameter groups are estimated separately.
- The SLR constellation used in this study comprises up to 11 geodetic satellites with different orbit characteristics and allows a common estimation of all parameters.

- Which satellite constellation yields optimum results for the estimated parameters?
- Within this presentation, we address the following points:
  - Decorrelation of parameter groups in the normal equation (NEQ)
  - Determination of terrestrial reference frames (TRF) and Earth orientation parameters (EOP)
  - Sensitivity of SLR observations to the coefficients of the Earth’s gravity field model (GFC)
  - Improvement of the estimated GFC by using additional satellites
SLR multi-satellite solution

- SLR observations to up to 11 geodetic satellites in different orbits (1979–2017)
- Combination of single-satellite solutions on the normal equation level

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Inclination [deg]</th>
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<tbody>
<tr>
<td>Etalon-1 (ET-1)</td>
<td>65.3</td>
</tr>
<tr>
<td>Etalon-2 (ET-2)</td>
<td>64.4</td>
</tr>
<tr>
<td>LAGEOS-1 (LA-1)</td>
<td>109.8</td>
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<tr>
<td>LAGEOS-2 (LA-2)</td>
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<tr>
<td>Ajisai (AJI)</td>
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</tr>
<tr>
<td>LARES (LRS)</td>
<td>69.5</td>
</tr>
<tr>
<td>Starlette (STA)</td>
<td>49.8</td>
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<td>WESTPAC (WP1)</td>
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<td>BLITS (BTS)</td>
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<td>Stella (STE)</td>
<td>98.4</td>
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<tr>
<td>Larets (LTS)</td>
<td>98.0</td>
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</table>

- Comparison of the 4-satellite standard solution to several 5-satellite solutions and a solution combined from all 11 satellites
- We used the DOGS software developed at DGFI-TUM

Current ILRS standard setup
Estimated parameters

- Weekly 3-D station Cartesian coordinates
- Daily EOP \((x_{\text{Pol}}, y_{\text{Pol}}, \Delta LOD)\)
- Selected weekly Stokes coefficients up to degree 6

Correlations between parameters

Correlation matrix of combined solution comprising orbit parameters (LA-1 only), GFC and EOP

<table>
<thead>
<tr>
<th></th>
<th>orbital elements and $p_{\text{albe}}$</th>
<th>Stokes coefficients</th>
<th>EOP and SRP scaling factor ($p_{\text{rad}}$)</th>
<th>correlations between parameter groups</th>
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<tr>
<td>1</td>
<td>$p_{\text{albe}}$</td>
<td></td>
<td></td>
<td>(a)</td>
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<tr>
<td>2</td>
<td>$a, e, i, \Omega, M_0, \omega$</td>
<td></td>
<td></td>
<td>(b)</td>
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<td>3</td>
<td>Empirical accelerations</td>
<td></td>
<td></td>
<td>(c)</td>
</tr>
<tr>
<td>4</td>
<td>Stokes coefficients</td>
<td></td>
<td></td>
<td>(d), (e), (f)</td>
</tr>
<tr>
<td>5</td>
<td>$x$-pole coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$y$-pole coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$p_{\text{rad}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\Delta$LOD</td>
<td></td>
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</table>
Correlations between parameters

Correlation matrix of combined solution comprising orbit parameters (LA-1 only), GFC and EOP

- Orbital elements and $p_{albe}$
- Stokes coefficients
- EOP and SRP scaling factor ($p_{rad}$)
- Correlations between parameter groups

LA-1/2, ET-1/2
(standard ILRS constellation)

Reduced correlations of the parameter groups
Correlations between parameters

Correlation matrix of combined solution comprising orbit parameters (LA-1 only), GFC and EOP

- Different Parameter groups are decorrelated

(a) Orbital elements and $p_{\text{albe}}$
(b) Stokes coefficients
(c) EOP and SRP scaling factor ($p_{\text{rad}}$)
(d), (e), (f) correlations between parameter groups

up to 11 satellites
Results: Terrestrial Reference Frames

Weekly 7-parameter Helmert transformation w.r.t. SLRF2014

- Left: reduction of the scatter of TRF datum parameters by up to 35%
- Right: WRMS of the transformation residuals reduced by up to 22%
Results: Terrestrial Reference Frames

Weekly 7-parameter Helmert transformation w.r.t. SLRF2014

- Left: reduction of the scatter of TRF datum parameters by up to 35%
- Right: WRMS of the transformation residuals reduced by up to 22%

WRMS improvement of $t_z$

w.r.t. 4-satellite solution

4-sat. + LRS  8.2 %
11-sat.  34.7 %
Results: Earth Orientation Parameters

Differences of estimated EOP w.r.t. IERS 08 C04

- WRMS of the EOP reduced by up to 26 %
- Weighted mean value reduced by up to 96 %
Results: Earth Orientation Parameters

WRMS improvement of $y_{\text{pole}}$ w.r.t. 4-satellite solution
- 4-sat. + LRS: 0.8%
- 11-sat.: 21.2%

Differences of estimated EOP w.r.t. IERS 08 C04
- WRMS of the EOP reduced by up to 26%
- Weighted mean value reduced by up to 96%
Sensitivity analysis

Sensitivity of the observations to gravity field coefficients

- With an increased number of used satellites, the solution gains sensitivity w.r.t. the Earth’s gravity field.
- 4 satellites: up to d/o 3
- 5 satellites: up to d/o 6 (higher d/o for tesseral coefficients)
- Max. constellation: up to d/o 12

But:

- Not all coefficients can be determined reliably due to remaining correlations!
Correlations between gravity field coefficients

- The correlation between $C_{2,0}$ and $C_{3,0}$ can be reduced significantly by using additional satellites.
- In an 11-satellite constellation, both parameters are decorrelated.

- The correlation between $C_{2,0}$ and $C_{4,0}$ cannot be eliminated.
- Reason: there is no satellite orbit sensitive to only one of these coefficients (geometrical correlation of both coefficients).
Results: Improvement of gravity field coefficients

- Improvement of gravity field coefficients
- Improvement of the mean WRMS values w.r.t. 4-satellite solution

- 5-satellite solution: different improvement patterns depending on the orbit of the additional satellite
- 11-satellite solution: improvement by up to 93 %

### Average WRMS improvement of the GFC w.r.t. 4-satellite solution

- 4-sat. + LRS: 61.7 %
- 4-sat. + LTS: 41.9 %
- 11-sat.: 79.3 %
Scatter of the centered degree-1 Stokes coefficient solutions

- CSR SLR RL05: 5-satellite solution (Cheng et al., 2013)
- AIUB SLR: 8-satellite solution (Sosnica et al., 2015)
- DGFI-TUM SLR: 11-satellite solution, TRF and EOP fixed (Bloßfeld et al., submitted)
- Swenson (2008): GRACE, geophysical model for ocean bottom pressure (OBP)
- Rietbroek (2016): GRACE, OBP, GPS
Summary and Outlook

- The inclusion of additional satellites at various altitudes and orbit inclinations into the SLR solutions reduces correlations between the estimated parameters and increases the sensitivity of the solution to higher degree GFC.

- An 11-satellite setup allows for a reliable estimation of all parameters in a common adjustment.

- TRF datum parameters are improved by up to 35 %, station repeatability - by up to 22 %
- EOP are improved by up to 22 %
- GFC are improved by up to 94 %

- As the 5-satellite setup including LARES performs well, these results support the plan to include this satellite into the ILRS standard setup.

- The DGFI-TUM solution of the geocenter motion derived from 11-satellite SLR constellation is comparable to other geocenter motion solutions derived from SLR data.

A publication to the subject:
Bloßfeld et al.: “Consistent estimation of geodetic parameters from SLR satellite constellation measurements”, submitted to “Journal of Geodesy” (in review)
Backup slides
### Summary

<table>
<thead>
<tr>
<th></th>
<th>4-sat. (ref.)</th>
<th>11-sat. value [%]</th>
<th>4-sat. + AJI value [%]</th>
<th>4-sat. + STA value [%]</th>
<th>4-sat. + STE value [%]</th>
<th>4-sat. + LTS value [%]</th>
<th>4-sat. + LRS value [%]</th>
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<td>11.8</td>
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<td>0.043</td>
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<td>41.9</td>
<td>61.7</td>
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</tbody>
</table>
Correlations between parameters

(1) \( p_{albe} \)
(2) \( a, e, i, \Omega, M_0, \omega \)
(3) Empirical accelerations
(4) Stokes coefficients
(5) \( x \)-pole coordinates
(6) \( y \)-pole coordinates
(7) \( P_{rad} \)
(8) \( \Delta \text{LOD} \)

Empirical accelerations mathematically correlated
Correlations between parameters

- High correlation between certain gravity field coefficients due to mathematical modelling.
Correlations between parameters

- High correlations between pole coordinates and LOD parameters

1. \( p_{\text{hel}} \)
2. \( a, e, i, \Omega, M_0, \omega \)
3. Empirical accelerations
4. Stokes coefficients
5. \( x \)-pole coordinates
6. \( y \)-pole coordinates
7. \( P_{\text{rad}} \)
8. \( \Delta \text{LOD} \)