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Vertical price dependence structures: copula-based evidence from the beef supply chain in the USA

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Abstract

The objective of this paper is to assess the degree and the structure of price dependence along the beef supply chain in the USA. This is pursued using the statistical tool of copulas, and monthly rates of price changes over the period 2000–2013. The analysis considers two pairs of markets, namely the pair farm–wholesale and the pair wholesale–retail. It turned out that price co-movement for the pair farm–wholesale is relatively strong and it is described with the Gumbel–Clayton copula, while that for the pair wholesale–retail is rather weak and it is described by the Gumbel copula. The empirical findings point to the existence of price transmission asymmetry, which is much more important for the pair wholesale–retail.

Keywords: price dependence, US beef, copulas

JEL classification: Q11, C13

1. Introduction

The analysis of vertical price linkages has been an important topic in agricultural and food economics over a long period of time. Price inertia and incomplete pass-through may be indicators of market inefficiency and as such they attract the attention of economists and policy-makers. In well-functioning (integrated) markets, price shocks in any market level are transmitted to other market levels; primary producers benefit from price increases at the wholesale and the retail levels and final consumers benefit from cost reductions upstream. The efficiency of a food supply chain is, therefore, crucial to maintain a sustainable distribution of value added and of benefits among the stakeholders (farmers, processors, wholesalers, retailers and consumers).

Empirical investigations of vertical price transmission have been conducted with a variety of quantitative tools, ranging from simple correlation analysis and

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regression models to recently developed econometric approaches such as the linear error correction model (ECM), the asymmetric/non-linear ECM, the threshold vector ECM, the smooth transition cointegration model and the Markov-switching ECM. Most researchers have focused their attention to potential asymmetries in the speed of price transmission (e.g. Goodwin and Holt, 1999; Goodwin and Harper, 2000; Ben-Kaabia and Gil, 2007; European Commission, 2009). Fewer studies allowed for asymmetries in both the speed and the magnitude of price transmission (e.g. Lass, 2005; Gervais, 2011). Although the findings appear to depend on the methods employed, the time period considered and the type of data used, the majority of earlier empirical works has obtained some evidence of asymmetric price transmission regarding either its speed or its magnitude or even both. As a rule, price increases at the primary (farm) level have been found to be transmitted to the wholesale and to the retail levels faster and/or more fully than price decreases.

Against this background, the objective of the present work is to investigate vertical price transmission in the US beef industry using monthly prices at three market levels (farm, wholesale and retail) over the period 2000:1 to 2013:6 and the statistical tool of copulas. The beef supply chain in the USA connects the agricultural sector, the food processing sector and the food distribution sector. It is quite complex and heterogeneous exhibiting a diversity of products, enterprises and markets. More importantly, it has often times come under the scrutiny of antitrust authorities because of buyers' market strategies (e.g. captive supplies) as well as of very high levels of concentration in beef processing and packing (Ward, 2010).

Copulas offer an alternative and a very flexible way to analyse price dependencies/co-movements, particularly during extreme market events (upturns and downturns). The use of copulas for modelling linkages among random processes gained momentum in the late 1990s especially in engineering, risk management and finance, but only very recently has found its way into applied and agricultural economics. Standard tools for the analysis of multivariate structures assume that the marginal distributions belong to the same family (typically the normal) and often that the dependence structure follows a linear relationship. A distinct advantage of copulas is that they allow the joint behaviour of random processes to be modelled independently of the marginal distributions, offering, thus, considerable flexibility in empirical research (e.g. Nelsen, 2006; Patton, 2012). With regard to vertical (or to spatial) price interrelationships, the notion behind employing copulas to characterise co-movement (dependence) is that in well-integrated markets prices move together; specifically, they boom and they crash together. Copulas are especially suitable for modelling the joint behaviour of random processes during extreme events making it possible to assess whether prices are linked with the same intensity at extreme market upswings and downswings (Reboredo, 2011).

There is a number of recent empirical works which have investigated market integration with copulas. Reboredo (2011) assessed price dependence in four regional crude oil markets; Reboredo (2012) examined co-movement between international food (corn, soya beans and wheat) and oil prices; Serra and Gill

(2012) investigated the linkages between biodiesel, diesel and crude oil prices in Spain; Bhatti and Nguyen (2012) focussed on the linkages between oil and stock markets; Wen, Wie and Huang (2012), Czado, Schepsmeier, Min (2012) and Aloui, Hammoudeh and Nguyen (2013) examined dependence between stock markets; Reboredo (2013) investigated dependence between gold and exchange rates and Zimmer (2013) analysed co-movement of housing prices in four US census divisions. To the best of our knowledge, there has been no published work which has examined price linkages along a vertical structure such as the beef supply chain in the USA.

The structure of the present work is as follows: Section 2 contains the analytical framework and Section 3 presents the data, the empirical models and the empirical results. Section 4 offers conclusions.

2. Analytical framework

2.1. Modelling dependence with copulas

The use of copulas to assess general (both linear and non-linear) dependence structures has its roots in Sklar's (1959) theorem according to which a multivariate distribution of a vector of random variables is fully specified by the individual marginal densities and a joining function known as *copula*. In the simple bivariate case, let the joint cumulative density function (cdf) of a pair of random variables (X_1, X_2) be $F(x_1, x_2)$ and the marginal cdfs be $F_1(x_1)$ and $F_2(x_2)$, respectively. Then, Sklar's theorem states that

$$F(x_1, x_2) = C\{F_1(x_1), F_2(x_2)\}, \tag{1}$$

where C is the copula function. Provided that the marginal distributions are continuous, C, F_1 and F_2 are uniquely determined by $F(x_1, x_2)$. Conversely, for any pair (F_1, F_2) and for any copula C, the function F given in equation (1) defines a valid joint cdf for (X_1, X_2) with margins F_1 and F_2.

The copula is a bivariate cdf with uniform margins, $C : [0, 1]^2 \rightarrow [0, 1]$, and it can be obtained from equation (1) as

$$C(u_1, u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)) \tag{2}$$

where F_i^{-1} $(i = 1, 2)$ are marginal quantile functions and u_i are probabilities (quantiles) on U[0,1]. The joint probability density function (pdf) associated with C is obtained as

$$c(u_1, u_2) = \frac{\partial^2 C}{\partial u_1 \partial u_2} = \frac{f(F_1^{-1}(u_1), F_2^{-1}(u_2))}{f_1(F_1^{-1}(u_1))f_2(F_2^{-1}(u_2))} = \frac{f(x_1, x_2)}{f_1(x_1)f_2(x_2)} \tag{3}$$

where f is the joint pdf associated with F, and f_1 and f_2 are, respectively, the marginal pdfs of X_1 and X_2.

From equation (3) it follows that

$$f(x_1, x_2) = c(F_1(x_1), F_2(x_2))f_1(x_1)f_2(x_2). \tag{4}$$

A joint pdf contains information on the marginal behaviour of each random variable as well as on the dependence between them. In $c(F_1(x_1), F_2(x_2))$, each random variable is fed into its own cdf. In this way, all information contained in the marginal distributions is swept away and what is left in c (and in C) is the pure joint information between X_1 and X_2. From relation (4) it is obvious that the copula function fully characterises the co-movement (dependence) of the random variables by capturing the information missing from the marginal distributions to complete the joint distribution (Meucci, 2011).

Using copulas to assess dependence (co-movement) offers a number of distinct advantages. First, just as marginal distributions provide an exhaustive description of the behaviour of two random processes when considered separately, copulas fully and uniquely characterise the dependence structure between them. Second, copulas (due to the converse of Sklar's (1959) theorem) are able to model co-movement independently of the marginal distributions. Third, copulas provide information on the degree as well as the structure of dependence; as known, standard measures of co-movement such as Pearson's correlation coefficient provide information on whether two random processes are linearly related. Copulas, in contrast, allow for more general forms of functional dependence, with linear co-movement being a special case. To express it in a different way, while Pearson's correlation coefficient measures only linear dependence as it only considers second-order moments, the copula, by fully characterising the probabilistic co-movement between stochastic processes, is a flexible tool capable of measuring a variety of dependence structures (linear or non-linear) across the entire joint distribution of the random variables. Fourth, because copulas are based on the ranks of random processes, they are invariant to continuous and monotonically increasing transformations of them.

2.2. Certain bivariate copula families and their implied dependence structures

Given that multivariate stochastic processes may have quite different properties, it is highly desirable for a researcher to have at her (his) disposal a variety of copulas to capture adequately the salient characteristics (e.g. asymmetries and heavy tails) of the processes to be modelled. For Durante and Sempi (2010) a 'good' family of copulas is: (a) interpretable, meaning that its members have a probabilistic interpretation suggesting 'natural' situations where this family could be considered; (b) flexible, meaning that its members are capable of representing many possible types and degrees of co-movement; (c) easy to handle, meaning that the family members are expressed in a closed form or, at least, are easily simulated by means of some known algorithm.

The theoretical work on this topic has led to a large number of copula families with desirable properties. In the following, we discuss only the families employed in the present study which, however, are among those typically considered in economics and finance (e.g. Embrechts, McNeil and Straumann, 2002; Busetti and Harvey, 2011; Reboredo, 2011; Czado, Schepsmeier, Min, 2012; Serra and Gil, 2012; Patton, 2012; Zimmer, 2013).

The Gaussian and the Student-t are members in the family of *elliptical* copulas. The former contains a single dependence parameter, ρ (the linear correlation coefficient corresponding to the bivariate normal distribution). The Student-t copula involves two parameters, namely the linear correlation coefficient and the degrees of freedom (denoted as ν). When $\nu \geq 30$ the Student-t copula becomes indistinguishable from the Gaussian one. The Clayton, the Gumbel and the Gumbel–Clayton are *Archimedean* copulas. The first two involve a single dependence parameter (denoted as θ) whereas the third involves two dependence parameters (denoted as θ_1 and θ_2). The key advantages of the Archimedean copulas over the elliptical ones are: (a) they can be written in explicit forms and (b) they are not restricted to radial symmetry, offering, thus, a great flexibility in modelling different kinds of dependence. The key advantages of the elliptical over the Archimedean ones are: (a) they are more suitable in modelling high-dimensional dependence structures and (b) they can specify different levels of correlation between marginal distributions (e.g. Hofert, 2010; Fang, 2012).

Direct comparisons of dependence parameters across copula families are (in most cases) meaningless. Hence, a number of alternative measures of co-movement have been proposed in the literature. A standard rank-based measure of functional dependence is *Kendall's tau*; it is calculated from the number of concordant and discordant pairs of observations and it provides information on co-movement across the entire joint distribution function (both at the centre and at the tails of it) (e.g. Genest and Favre, 2007).

Often though, information concerning dependence at the tails (at the lowest and the highest ranks) is extremely useful for economists, managers and policy makers. Tail (extreme) co-movement is measured by the *upper*, λ_{U}, and the *lower*, λ_{L}, *dependence coefficients* defined as

$$\begin{aligned}\lambda_{\mathrm{U}} &= \lim_{u\to 1} \mathrm{prob}(U_1 > u | U_2 > u)\\ &= \lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u} \in [0,1]\end{aligned} \tag{5}$$

and

$$\begin{aligned}\lambda_{\mathrm{L}} &= \lim_{u\to 0} \mathrm{prob}(U_1 < u | U_2 < u)\\ &= \lim_{u\to 0} \frac{C(u,u)}{u} \in [0,1].\end{aligned} \tag{6}$$

where λ_{U} measures the probability that X_1 is above a high quantile given that X_2 is also above that high quantile, while λ_{L} measures the probability that X_1 is below a low quantile given that X_2 is also below that low quantile. In other words, the two measures of tail dependence provide information about the likelihood for the two random variables to boom and to crash together, respectively. Note that since λ_U and λ_L in equations (5) and (6) are expressed via copula, certain properties of copulas (e.g. invariance to monotonically increasing transformations of the underlying random processes) apply to tail coefficients as well.

The Gaussian copula is symmetric and exhibits zero tail dependence. That is, irrespective of the degree of the overall dependence, extreme changes in one random variable are not associated with extreme changes in the other random variable. The Student-*t* copula exhibits symmetric non-zero tail dependence (joint booms and crashes have the same probability of occurrence). The Clayton copula exhibits only left co-movement (lower tail dependence), the Gumbel exhibits only right co-movement (upper tail dependence) and the Gumbel–Clayton copula allows for (potentially asymmetric) both right and left co-movement. Table A1 presents the way the parameters of different copula families are related to Kendall's tau and to tail dependence coefficients.

2.3. Testing for time-varying dependence

Before specifying a functional form for a copula, it is informative to test for time-varying dependence, i.e. whether its parameter(s) have remained constant over the time period considered. There has been some empirical evidence that the degree of co-movement between random processes may change with market conditions (tranquil vs turmoil periods). Recently, a number of tests for time-varying dependence have been proposed in the literature (e.g. Manner and Reznikova, 2012; Patton, 2012). Here, we employ the tests of Busetti and Harvey (2011), which are based on the sample τ-*quantics* and the sample τ-*biquantics*, for individual and for bivariate time series, respectively.

Let x_t be the value of an individual time series X at time $t = 1, 2, \ldots, T$. Let also $\tilde{\xi}(\tau)$ be the empirical (sample) quantile of X, with $0 < \tau < 1$. The τ-quantile indicator (τ-quantic) is defined as

$$IQ(\tau)(x_t - \tilde{\xi}(\tau)) = \begin{cases} \tau - 1, & \text{for } x_t < \tilde{\xi}(\tau) \\ \tau, & \text{for } x_t > \tilde{\xi}(\tau) \end{cases} \quad (7)$$

and it has mean zero and variance $\tau(1 - \tau)$. Under the null hypothesis of stationarity, the τ-quantile is constant; under the alternative, it is subject to breaks at unknown points or to gradual but persistent changes. The relevant test statistic is

$$n_\tau(Q) = \frac{\sum_{i=1}^{T} \left(\sum_{i=1}^{t} IQ(x_i - \tilde{\xi}(\tau)) \right)^2}{T^2 \tau(1 - \tau)}, \quad (8)$$

which, under the null hypothesis, follows the Cramer–von Mises (CvM) distribution with one degree of freedom.

Let a bivariate time series $X = (X_1, X_2)$. Let also the empirical copula $C_T(\tau, \tau)$ which gives the proportion of cases both series in pair are less than or equal to particular quantiles, $\tilde{\xi}_1(\tau)$ and $\tilde{\xi}_2(\tau)$, respectively. The τ-biquantic is defined as

$$BIQ(x_{1t} \leq \tilde{\xi}_1(\tau), x_{2t} \leq \tilde{\xi}_2(\tau)) = C_T(\tau, \tau) - I(x_{1t} \leq \tilde{\xi}_1(\tau))I(x_{2t} \leq \tilde{\xi}_2(\tau)) \quad (9)$$

where $I(\cdot)$ is an indicator function. The BIQ has zero mean and (for *i.i.d.* individual series) variance $C_T(\tau, \tau)(1 - C_T(\tau, \tau))$. Under the null hypothesis of stationarity, the quantile τ of the bivariate empirical copula is constant; under the alternative it is subject to breaks or it is changing at a slow but persistent way. The relevant test statistic is

$$n_\tau(BQ) = \frac{\sum_{i=1}^{T} \left(\sum_{i=1}^{t} BIQ_i(\tau)\right)^2}{T^2 C_T(\tau, \tau)(1 - C_T(\tau, \tau))}, \quad (10)$$

which, under the null hypothesis, follows the Cramer–von Mises distribution with one degree of freedom.[1]

3. The data, the empirical models and the empirical results

The data for the empirical analysis are monthly beef prices at the farm, the wholesale and at the retail level of the US beef industry; they refer to the period 2000:1 to 2013:6 and have been obtained from the Economic Research Service of the United Stated Department of Agriculture (ERS-USDA). Figure 1 presents the three price series. We observe that the prices at the farm and at the wholesale level moved, to a large extend, together over time; as a result, the margin in the pair farm–wholesale did not show any clear upward or downward trend. The prices at the retail level, however, diverge from those at the wholesale level and the relevant margin has almost doubled from the earlier to the most recent periods.

Here, we are interested in assessing the degree and the structure of price dependence at the different levels of the US beef supply chain. Following the approach adopted in earlier works on price dependence (e.g. Reboredo, 2011; Serra and Gil, 2012; Zimmer, 2013), we model the co-movements between the rates of price change at the farm, the wholesale and at the retail level using the statistical tool of copulas. In this framework, an empirical finding (say) that a Student-*t* copula adequately represents the dependence structure

1 Non-parametric tests, such as the test by Busetti and Harvey (2011), have a notable advantage over parametric ones. That is, the outcome of a non-parametric test does not depend on the specification of a parametric model. One could have alternatively specified a parametric time-varying copula model and have tested for the statistical significance of parameters associated with time-varying dependence. The null hypothesis in that case, however, would be too narrow (joint), as it would concern both the specification (functional form/copula family) of the parametric model and the parameters associated with time-varying dependence.

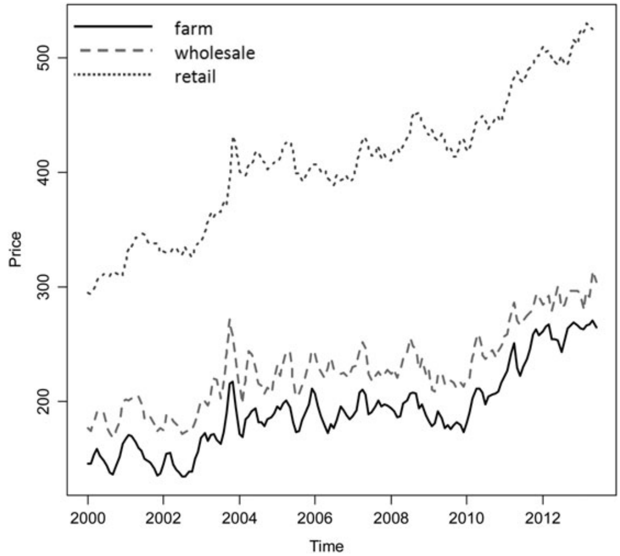

Fig. 1. The price series at the different levels of the US beef supply chain.

of a bivariate random process will imply that positive and negative price shocks are likely to be transmitted from one market level to the other with the same intensity. An empirical finding, however, that a Clayton copula adequately represents the dependence structure will imply that negative price shocks are transmitted from one market level to the other, but not the positive ones.

For the empirical implementation, we employ the semi-parametric approach proposed by Chen and Fan (2006a, 2006b) which involves three steps: (a) an ARMA–generalized autoregressive conditional heteroskedasticity (GARCH) model is fit to each series of the rates of price change (innovation series).[2] (b) The obtained standardised residuals (*filtered* data) are then used to calculate the respective empirical distribution functions (*copula data* and meaning data on (0,1)). (c) The estimation of copula models is conducted by applying the maximum likelihood (ML) estimator to the copula data (Canonical ML). The semi-parametric approach exploits the fact that the copula and the margins can be estimated separately using potentially different methods. The Canonical ML copula estimator is consistent but less efficient relative to the fully parametric one. Therefore, the asymptotic distributions of the copula parameters and the dependence measures (such as the Kendall's tau and the tail coefficients) should be approximated using resampling methods (e.g. Fermanian and Scaillet, 2004;

2 ARMA–GARCH models are commonly used to obtain filtered data in empirical investigations of dependence among stochastic process with copulas (e.g. Rémillard, 2010; Czado, Schepsmeier, Min, 2012; Aloui, Hammoudeh and Nguyen, 2013; Brechmann and Schepsmeier, 2013; Zimmer, 2013). Alternative specifications include extensions of the GARCH model such as the TGARCH and the GJR, the ARMA–TGARCH and the ARMA–GJR models as well as the use of a VEC model in the first stage and the application of the GARCH model to the individual series of VEC residuals in the second stage (e.g. Bhatti and Nguyen, 2012; Patton, 2012; Serra and Gil, 2012).

Table 1. *p*-Values of the tests for autocorrelation and ARCH effects

	Box–Pierce			ARCH–LM		
	No. of lags			No. of lags		
Filtered rates of price change	1	6	12	1	6	12
Farm	0.518	0.859	0.233	0.519	0.839	0.354
Wholesale	0.961	0.764	0.970	0.215	0.804	0.883
Retail	0.759	0.608	0.816	0.515	0.796	0.857

Choros, Ibragimov and Permiakova, 2010; Gaißer, Ruppert and Schmid, 2010; Patton, 2013).

To obtain the filtered rates of price change, an ARMA(2,1)–GARCH(1,1) model has been fit to each of the innovation series (parameter estimates and assessment of goodness of fit for the marginal models are presented in Table A2). Table 1 presents the *p*-values resulting from the application of the Box–Pierce and the autoregressive conditional heteroskedasticity–Lagrange multiplier (ARCH–LM) tests to the filtered data at various lag lengths. It appears that these are free from autocorrelation and from ARCH effects.[3]

Table 2 presents the constancy tests for three quantiles (0.25, 0.5 and 0.75) of the individual time series (filtered rates of price changes).[4] The empirical values of the $n_\tau(Q)$ statistics are in all cases below the 5 per cent critical value (0.461), suggesting that the null of constancy is consistent with the data. Table 3 presents the constancy tests for the above three quantiles of the bivariate empirical copulas. The empirical values of the $n_\tau(BQ)$ statistics are again in all cases below the 5 per cent critical value. Thus, there is not sufficient statistical evidence for breaks and/or gradual but persistent shifts in the respective empirical copulas.

Figure 2 presents scatterplots of the copula data for the pairs farm–wholesale and wholesale–retail. Starting with the pair farm–wholesale, the majority of observations lies close and along the positive diagonal suggesting a positive association between the rates of price change at the farm and at the wholesale level. The picture, however, is quite different for the pair wholesale–retail where there is a considerable dispersion of points along the positive diagonal and also a sizable part of observations lies along the negative diagonal. This is an indication that the wholesale and the retail levels of the beef supply chain in the USA are not as strongly interconnected as the farm and the wholesale levels.

Table 4 presents the total scores from the comparisons of the five copula models using Clarke's (2007) test (Appendix C). For the pair farm–wholesale, the Gaussian, the Student-*t* and the Gumbel–Clayton are statistically indistinguishable from each other and superior to the Clayton and to the Gumbel copulas. The degrees of freedom for the Student-*t*, however, turned out to be

3 All estimations, testing, and resampling involved in this work have been carried out using R.

4 Note that exactly the same quantiles have been considered by Busetti and Harvey (2011).

Table 2. Tests for the constancy of the quantiles of the individual series[a]

	Empirical values of the $n_\tau(Q)$ statistics		
Filtered rates of price change	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$
Farm	0.319	0.325	0.061
Wholesale	0.18	0.394	0.072
Retail	0.126	0.101	0.096

[a]The critical values are 0.743, 0.461 and 0.347, at the 1 , 5 and 10 per cent levels, respectively.

Table 3. Tests for the constancy of the quantiles of the empirical copula[a]

	Empirical values of the $n_\tau(BQ)$ statistics		
Filtered rates of price change	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$
Farm–wholesale	0.051	0.358	0.107
Wholesale–retail	0.065	0.453	0.073

[a]The critical values are 0.743, 0.461 and 0.347, at the 1, 5 and 10 per cent levels, respectively.

above 30, implying that it is indistinguishable from a Gaussian copula. For the pair wholesale–retail, the degrees of freedom for the Student-*t* turned out again to be above 30 and the Clarke's test has selected the Gumbel copula. Given that Clarke's (2007) test could not discriminate between the Gaussian and the Gumbel–Clayton for the pair farm–wholesale, we have resorted to the rank-based Cramer–von Mises (*CvM*) and Kolmogorov–Smirnov (*KS*) goodness of fit tests (Appendix D). Table 5 presents the *p*-values from the application of the *CvM* and the *KS* tests.[5] For the wholesale–retail pair, the Gumbel copula appears to fit the data quite well. However, for the farm–wholesale pair both the *CvM* and the *KS* tests do not reject the null of a Gaussion or a Gumbel–Clayton copula. To choose between them, we use the observed significance level (*p*-value) as the selection criterion. Since a larger *p*-value indicates a smaller departure from the null than a smaller *p*-value, we select the copula model against which the sample data provide the less evidence, i.e. the model with the maximum *p*-value; for the farm–wholesale pair, both goodness-of-fit tests point to the Gumbel–Clayton copula.

Table 6 presents the estimation results for the selected copula models.[6] The standard errors of the copula parameters and dependence measures have been

5 It is sensible to use a goodness of fit test even in cases where the Clarke's (2007) (or any other comparison) test selects a single model. The fact that a copula model is found superior to other copula models does not necessarily imply that it is itself an adequate model (that means, one with a good fit on the data).

6 Although our primary focus are the transmissions of price changes from the farm to the wholesale and from the wholesale to retail level, for completeness and also in order to facilitate comparisons with other relevant empirical works (which typically leave the wholesale level out of the analysis)

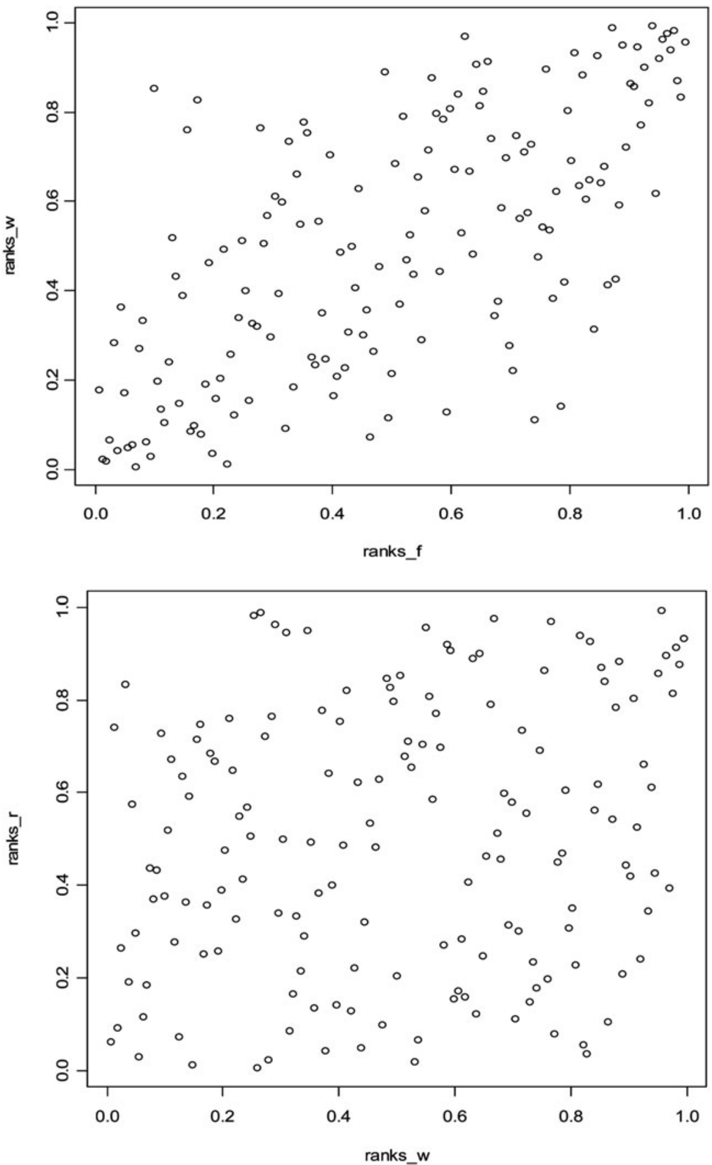

Fig. 2. Scatterplots of the copula data (top: farm–wholesale and bottom: wholesale–retail).

we have examined the pair farm-retail using exactly the same sequence of tests as for the other two pairs. The empirical copula for that pair has passed the constancy tests and the model selection and goodness-of-fit tests have pointed to the Gumbel copula. The application of the independence test (Appendix E), however, has failed to reject the null at the 10 per cent level or less. For this reason, we do not report any results for that pair. They are, however, available upon request. For the Gumbel–Clayton copula (pair farm-wholesale) and for the Gumbel copula (pair wholesale-retail) the *p*-values of the independence test are 0 and 0.012, respectively.

Table 4. Total scores from all pairwise comparisons of the five copula models considered

	Copula model				
Filtered rates of price change	Gaussian	Student-*t*	Clayton	Gumbel	Gumbel–Clayton
Farm–wholesale	2	2	−3	−3	2
Wholesale–retail	−1	0	−2	2	1

Table 5. *p*-Values of the goodness of fit tests[a]

	Gaussian		Gumbel–Clayton	
Filtered rates of price change	*CvM*	*KS*	*CvM*	*KS*
Farm–wholesale	0.832	0.604	0.892	0.836
	Gumbel			
	CvM	*KS*		
Wholesale–retail	0.644	0.384		

[a]The *p*-values have been obtained by the bootstrap proposed by Genest, Remillard and Beaudoin (2009) using 250 replications.

Table 6. Estimates of the parameters and the dependence measures of the selected copula models[a]

Filtered rates of price change	Selected copula	Parameter estimates	Kendall's tau	λ_L	λ_U
Farm–wholesale	Gumbel–Clayton	$\hat{\theta}_1 = 0.437^{***}$ (0.076) $\hat{\theta}_2 = 1.579^{***}$ (0.151)	0.480*** (0.036)	0.366*** (0.139)	0.449*** (0.067)
Wholesale–retail	Gumbel	$\hat{\theta} = 1.156$*** (0.063)	0.135*** (0.047)	–	0.178*** (0.059)

[a]The Kendall's τ and the tail dependence coefficients with their respective standard errors (in parentheses) as well as the standard errors of the copula parameters have been obtained using *i.i.d.* bootstrap with 10,000 replications.
***Statistical significance at the 1 per cent level (or less).

obtained using the bootstrap method (Appendix E). Starting with the pair farm–wholesale, the Kendall's tau is 0.48 and statistically significant at any reasonable level indicating that the number of concordant pairs of observations exceeds by far the number of discordant ones. The coefficients of tail dependence are both statistically significant at any reasonable level as well. The value of λ_U implies that with probability 0.45 a price boom at the farm level will be associated with a price boom at the wholesale level. The value of λ_L implies that with probability 0.37 a price crash at the farm level will be associated with a price crash at the retail level. The difference between the two probabilities offers an indication that price shocks during market upswings are likely to be transmitted with a higher intensity compared with price shocks during market downswings. All these taken together suggest that the farm and the wholesale levels of the beef supply chain in the USA have been integrated to a considerable degree over the period examined in this work. The results from the estimation of the copula model are therefore consistent both with the relative stability of margins (Figure 1) and with the positive interconnection of the underlying filtered series of price changes (Figure 2, top).

Turning to the pair wholesale–retail, the Kendall's tau (although statistically significant at the 5 per cent level or less) is rather low, indicating a limiting degree of dependence over the entire joint distribution of the filtered series of price changes. Extreme left co-movement is zero, while, on the basis of the upper dependence coefficient, the probability that a price boom at the wholesale level will be associated with a price boom the retail level is only 0.18. The results are again consistent with the widening of margins (Figure 1) and the rather limited interconnection of the underlying filtered series (Figure 2, bottom). Given the rather weak tendency of price changes to co-move for the pair wholesale–retail, it is not very surprising that the relationship between price changes for the pair farm–retail can be represented by the independence copula.[7]

To interpret the estimated dependence patterns and, more importantly, to be able to speculate about their potential implications for the various stakeholders of the beef supply chain, some information with respect to causal markets is necessary. Causal is the market at which the price of a commodity is established (or equivalently the market from which price shocks emanate). For the meat supply chains in the USA there has been considerable empirical evidence (through tests of Granger causality and/or tests of weak exogeneity in the context of co-integration analysis) in favour of uni-directional causality from the farm to the downstream market levels (e.g. Heien, 1980; Goodwin and Holt, 1999; Goodwin and Harper, 2000; Goodwin and Piggott, 2001). A common explanation for the relevant empirical findings has been that supply shocks are more frequent than the demand shocks and that sellers often adopt fixed mark-up pricing.

In the light of this information, the co-movement pattern at the tails for the pair farm–wholesale implies that extreme positive rates of price change at the farm level are likely to be transmitted to the wholesale level with a higher intensity than negative ones. The actors at the wholesale level appear, thus, to enjoy a certain (limited) advantage relative to those at the farm level in the sense that *ceteris paribus* their gross margin will tend to remain the same when there is an increase in the price of the primary commodity, and it will tend to expand when there is a decrease in that price. In exactly the same way, the empirical finding of a zero extreme left co-movement and a positive

7 Following the suggestion from an anonymous reviewer, we also checked the consistency between the results from bivariate and trivariate elliptical copula models, and found no qualitative differences. Relevant results can be made available upon request.

extreme right co-movement for the pair wholesale–retail indicates that actors at the retail level are likely to enjoy an advantage relative to those at the wholesale level. Also, from the estimated dependence patterns at both the farm–wholesale and the wholesale–retail pair it appears that consumers are more likely to feel an increase of prices upstream than a decrease.

At the first stage of beef supply chain (farm level) are the owners of the cow/calf operations, of the stocker operations and of the feedlot operations. Given that for a company to undertake the whole cattle lifecycle (stocking, slaughtering and processing) requires extremely large amounts of capital, there is less vertical integration in the US beef industry relative to the pork and the poultry industries. Cattle producers are typically small and independent and as such they cannot have much leverage on the distribution of added-value along the beef supply chain (Lowe and Gereffi, 2009). At the second stage of the beef supply chain (wholesale) are active the leading packing companies which perform beef processing and are also connected directly with the retailers, eliminating the need for a middleman. The CR4 for the biggest beef packers (Tyson, Cargill Meat Solutions, JBS Swift and National Beef) is close to 0.8. Those are actors with a potentially very high degree of leverage. At the last stage (retailing, food service and restaurants), there are also high levels of concentration and actors with significant name recognition (e.g. Kroger, Compass Group PLG, McDonald's, Burger King, etc.) and, therefore, with a potentially high degree of leverage as well.

It is worth noting that the type of price transmission asymmetry where positive shocks are transmitted with higher intensity compared with negative ones is consistent with a transmission function which is convex in (logarithmic) prices. Azzam (1999) showed that a convex price transmission function can be derived under seller power and a strict concave demand function downstream; Xia (2009) obtained exactly the same price transmission function under buyer power and a strictly concave input supply function upstream.

4. Conclusions

The smooth transmission of price shocks along food supply chains is a necessary condition for all stakeholders to benefit from the agricultural policy reforms towards more market orientation. Price rigidities and/or incomplete pass through are considered as indications of inefficiency and they attract the attention of both researchers and antitrust authorities. In this context, the objective of the present paper has been to assess the degree and the structure of price dependence in the US beef supply chain. This objective has been pursued using monthly price data from the farm, wholesale and retail levels, and the statistical tool of copulas.

According to our results:

(a) The dependence between farm and wholesale prices is relatively strong and it is best described by a Gumbel–Clayton copula with a certain degree of tail asymmetry; positive price shocks are transmitted with higher intensity compared with negative ones. Given this pattern of price transmission, beef processors and wholesalers may benefit at the expense of primary producers.

(b) The co-movement between wholesale and retail prices is rather weak and is best described by the Gumbel copula which is consistent with zero dependence at the lower tail. Given this pattern of price transmission, retailers appear to have an advantage over beef processors and wholesalers. In both processing and retailing of beef in the USA operate very large firms with significant name recognition. The finding that the advantage lies with the retailers may be an indication of a change in the balance of power between those two actors; it is well known that the food retailers' bargaining power has increased significantly in the most recent years.

(c) The dependence between farm and retail prices is best described by the independence copula (meaning, it is zero from a statistical viewpoint). An advantage of this work relative to others (which have focussed on two market levels only, typically the farm and the retail ones) is that it is capable of identifying the pair of markets where the problem with the asymmetry of price transmission lies.

The empirical findings of the present study, when viewed in the light of the potential leverage of various actors in the US beef industry and the theoretical contributions of Azzam (1999) and Xia (2009), appear to raise concerns about the efficiency (integration) of the beef supply chain in the USA and to point to the market power as a possible source of it. However, one should interpret them with care. Ward (2010), who reviewed a large number of studies on the behaviour and conduct of beef packing firms in the USA, notes that the evidence was insufficient to persuasively argue that the meat packing industry was not competitive; as a matter of fact, certain studies showed that the structural changes over the last 30 years that have led to considerable concentration were on the balance beneficial from the efficiency standpoint. Moreover, economic phenomena such as an increase of price margins (or of price ratios) over time may arise under perfect competition as well. Gardner (1975) showed that an increase (decrease) in supply of farm products will work towards a smaller (larger) margin. Also, an increase in demand for food will increase the retail–farm price ratio if marketing inputs are less elastic in supply than farm products. Finally, market power at one level of the food supply chain may be counterbalanced by the presence of market power at another level of it. Therefore, further work on this elaborate topic is necessary.

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Appendix A. Relationships between copula parameters and dependence measures

Table A1. Copula parameters, Kendall's τ, and tail dependence (Brechmann and Schepsmeier, 2013)

Copulas	Parameters	Kendall's τ	Tail dependence (lower and upper)
Gaussian	$\theta \in (-1, 1)$	$(2/\pi)\arcsin(\theta)$	(0,0)
Student-*t*	$\theta \in (-1, 1),\ \nu > 2$	$(2/\pi)\arcsin(\theta)$	$\left(2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{(1-\theta)/(1+\theta)}\right), 2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{(1-\theta)/(1+\theta)}\right)\right)$
Clayton	$\theta > 0$	$\theta/(\theta+2)$	$(2^{-(1/\theta)}, 0)$
Gumbel	$\theta \geq 1$	$1-(1/\theta)$	$(0, 2-2^{1/\theta})$
Gumbel–Clayton	$\theta_1 > 0,\ \theta_2 \geq 1$	$1-(2/(\theta_2(\theta_1+2)))$	$(2^{-(1/\theta_1\theta_2)}, 2-2^{1/\theta_2})$

Appendix B. Marginal models: estimation results

Table A2. Parameter estimates and goodness of fit measures from the marginal models[a]

	Farm	Wholesale	Retail
Mean equation			
Constant	0.0014 (0.0011)	0.0013 (0.0009)	0.0007 (0.001)
AR(1)	0.9999[b] (0.1202)	0.8743[b] (0.1119)	0.9993[b] (0.337)
AR(2)	−0.4901[b] (0.0740)	−0.3783[b] (0.077)	−0.2244[b] (0.875)
MA(1)	−0.6092[b] (0.1295)	−0.6984[b] (0.1144)	−0.7913[b] (0.3679)
Volatility equation			
Constant	0.0002 (0.0003)	0.0004[b] (0.0002)	0.0001[b] (0.00005)
ARCH	0.0412 (0.0603)	0.1455 (0.0868)	0.2032[b] (0.0951)
GARCH	0.7876[b] (0.2851)	0.5771[b] (0.1971)	0.3411 (0.234)
Model evaluation			
BIC	−3.7173	−3.513	−5.206
Adjusted R^2	0.723	0.813	0.927

[a]Applied to the raw data (price rates of change). The selection of lag orders has been based on the BIC. The use of adjusted R^2 to evaluate the fit of ARMA–GARCH models has been proposed by Liu, Erdem and Shi (2011). The values of adjusted R^2 should be interpreted with care since GARCH models are non-linear (the adjusted R^2 can provide only approximate results).
[b]Statistically significant at the 5 per cent level.

Appendix C. The Clarke's (2007) test of bivariate copula selection

It is a likelihood ratio test based on the Kullback–Leibler information criterion, which measures the distance between two statistical models. Let C_1 and C_2 two non-nested copula models with estimated parameter vectors $\hat{\theta}_1$ and $\hat{\theta}_2$, respectively. Let also the log differences of their pointwise likelihoods $m_t = \ln(C_1(u_{t1}, u_{t2}; \widehat{\theta}_1)/C_2(u_{t1}, u_{t2}; \widehat{\theta}_2))$, for $t = 1, 2, \ldots, T$. The null hypothesis that the two models are statistically indistinguishable is specified as H_0: $P(m_t > 0) = 0.5,\ \forall t = 1, ..., T$. The intuition behind this specification of the null is that under statistical equivalence the log-likelihood ratios of the single observations are uniformly distributed around zero and, in expectation, 50 per cent of them are greater than zero. The relevant test statistic is $B = \sum_{t=1}^{T} I(m_t > 0)$, where $I(\cdot)$ is the indicator function, and it follows the binomial distribution with parameters T and $p = 0.5$. The null is rejected for values of B that are significantly different (smaller or larger) than $T/2$, which is the expected value under the null. For a one-sided test at significance level α, the critical value, c_α, is defined as the smallest (for an upper-tail test) or largest (for a lower-tail test) integer for which

$$\sum_{c=C_1}^{C_2} \binom{T}{c} 0.5^T \leq a,$$

where $(C_1, C_2) = (c_a, T)$ if the test is upper-tail, and $(C_1, C_2) = (0, c_a)$ if it is lower-tail.

When there are more than two competing models, a bivariate copula model is compared with all possible bivariate copula models under consideration at a chosen significance level. If the model is superior (inferior) to another, a score '$+1$' ('-1') is assigned to it; no score is assigned to statistically indistinguishable models. For each model, its total score is the sum of the scores from all pairwise comparisons. The best model is the one that achieves the highest total score (Belgorodski, 2010; Brechmann and Schepsmeier, 2013).

Alternative specification tests and alternative selection algorithms with more than two rival models have been proposed in the literature. Clarke (2007) showed that his test, because it is distribution free, has more discriminatory power under common research situations than the widely used Vuong (1989) test. The term 'common research situations' refers to non-normal distributions of the log differences of the pointwise likelihoods obtained from two rival copula models. Here, the application of the non-parametric Shapiro–Wilks (1965) test suggested that all time series of log differences depart strongly from normality. These results imply that for our data and for the rival models considered, Clarke's (2007) test is definitely more efficient than Vuong's (1989) test; it is also very likely to be more efficient than the pseudo-likelihood ratio test by Chen and Fan (2006b) which, like Vuong's (1989) test, is not distribution free.

Chen and Fan (2006b) proposed a selection algorithm in which the null hypothesis is that a given model ('benchmark') performs at least as well as the remaining models ('candidates') and the alternative hypothesis is that there is at least one 'candidate' which is better than the 'benchmark'. How a 'benchmark' model is chosen and the potential implications of that choice on the final outcome of the selection process have not been addressed in the relevant literature. The procedure by Belgorodski (2010) and by Brechmann and Schepsmeier (2013) dispenses with the need to choose arbitrarily 'benchmarks' and 'candidates'.

Appendix D. The *CvM* and *KS* goodness-of-fit tests

The rank-based *KS* and *CvM* tests compare the distance between C_T and $C_{T,\theta}$ where the last is an estimate of C_θ under the null of $C = C_\theta$. The distance is defined as $D_T = \sqrt{T}(C_T - C_{T,\theta})$. The Kolmogorov–Smirnov statistic is $KS_T = \sup_{u\in[0,1]^2}|D_T(u)|$ and the *CvM* statistic is $CvM_T = \int_{[0,1]^2} D_T(u)^2 dC_T(u)$. The two tests are consistent (i.e. if the null is in fact false, then it is rejected with probability 1 as the sample size increases).

Appendix E. Test of independence for bivariate copula data

The test of asymptotic independence for a pair of copula data series is based on the sample estimate of Kendall's tau, $\hat{\tau}$, and the test statistic $T = \sqrt{(9n(n-1))/(2(2n+5))} \times \hat{\tau}$, with n the length of the copula data series. Under the null of independence, T follows asymptotically the standard normal distribution (e.g. see Prokhorov, 2001). Thus, for the two-sided alternative, the asymptotic p-value equals $2(1 - \Phi(|T|))$, with $\Phi(\cdot)$ the standard normal cdf.

Appendix F. Estimation of the standard errors for the copula parameters and for the dependence measures

Following the works of Chen and Fan (2006b) and Rémillard (2010), Patton (2013) suggests the use of a simple *i.i.d.* bootstrap approach to obtain confidence intervals for copula parameters and dependence measures, when the model is semi-parametric and the copula is constant over time. If the latter condition does not hold, he proposes the use of block bootstrap (e.g. Politis and Romano, 1994). Implementation details for these two bootstrap approaches are given in Patton (2013: 12–13, 23–25). As our data provide strong evidence for time-invariant copulas, our results (presented in Table 6) are based on the simple *i.i.d.* bootstrap. However, as a safeguard against a Type II error for the copula time constancy test, we also applied several variants of the block bootstrap (e.g. with fixed and varying block-sizes). Results turn out to be qualitatively the same in all cases: copula parameters and dependence measures are statistically significant at any reasonable level.