Reliability and Stability of VLBI-derived Sub-Daily EOP Models

Thomas Artz [1], *Sarah Böckmann* [1], *Laura Jensen* [1], *Axel Nothnagel* [1], *Peter Steigenberger* [2]

[1] *Institut für Geodäsie und Geoinformation, Universität Bonn*

[2] *Institut für Astronomische und Physikalische Geodäsie, Technische Universität München*

Contact author: Thomas Artz, e-mail: `artz@igg.geod.uni-bonn.de`

Abstract

Recent investigations have shown significant shortcomings in the model which is proposed by the IERS to account for the variations in the Earth's rotation with periods around one day and less. To overcome this, an empirical model can be estimated more or less directly from the observations of space geodetic techniques.

The aim of this paper is to evaluate the quality and reliability of such a model based on VLBI observations. Therefore, the impact of the estimation method and the analysis options as well as the temporal stability are investigated.

It turned out that to provide a realistic accuracy measure of the model coefficients, the formal errors should be inflated by a factor of three. This coincides with the noise floor and the repeatability of the model coefficients and it captures almost all of the differences that are caused by different estimation techniques. The impact of analysis options is small but significant when changing troposphere parameterization or including harmonic station position variations.

1. Introduction

In the analysis of Very Long Baseline Interferometry (VLBI) observations, a model is used to correct the delays for inter-day Earth's orientation variations. When different models are used, differences in the Earth orientation parameters (EOPs) –especially the rates– with an annual (and semi-annual) signature appear. Their amplitudes can easily reach the 100 μas level.

The International Earth Rotation and Reference Systems Service (IERS) recommends a model based on an ocean tidal model plus components originating from nutation (Tab. 8 and Tab. 5 of the IERS Conventions 2003 [3]). However, VLBI is sensitive to the integral effect of all variations at each tidal line, including e.g., the tidal and thermally driven S1-excitations. Thus, the IERS model is not sufficient. Beside the dominating oceanic part, other forces should be applied, e.g. tidal variations in the atmosphere or variations of non-tidal origins. As these are not completely modelled physically, the next best option is to estimate a model based on space geodetic techniques.

Several investigations were performed, e.g., on the basis of observations of VLBI or Global Navigation Satellite Systems. However, the results are not homogeneous as different observations and solution set-ups were used. A model for the sub-daily variation of the EOPs (sub-daily EOP model) is composed of poly-harmonic functions for polar motion (PM) and universal time (UT1):

$$\Delta X(t) = \sum_{j=1}^{n} -p_j^c \cos\psi_j(t) + p_j^s \sin\psi_j(t) \tag{1}$$

$$\Delta Y(t) = \sum_{j=1}^{n} p_j^c \sin\psi_j(t) + p_j^s \cos\psi_j(t) \tag{2}$$

$$\Delta UT1(t) = \sum_{j=1}^{n} u_j^c \cos\psi_j(t) + u_j^s \sin\psi_j(t) \quad (3)$$

where n describes the number of tidal terms of the model and $\psi_j(t)$ are the corresponding angular arguments. The model coefficients p_j^c, p_j^s, u_j^c and u_j^s can be estimated directly from the VLBI observations. This approach has been, used, e.g., by [2]. Another possibility –applied, e.g., by [5]– is to solve for highly-resolved EOP time-series in a first step, to generate the left hand side of eq. (1)-(3). In a second step, these time-series are used as pseudo-observations to solve for the model coefficients. Finally, a solution on the normal equation (NEQ) level can be performed. In this study, NEQs are, therefore, built as in the first step of the solution level approach. However, this equation system is not solved directly. The NEQs are transformed to change the parameterization from highly-resolved EOPs to the model coefficients. Subsequently, all modified NEQ systems are added together and solved. The method of parameter transformation is described, e.g., in [1].

To assess the stability of such empirical sub-daily EOP models, the temporal behavior of the estimated model terms is analyzed in this paper. Furthermore, the noise floor and the formal errors are investigated to define the precision of the estimated model. In addition, several solution set-ups and estimation techniques are compared to describe the reliability of VLBI-derived models.

2. Solution description of the standard approaches

Altogether, three solutions have been performed with almost identical features: for all three approaches, the basic parameterization and modeling is identical. The celestial and terrestrial reference frames are estimated. Axis offsets are estimated if no information based on local surveying is available. Furthermore, the zenith wet delays (ZWD) are estimated with a temporal resolution of 20 min while troposphere gradients are resolved with a 6 h resolution. Stabilizing rate constraints of 50 ps/h are imposed on ZWDs. For the gradients, rate constraints of 2 mm/d and offset constraints of 0.5 mm are added. The nutation is fixed to estimates from a global VLBI solution.

The only difference of the three solutions is characterized by the method to derive the coefficients of the sub-daily EOP model. The coefficients of the observation-level model as well as an hourly EOP solution are estimated in a global solution with Calc/Solve [4], i.e., simultaneously with all other parameters. Subsequently, the time series of hourly spaced EOPs is used as pseudo observations in a separate estimation process to derive the model on the solution level. For the solution on the NEQ-level, session-wise NEQs are generated with Calc/Solve and then transformed as described above. These NEQs are, in turn, added to a global solution and the reference frames and the model are estimated simultaneously as well.

3. Stability and accuracy of the model coefficients

To investigate the stability of VLBI-derived models, the most stable solution should be found first. To achieve such a kind of solution, altogether 25 solutions are calculated: the first with observations only from the year 2009, then the observations of 2008 are added for the second solution and so on. Finally, the last solution is calculated with all observations from 1984 to 2009. For solutions with observations of less than 18.6 years, terms that differ from each other by less than one cycle in the considered time span are constrained as described by [2].

By analyzing two subsequent solutions, a solution over a time span of 14 years is considered

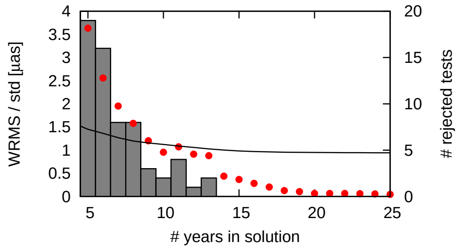

Figure 1. Mean standard deviation (solid line) and WRMS of the differences of one limited solution to the prior one (dots); the number of terms that are different from those in the prior solution with a reliability of 95% for PM (bars).

to be sufficiently stable. The mean standard deviation of the model terms' differences is almost constant from there on (see Fig. 1). Furthermore, the WRMS of the model differences is clearly below the standard deviations. Finally, a simple test indicates that with more than 13 years of data adding one more year does not yield significant model differences. The grey bars in Fig. 1 indicate how many coefficients differ between two subsequent solutions with a probability of 95%.

To test the stability, 13 models have been estimated with 14 years of observations –starting in 1984– each shifted by one year. For every coefficient, a WRMS value of the difference to the mean value has been calculated. A large WRMS of an individual coefficient is caused by a bad repeatability/stability indicating that this particular coefficient cannot be well determined or is not sufficiently described by a constant amplitude. Figure 2 shows these WRMS values for the PM and UT1 terms. For PM, the repeatabilities are in general below 3 μas but there are some exceptions. These are the M2 coefficients in the retrograde and the K2 coefficients in the prograde semi-diurnal band. In the prograde diurnal band, bigger deviations appear at K1 and its sidebands, 2Q1 and Q1. For UT1, the repeatability is below 0.2 μs for most of the terms. Here, bigger differences can be seen at K1 and K2 and their sidebands. The presence of the sidebands –which would not be separable from the major terms without constraints in this limited time span– might be the reason for the lower stability of these coefficients. Furthermore, for more than one third of the coefficients, a linear evolution can be recognized in the time series. This leads to the suggestion that the estimated coefficients are not constant in time. Besides the information on the repeatability of the estimated terms, these values serve as an additional accuracy measure. This empirically derived standard deviation is approximately three times larger than the formal errors of the least squares adjustment and, thus, indicates too optimistic formal errors.

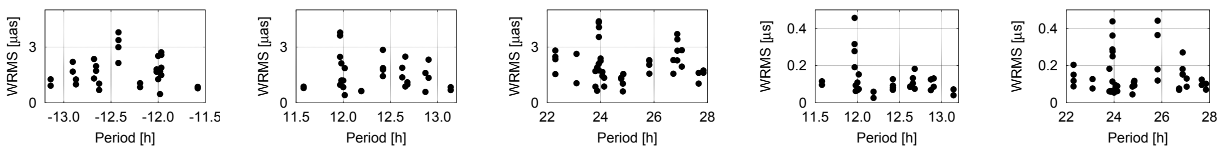

Figure 2. WRMS values of the estimated coefficients. From left to right: retrograde and prograde semi-diurnal PM, diurnal PM, semi-diurnal and diurnal UT1.

A final measure for the accuracy can be derived by indicating the noise floor of the complete model (not shown here). This noise floor can be evaluated by adding some terms to the model, for which no impact on the EOPs is expected. Following [2], nine terms have been chosen for the PM model and six for UT1. The semi-diurnal band exhibits a lower noise level compared to the diurnal band. For the complete solution, this level is below 6 μas (PM) and 0.35 μs (UT1). For both, the term with a period of 27.04 h is significantly larger compared to the other ones.

4. Reliability of the estimated coefficients

As shown in Sec. 3, almost all coefficients can be estimated producing stable results and the accuracy of the model coefficients can be reliably described by three times the formal errors. However, this does not yet comprise any information on the reliability of these terms. For this purpose, the impact of changes in the solution set-up and the method of estimating the sub-daily EOP model can be analyzed.

4.1. Impact of Analysis Options

Changes of the in the analysis set-up, which are not related to the parameterization or modeling of the EOP, should not change the estimated coefficients significantly. Only modifications in the handling of the station or source positions might affect the estimated model. Especially the variations in the settings which are not based on common rules is of interest as this describes the impact of the analyst's noise.

Most modifications as fixing the reference frames or nutation to results from a formerly performed global VLBI solution do not lead to significant changes of the model. Moreover, using only VLBI observations since 1990 has no recognizable influence. Furthermore, extending the resolution of the ZWD from 20 min to 60 min and the one of the troposphere gradients from 6 h to 24 h, leads to significant differences which are shown in Fig. 3. It has to be mentioned that the higher resolution for the troposphere leads to a slightly better agreement with the IERS model (not shown here) and changing ZWD or the gradients alone does not have a comparable impact.

The biggest effect results from the introduction of non-linear station motion. Calc/Solve provides the opportunity to estimate harmonic station variations. This has been done for the stations with sufficient observations for annual (Sa), semiannual (Ssa), diurnal (S1) and semi-diurnal (S2) components. Subsequently, these station motions are used as a priori information for the global solution to estimate the EOP model. This procedure leads to significant differences of the S1 and S2 coefficients which are shown in Fig. 3 for PM exemplarily. Obviously, station motions are able to absorb EOP variations and vice versa.

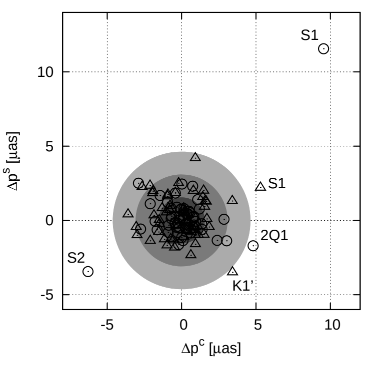

Figure 3. Differences of the PM model terms of the standard solution to a solution where harmonic site positions (circles) or a different parameterization of the troposphere (triangles) is used. The grey colored areas denote 1 (dark) over 2 to 3 σ (light).

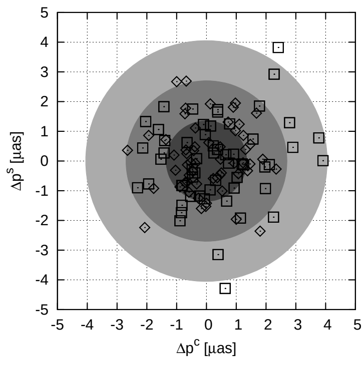

Figure 4. PM model differences of the observation level approach to the NEQ (diamonds) and solution (squares) level approaches.

4.2. Model Comparisons

To assess the impact of the model estimation technique, the NEQ and solution level approaches are compared to the observation level approach. The latter one is the most strict one as the information of each observation is preserved. For the two other approaches, a discretization to hourly spaced EOPs is done first, thus, information is lost. The advantage of the NEQ above the solution level approach is the conservation of the full variance and covariance information.

The differences are shown in Fig. 4. These are, in general, below the 3-σ level and thus, in the range of the model precision as shown in Sec. 3. Furthermore, the assumptions concerning the strictness of the estimation procedure are fulfilled as the model on the observation level is slightly closer to the one on the NEQ level compared to the solution level.

5. Conclusions

The general stability and accuracy of coefficients of a sub-daily EOP model is in the range of three times the formal errors. This is confirmed by the repeatability of the model coefficients from shifted stable (12-year) solutions and by the noise floor of the complete model. For some terms, less stable results are achieved. These are terms, for which sidebands are not separable from the major term with observations of less than 18.6 years.

Furthermore, different estimation techniques have been compared. The approach on the observation level and on the NEQ level do comprise almost no differences within the derived accuracy level. The differences of the solution level approach are only slightly bigger.

Finally, it was shown that the impact of various analysis options is low but not negligible. Especially the troposphere parameterization leads to significant model differences. Furthermore, harmonic station position variations are able to absorb EOP variations at the same period or vice versa. It's not yet clear, whether it is correct to consider such station motions or not.

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