Optimized Combination of Conventional and Constrained Massive MIMO Arrays

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***Abstract*—A combination of low cost constrained massive multiple-input-multiple-output (MIMO) and conventional arrays for constructing receiver signal points is investigated. The constrained radio frequency (RF) front ends are limited to on/off switching of antenna elements. A few full RF frontends are shown to compensate for the signal errors of the constrained RF frontends for various scenarios. An algorithm for such a hybrid RF (HRF) system is developed that optimizes the performance with respect to the mean square error of the constructed receiver signals for a Rayleigh fading channel model.**

I. Introduction

Massive MIMO promises to deliver high spectral efficiency with low energy consumption [1], [2]. The theory for massive MIMO is studied in [3], [4] and implementation issues such as hardware costs and dimension are assessed in [5]. An uplink system with 1 bit analog-to-digital converters (ADC)s at the receiver approaches capacity when using QPSK [6], [7], [8], [9]. Several approaches simplify the implementation of massive MIMO while preserving some of the gains. For example, a simple idea is to use only one antenna at the base station (BS) at a certain time instant to transmit [10] so that the transmitter (Tx) needs only one RF-chain. As another example, hybrid beamforming reduces the total number of RF-chains to decrease the cost and power consumption of the BS [11].

We propose a design where a large number of low cost, constrained RF-chains (CRF)s cooperate with a small number of full RF-chains (FRF)s. In the simplest case, the CRFs use on/off switching, thereby requiring minimum functionality like a single bit DAC, a power amplifier (PA) with relaxed linearity constraints, to achieve high power-added efficiencies (PAE), less stringent filter requirements, etc.. One interesting use case is to add booster arrays with a large number of CRFs to existing macro sites - the FRFs - to form a HRF massive MIMO array. This is depicted in Figure 1. An HRF system can compensate for the non idealities of the CRFs, as long as the number of FRFs is larger than that of the served data streams. Compared to hybrid analogue digital beamforming one can avoid an analogue network and retain full precoding flexibility.

Here we investigate systems with one-bit DACs. We develop an algorithm to optimize an HRF system and show simulation results for mean squared error (MSE) and power consumption. We focus on the best combining strategy of CRF and FRF frontends to generate single QAM time samples under the assumption of Rayleigh fading channels.

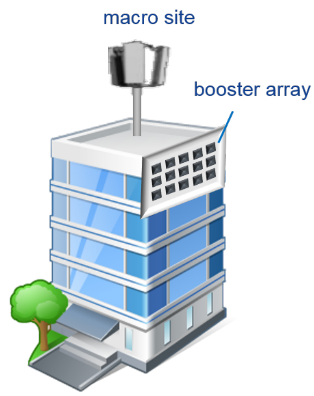

Fig. 1: Use cases for the CRFs.

Notation: We use boldface lowercase and uppercase letters to denote column vectors and matrices, respectively. For a matrix $\boldsymbol{A}$, we denote its transpose and the Hermitian transpose as $\boldsymbol{A}^T$ and $\boldsymbol{A}^H$, respectively. The i-th column of a matrix $\boldsymbol{A}$ is denoted as $\boldsymbol{a}_i$ and the i-th entry of a vector $\boldsymbol{a}$ as a_i.

II. Optimized HRF Signal Generation

A. System Model

Consider a one-cell downlink with K single antenna users and one base station (BS) equipped with M antennas with $M \gg K$. The discrete-time complex received signal is

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{K\times M}$ is the channel matrix from the BS to the K users. The entry h_{ij} of $\boldsymbol{H}$ is the channel coefficient between the j-th antenna of the BS and the i-th user equipment (UE). We consider Rayleigh fading where the channel coefficients h_{ij} are independent $\mathcal{CN}(0,1)$ random variables. $\boldsymbol{x} \in \mathbb{C}^{M\times 1}$ is the transmit signal. The entries of $\boldsymbol{n} \in \mathbb{C}^{K\times 1}$ are independent circularly symmetric Gaussian random variables. For the HRF scheme we define the sub-matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, where $\boldsymbol{H}_1$ contains the first M_1 columns of $\boldsymbol{H}$ representing the channel coefficients of the CRFs and $\boldsymbol{H}_2$ containing the last M_2 columns of $\boldsymbol{H}$ representing the FRFs. The transmit signal $\boldsymbol{x}$ is constrained as follows

$$\begin{bmatrix} x_1, & \dots, & x_{M_1} \end{bmatrix} \in \left\{0, \frac{1}{\sqrt{M_1}}\right\} \tag{2}$$

$$\begin{bmatrix} x_{M_1+1}, & \dots, & x_M \end{bmatrix} \in \mathbb{C}. \tag{3}$$

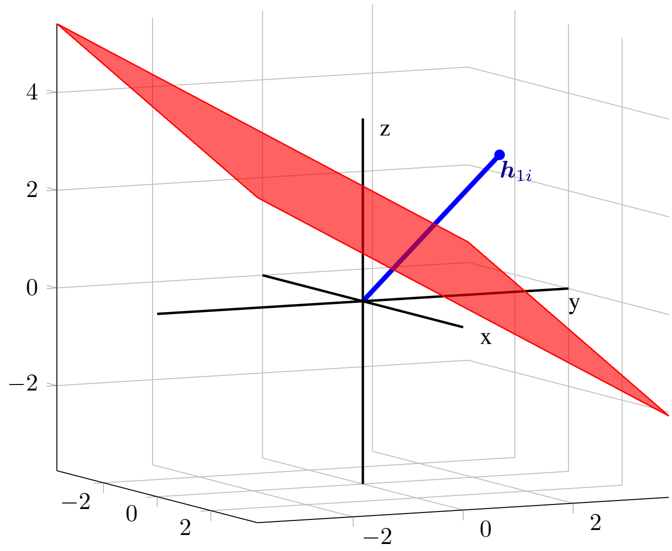

Fig. 2: Subspace spanned by 2 FRF antennas in a scenario with 3 UEs.

The constraint in (2) is needed to fulfill the power constraint when all CRF antennas transmit. In this case the power constraint is 1.

The concept of over the air signal generation aims at generating the desired signal (QAM symbols or time samples of an OFDM symbol) at the receiver by transmitting signals from antenna elements such that, when multiplied by the channel response, they result in the desired signal at the receiver [12]. For multiple receivers, the desired signals need to be generated simultaneously at all the receivers. The signal samples from all the UEs at any particular time instant can be represented by a signal (vector) of dimension n. Switching on any one of the transmitter antenna elements results in a signal point in this n-dimensional space which corresponds to the channel response at that time instant. A knapsack-like algorithm can be used to find an antenna combination to get close to the desired signal vector [12].

B. Algorithm

Adding FRF-chains to reduce the error after applying the knapsack-like algorithm leads to a transmit power saving and/or a reduction in the MSE. Our approach is as follows. Instead of sequentially using the knapsack-like algorithm and then reducing the remaining error with the FRFs [12] we optimize both in one step. When we assume no power constraints for the FRFs we can view the points from where we can perfectly achieve the desired point as the subspace spanned by the columns $\boldsymbol{H}_2$. An example with real numbers is depicted in Figure 2. Here the assumptions are as follows. We have 3 UEs and therefore 3 dimensions. There are only 2 FRFs, therefore only the red subspace can be achieved by these antennas. The subspace is created by taking the span of the basis for the channel matrix $\boldsymbol{H}_2$ and shifting it by the desired signal vector of the UEs. $\boldsymbol{h}_{1i}$ is the received vector created by activating one CRF antenna. The closest distance is found by projecting the point onto the subspace. The knapsack-like algorithm can be extended to use this information. Instead of choosing the antenna combination to minimize the distance to a desired point we minimize the distance to a subspace.

Formally the algorithm can be described as follows:

We want to find the minimum distance d between a point $\boldsymbol{z}$ in $\mathbb{C}^K$ and a subspace in $\mathbb{C}^K$ with basis $\boldsymbol{A} \in \mathbb{C}^{K\times M_2}$:

$$d = \min_{\boldsymbol{\alpha}} ||\boldsymbol{z} - \boldsymbol{A}\boldsymbol{\alpha}|| \tag{4}$$

$\boldsymbol{A}$ can be calculated from $\boldsymbol{H}_2$ with standard methods, e.g., LU decomposition. The minimum distance is calculated using the singular value decomposition (SVD) [13]

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}$$

where $\boldsymbol{U} \in \mathbb{C}^{K\times K}$ and $\boldsymbol{V} \in \mathbb{C}^{M_2\times M_2}$ are unitary and $\boldsymbol{\Sigma} \in \mathbb{C}^{K\times M_2}$ contains M_2 nonnegative values $\sigma_1, \dots, \sigma_{M_2}$ on the diagonal. The objective of (4) can be rewritten as

$$||\boldsymbol{z} - \boldsymbol{A}\boldsymbol{\alpha}|| = ||\boldsymbol{z} - \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}\boldsymbol{\alpha}|| \tag{5}$$

$$= ||\boldsymbol{w} - \boldsymbol{\Sigma}\boldsymbol{o}|| \tag{6}$$

with $\boldsymbol{w} = \boldsymbol{U}^*\boldsymbol{z}$ and $\boldsymbol{o} = \boldsymbol{V}\boldsymbol{\alpha}$. Writing out the expression in vector form we have

$$\boldsymbol{w} - \boldsymbol{\Sigma}\boldsymbol{o} = \begin{bmatrix} w_1 - \sigma_1 o_1 \\ \vdots \\ w_{M_2} - \sigma_{M_2} o_{M_2} \\ w_{M_2+1} \\ \vdots \\ w_K \end{bmatrix}. \tag{7}$$

To minimize the distance, the entries of $\boldsymbol{o}$ are chosen such that the first M_2 entries of (7) are zero. Therefore the minimum distance is

$$d = \sqrt{|w_{M_2+1}|^2 + \dots + |w_K|^2} \tag{8}$$

$$= \sqrt{|\boldsymbol{u}_{M_2+1}^H \boldsymbol{z}|^2 + \dots + |\boldsymbol{u}_K^H \boldsymbol{z}|^2} \tag{9}$$

where $\boldsymbol{u}_i$ is the i-th column of $\boldsymbol{U}$. In our case $\boldsymbol{z} = \boldsymbol{h}_{1i} - \boldsymbol{err}$, where $\boldsymbol{h}_{1i}$ is the i-th column of $\boldsymbol{H}_1$ and $\boldsymbol{err}$ is the remaining error vector. The subtraction of the remaining error vector is equivalent to shifting the origin of the subspace created by the columns of $\boldsymbol{H}_2$ to the remaining error vector. In pseudo code the knapsack-like algorithm with the subspace distance metric is shown in Algorithm 1. The algorithm is initiated by multiplying $\boldsymbol{H}$ with $\sqrt{P}$, the power constraint, and initializing the error with the desired receive vector $\boldsymbol{u}$, as all antennas are turned off at the beginning. In the next step the algorithm calculates the index of the column with minimal Euclidean distance to the subspace. If the updated error is larger than the new error, the algorithm stops. Otherwise the error vector is updated, the antenna corresponding to the column of $\boldsymbol{H}$ is activated and the column of $\boldsymbol{H}$ is set to "not a number" (NaN) in order to be ineligible in the following iterations. The algorithm has a complexity of $O(M^2 \log(M) + M^2 K)$.

Algorithm 1 KS with subspace distance metric

$\boldsymbol{H_1} = \boldsymbol{H_1}/\sqrt{M_1}$
$\boldsymbol{err} = \boldsymbol{u}$
for $i = 1 : M_1$ **do**
  for $j = 1 : M_1$ **do**
    $\boldsymbol{z}_j = \boldsymbol{h}_{1j} - \boldsymbol{err}$
  end
  j^*= argmin $\sqrt{|\boldsymbol{u}_{M_2+1}^H \boldsymbol{z}_j|^2 + \cdots + |\boldsymbol{u}_K^H \boldsymbol{z}_j|^2}$
  if $||\boldsymbol{err}|| < ||\boldsymbol{err} - \boldsymbol{h}_{1j^*}||$ **then** stop;
  $\boldsymbol{err} = \boldsymbol{err} - \boldsymbol{h}_{1j^*}$
  $\boldsymbol{x}(j^*) = \frac{1}{\sqrt{M}}$
  $\boldsymbol{h}_{1j^*} = NaN$
end

TABLE I: General system settings for the simulations.

# BS	1
# BS antennas (M)	20-100
BS sum power constraint (P)	1 (CRF), ∞ (FRF) and 1 for CRF+FRF
Noise power	0
# UE (K)	10
# UE antennas	1
Input alphabet	256 QAM
Quantization scheme	1 bit amplitude, 0 bit phase
Channel model	Rayleigh

III. Simulation Results

We compare our schemes under the Rayleigh fading channel model for various scenarios. We use Monte Carlo simulations to compare the schemes in terms of MSE and transmit power.

Consider the combination of many CRF-chains and some FRF-chains. For the first part we do not impose any power constraints on the FRF-chains, hence we can perfectly create the desired symbol at the receiver as long as the number of FRFs is larger than the number of UEs. Here we will only consider the more interesting case where the number of FRFs is smaller than the number of UEs. The underlying motivation is that FRFs are more costly compared to CRFs and their number should be therefore as small as possible. At the same time a high spectral efficiency requires a high number of spatially multiplexed UEs. The simulation parameters are given in Table I.

Figure 3 shows the MSE averaged over all UEs and many channel realizations for a Rayleigh fading scenario with 10 UEs. *KS* is the algorithm introduced in [12], where only low cost RFs are used to create the desired symbol points. Note that we do not use any phase modulation in this scenario. The red curves denoted with *seq* correspond to the case where we first use the KS algorithm and then minimize the remaining error with the FRFs by utilizing the least squares solution. The number of antennas on the x-axis corresponds to the total number of antennas, therefore *seq* with 4 FRFs has 16 CRF antennas when the total number is 20. Finally, the yellow curves show the performance of the extended knapsack algorithm, which is denoted as *opt*.

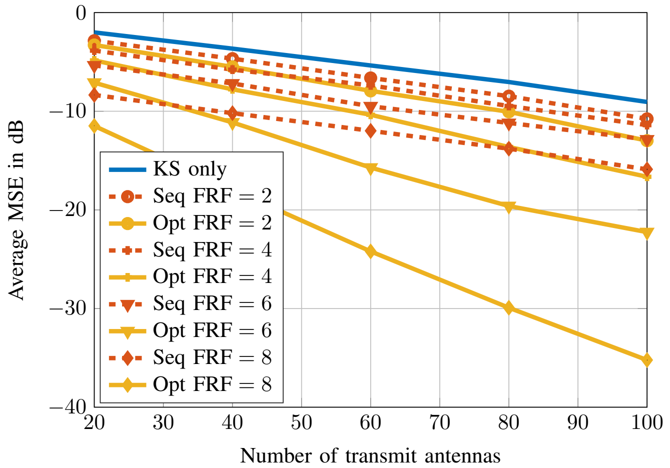

Fig. 3: KS algorithm, sequential algorithm (seq) and optimized algorithm (opt) for a Rayleigh fading channel with 10 UEs and no power constraints for the FRFs.

Increasing the number of FRFs in the *seq* case brings only minor gains. In comparison, the *opt* case improves significantly with the number of FRFs. For 256 QAM an MSE larger than 24 dB is needed to achieve a low error probability. With 6 FRFs, 80 total antennas are needed, which is a 12 dB gain compared to the KS only case. Figure 4 shows the total transmission power for the CRFs and FRFs combined. The power of the FRFs is unconstrained in this case and might become large as the signal vectors of two FRFs might partially cancel each other. The sum transmit power for the optimized scheme is therefore considerably larger than the transmit power for the sequential scheme. Moreover, the transmit power does not decrease when the number of CRFs increases. We conclude that the performance increase comes at the cost of higher power consumption.

Next, we compare the three schemes when a total power constraint of 0 dBW is applied for the antenna array. For the optimized scheme we calculate the projection onto the subspace and normalize it with the maximum transmit power. Then we calculate the MSE. This scenario is depicted in Figure 5. With only 2 FRFs the gains are minor and it would probably be beneficial to introduce scheduling instead of serving all 10 UEs at the same time and on the same frequency. For 8 FRFs we observe a larger performance gain when we compare the sequential and the optimized schemes. Compared to *KS* we gain about 4 dB and reduce the total transmit power by 12 dBW. The power is depicted in Figure 6. Now we see that for both combining schemes the power decreases when a larger number of CRFs is available.

IV. Conclusion

We have proposed an optimized algorithm for massive MIMO antenna arrays. Hybrid analogue digital beamforming - the conventional alternative - might become either complex or face performance limitations. An HRF solution that combines

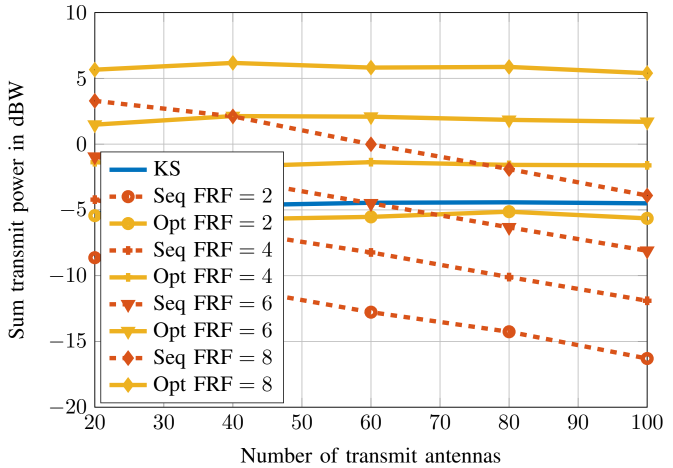

Fig. 4: KS algorithm, sequential algorithm (seq) and optimized algorithm (opt) for a Rayleigh fading channel with 10 UEs and no power constraints for the FRFs.

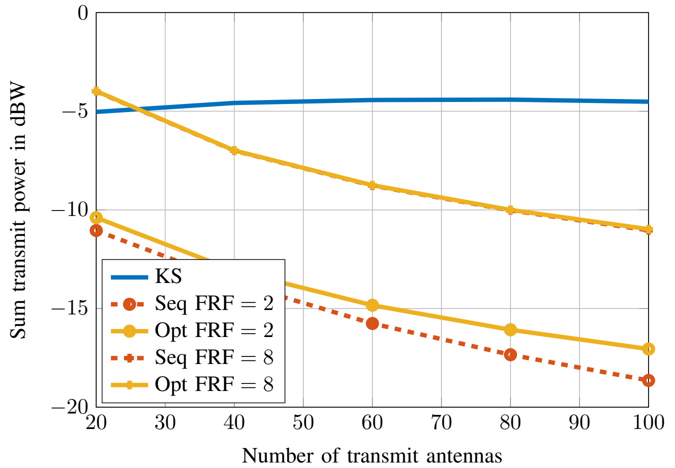

Fig. 6: KS algorithm, sequential algorithm (seq) and optimized algorithm (opt) for a Rayleigh fading channel with 10 UEs and power constraints for the FRFs.

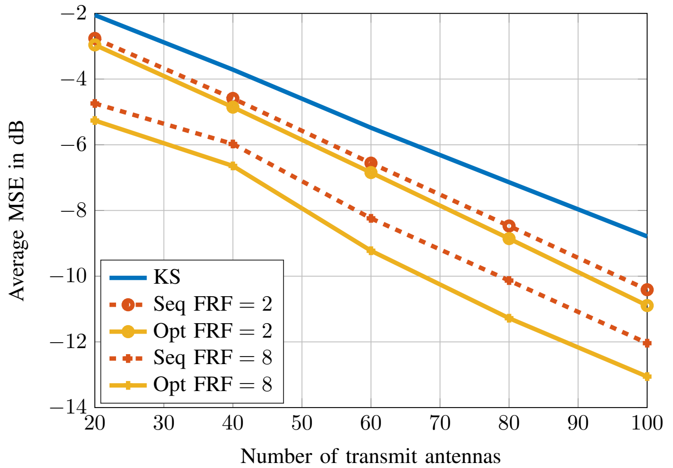

Fig. 5: KS algorithm, sequential algorithm (seq) and optimized algorithm (opt) for a Rayleigh fading channel with 10 UEs and power constraints for the FRFs.

a large number of low-cost low-size CRF-chains with a limited number of FRF-chains maintains full multi-user MIMO scheduling flexibility as well as massive MIMO benefits like improved energy efficiency. We proposed a KS-like algorithm that achieves near optimal results with respect to the MSE. The system can achieve MSE values supporting the highest modulation schemes used in LTE. The combined scheme can improve the MSE even when the number of FRF-chains is smaller than the number of UEs. This is an interesting use case for booster antennas added to current macro sites, having only four to eight FRF-chains. Future work will focus on channel estimation, further reductions in processing overhead, optimum combining schemes and system level aspects.

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