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A computer simulation of phosphate and nitrogen transport by water runoff[1])

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Introduction

Soil eroded by runoff may contribute a large amount of nutrients to surface waters, especially if the soil originates from fertilized fields. A description of the erosion process is to be found in *Baver*'s et al (1972) book or in *Hudson*'s (1971) book; the physical mechanisms involved were analyzed by *Zaslavsky* (1970). *Johnson* and *Moldenhauer* (1970) published a critical review of erosion models. A detailed erosion model was published by *David* and *Beer* (1975). Quantitative treatment of the flow process is based either on physical models or on statistical analysis of field observations (*Wischmeier* and *Mannering* (1969), Meyer and Wischmeier (1969).

In our model the descriptions of runoff and subsurface flow were based on the publication by *Ishihara* (1967).

Soil detachment and movement with water were modelled according to *Meyer* and *Wischmeier* (1969).

Ishihara (1967) proposed a mathematical model of surface runoff and surface flow. He considers a drainage basin consisting of (or equivalent to) a relatively dense subsoil of constant slope covered by a very porous layer of uniform thickness, as this is a common situation in agricultural land. Under such conditions, rain will immediately drain down to the subsoil. Some of the water penetrates into the subsoil and the rest remains in the porous layer, and flows within it. The water in the unsaturated part of the porous layer is neglected. After the porous layer is filled,

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surface flow will start. The region in which surface flow occurs is called "occurence domain of overland flow".

Meyer and *Wischmeier* (1969) propose four principal mechanisms of detachment and transport of soil with water: detachment by rainfall and by runoff, and transport by the same agents. This model is in agreement with views of *Zaslavsky* (1970) and *David* and *Beer* (1975).

Theoretical background of the erosion model

Computation of the surface runoff and subsurface flow in a porous layer (Fig. 1) is essential for calculating the erosion by runoff. *Ishihara*'s equation for the flow of water in permeable media is based on *Darcy*'s law and on the continuity equation:

$$(1) \quad \frac{\partial(\gamma H)}{\partial t} + \frac{\partial}{\partial x}(k \sin \alpha \cdot q) = r - f,$$

where: γ = effective porosity of surface stratum; H = depth of water in surface stratum; t = time; x = distance downslope from origin; k = conductivity coefficient of surface stratum; α = angle of the slope; q = discharge rate per unit width; r = rain intensity; f = infiltration capacity of the substratum. Figure 1 shows a schematic presentation of *Ishihara*'s model.

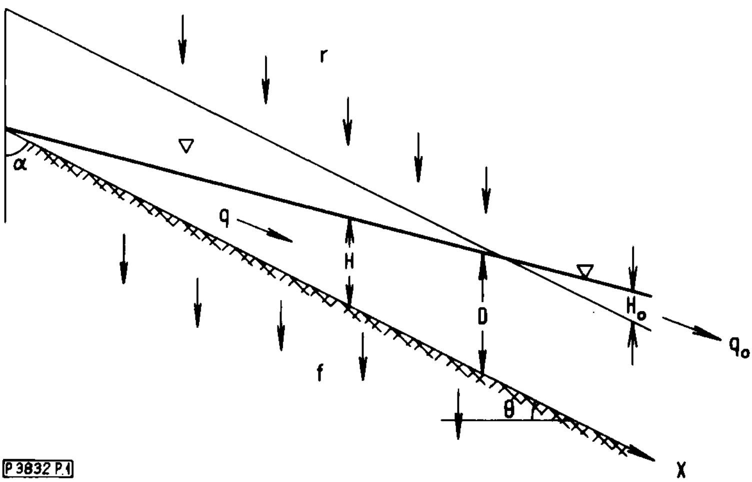

Figure 1: The scheme of flow on a slope containing a porous stratum and a substratum with low permeability for water (*Ishihara*, 1967)

Fließschema an einem Hang mit durchlässiger Schicht und darunter liegender Schicht mit geringer Wasserpermeabilität

Relaxing the condition of a constant slope, and assuming α and Θ as a function of distance x, and constant γ and k and after the differentiation we obtained:

$$(2) \quad \gamma \frac{\partial H}{\partial t} + k \left(\frac{\partial H}{\partial x} \sin \alpha + \frac{d\alpha}{dx} \cdot H \cdot \cos \alpha\right) = r - f$$

or by isolating the derivative of depth with respect to time:

(3) $$\frac{\partial H}{\partial t} = -\frac{1}{\gamma}\left[-k\left(\frac{\partial H}{\partial x}\sin\alpha + \frac{d\alpha}{dx}\cdot H\cdot\cos\alpha\right) + r - f\right]$$

When the porous layer is saturated H≡D (where D = depth of the porous layer) and surface flow begins. The calculation of this flow is based on *Ishihara*'s movement equation:

(4) $H_0 = K_0 \cdot q_0^p$ where p = 3/5 and

(5) $$K_0 = \left[\frac{N_0}{\sqrt{\sin\alpha}}\right]^p$$ (N = equivalent roughness coefficient)

The subscript $_0$ represents the hydraulic quantity for overland flow.

The flow q_0 is found by first substituting equation (5) into (4):

(6) $$H_0 = \left[\frac{N_0}{\sqrt{\sin\alpha}}\right]^p \cdot q_0^p$$

and differentiating H_0 with respect to time

(7) $$\frac{\partial H_0}{\partial t} = \left[\frac{N_0}{\sqrt{\sin\alpha}}\right]^p \cdot p \cdot q_0^{p-1} \cdot \frac{\partial q_0}{\partial t}$$

Equation (7) is substituted into the equation of continuity (8):

(8) $$\frac{\partial H_0}{\partial t} + \frac{\partial q_0}{\partial x} = r - f$$

and after separating the derivative of flow with respect to time and assigning its numerical value to p:

(9) $$\frac{\partial q_0}{\partial t} = 1.667 \cdot q_0^{0.4}\ (\sin\alpha)^{-0.3} \cdot (N_0)^{0.6} \cdot \left(\frac{\partial q_0}{\partial x} + r - f\right)$$

This equation (9) describes the flow rate on the surface. It is needed for the subsequent erosion computations.

Our erosion model was based on *Meyer* and *Wischmeier*'s (1969) equations:

(10) $D_R = S_{DR} \cdot A \cdot I^2$

where: D_R = Soil detached by rainfall; S_{DR} = a constant related to soil type; A = Increment of area of the slope; I = rainfall intensity.

(11) $D_F = S_{DF} \cdot A \cdot 1/2\,(S_S^{2/3}\, Q_S^{2/3} + S_E^{2/3} \cdot Q_E^{2/3})$

D_F = Soil detached by runoff; S_{DF} = a constant related to soil type; S = slope steepness; Q = flow rate; subscript S stands for the upper edge of the increment and E for the lower one.

(12) $T_R = S_{TR} \cdot S \cdot I$

T_R = transport capacity of rainfall; S_{TR} = a constant related to soil type;

(13) $T_F = S_{TF} \cdot S^{5/3} \cdot Q^{5/3}$

T_F = transportation capacity of runoff
S_{TF} = a constant related to soil type.

For a numerical solution of this equation system the field was divided into a number of subfields with an approximately constant slope. Each subfield was composed of an integer number of finite difference increments (Fig. 2)

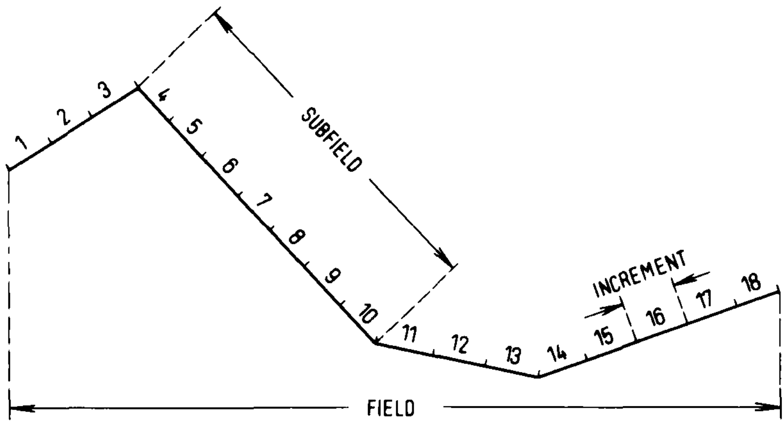

Figure 2: Schematic representation of a slope with more than one water flow direction

Schematische Darstellung eines Hanges mit mehr als einer Wasserflußrichtung

The boundary conditions of the subfields were defined according to the various possibilities as required by the slope changes in the field.

1. Boundary conditions at the edges of the field (increments 1 and 18 in Fig. 2 describe inflow or outflow of matter).

2. Boundary conditions at points at which the slope changes. These require special treatment. For example, the junction of subfields 3 and 4 is a ridge, and the water flows in opposite directions in these fields. Conversely, water flows into the junction of subfield 13 and 14 from both directions. There are a number of other possibilities of flow changes due to slope direction changes.

3. Boundary conditions at point where surface flow begins or disappears (at increment 8, Figure 2). These conditions change with time and are a function of water flow in the porous surface soil layer.

In principle, the described equations permit the calculation of the amount of eroded soil from a certain field, and therefore the amount of nutrient transported with it (assuming that the compositions of the soil is known).

Following the ideas of *Meyer* and *Wischmeier* (1969) the capacity of transport by runoff (T_F) and rainfall splashing (T_R) is limited. Therefore, part of the eroded soil may be deposited in the field.

The maximum amount of soil transported (T_T) is limited by the sum of rainfall splashing and runoff transport capacities:

(18) $T_T = T_F + T_R$

In cases where the amount of soil detached is larger than the total transport capacity, part of the soil will be deposited. Accordingly, if the soil deposition is defined as P, deposition will occur when $(T_T)_{i-1} \geqslant (T_T)_i$ (i represents an increment in the field (Fig. 2) and i-1 the uphill increment respectively to i).

The sedimentation at i will be:

(19) $(P)_i = (T_T)_{i-1} - (T_T)_i$

If the soil detachment is smaller than the transport:

(20) $(T_T)_i = (D_R)_i + (D_F)_i + (T_T)_{i-1}$

and in case of a larger detachment than the transport capacity:

(21) $(T_T)_i = (T_R)_i + (T_F)_i$

In our model, the whole system of differential equations is simultaneously integrated in respect to time by numerical routine in the CSMP programming language.

At the last stage of computation, the amounts of nutrients eroded and deposited are calculated.

The erosion process is selective because the organic matter and finer particles, which are relatively rich in plant nutrients, are more susceptible to it than the coarser soil fractions *Barrows* and *Kilmer* (1963). Accordingly, the eroded material will be usually richer in nutrients than the original soil. This is expressed by an enrichment ratio, M, specific for each nutrient and soil. Therefore, the amount of transported phosphorus (P) is obtained from the total amount of transported soil, the nutrient concentration in the original soil (C) and the enrichment ratio:

(22) $P = T_{tot} \cdot C_P \cdot M_P$

Simulation results and discussion

The sensitivity of the model may be tested by changing the value of one or more parameters and comparing the results.

The sensitivity of the model to rain quantities and intensities was tested by comparing runs 1,4 and 9 in Table 1. In run 1, the total quantity of rain was 59 mm, falling at irregular intensity over 24 hours. This is a rain pattern taken for one day in October 1976 in Haifa. In run 4, the rain intensities were doubled and thus a double total rain quantity was obtained. Doubling of the rainfall raised the underground water level from 12 cm in run 1 to 20 cm in run 4, which produced saturation of the upper soil layer with runoff and the resulting erosion (Fig. 3).

Comparison of a random distribution of rain intensity (run 4) to a uniform rain distribution over 24 hours (run 9) shows a later appearance of runoff and decrease in erosion (Fig. 4) in the uniform rain run.

It may be seen than an increase in rain intensity produces an increased runoff, a higher underground water level and greater erosion (Fig. 5).

The influence of changing the steepness of the slope under constant rain conditions is illustrated in runs 3 and 7. With increase of steepness from 10° to 30° the

Table 1: Parameters used in runs

Benutzte Parameter

number of run	angle of the slope degrees	effective porosity of the upper layer	hydraulic coefficient of the sublayer m/hour	soil erosion coefficients	depth of the upper soil layer m	accumulative amount of rain m	intensity of rain
1	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.20	58	not constant
2	10	0.20	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.20	58	not constant
3	30	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.20	58	constant
4	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.20	116	not constant
5	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.1	58	constant
6	30,10,12	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.2	58	not constant
7	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.3	58	constant
8	10,10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.1	58	not constant
9	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.2	116	constant
10	10	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.01 SDFC=0.0005	0.2	116	not constant
11	30	0.43	0.00036	STRC=0.01 STFC=0.0001 SDRC=0.001 SDFC=0.00005	0.2	116	not constant

Table 1: (cont.) Results of simulation runs

Ergebnisse der Simulationsläufe

time of Runoff beginning after rain start	underground water level m	maximum soil erosion 12 hours after run's start kg/100 m	maximum soil erosion 24 hours after run's start kg/100 m	amount of sediment 24 hours after run's start kg/100 m	sedimentation site, distance from the top of the slope m
none	0.12	4.5×10^{-3}	5.12×10^{-3}	none	–
12	0.2	4.56×10^{-3}	5.2×10^{-3}	none	–
none	0.11	5.4×10^{-3}	1.04×10^{-2}	none	–
12	0.2	9.44×10^{-3}	1.45×10^{-1}	1.75×10^{-4}	3
21.5	0.1	2.61×10^{-3}	5.4×10^{-3}	none	–
none	0.12	8.8×10^{-3}	1.04×10^{-2}	1.08×10^{2}	40
none	0.12	2.61×10^{-3}	5.2×10^{-3}	none	–
none	0.12	4.64×10^{-3}	5.4×10^{-3}	5.4×10^{-3}	50
19.5	0.2	5.2×10^{-3}	1.04×10^{-2}	none	–
none	0.16	8.96×10^{-3}	1.10×10^{-2}	none	–
none	0.09	9×10^{3}	1.08×10^{-2}	none	–

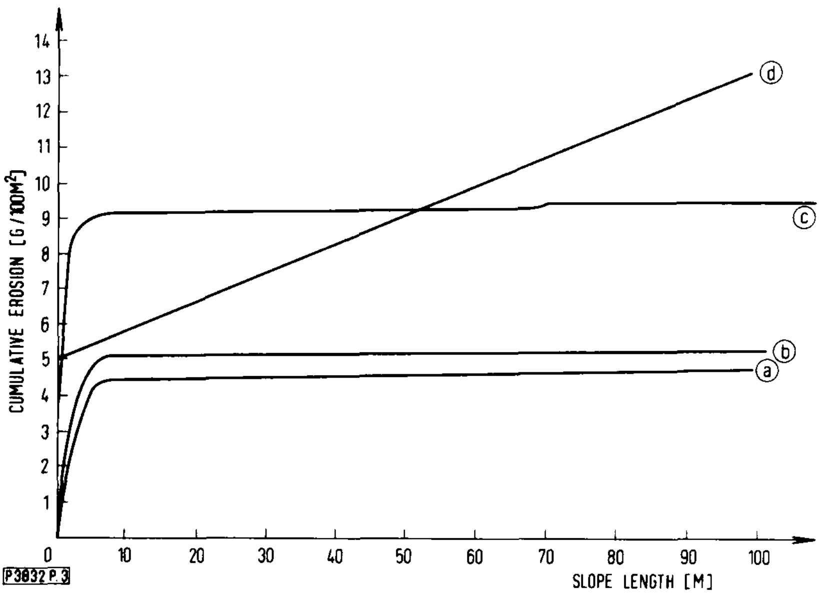

Figure 3: Simulated soil erosion on a 10° slope; (a) and (b) after 12 and 24 hours of variable rain, respectively (c) and (d) under double rain intensity

Simulierte Bodenerosion an einem Hang mit 10° Neigung; (a) und (b) jeweils nach 12 und 24 Stunden bei unterschiedlichem Regen, (c) und (d) bei doppelter Regenintensität

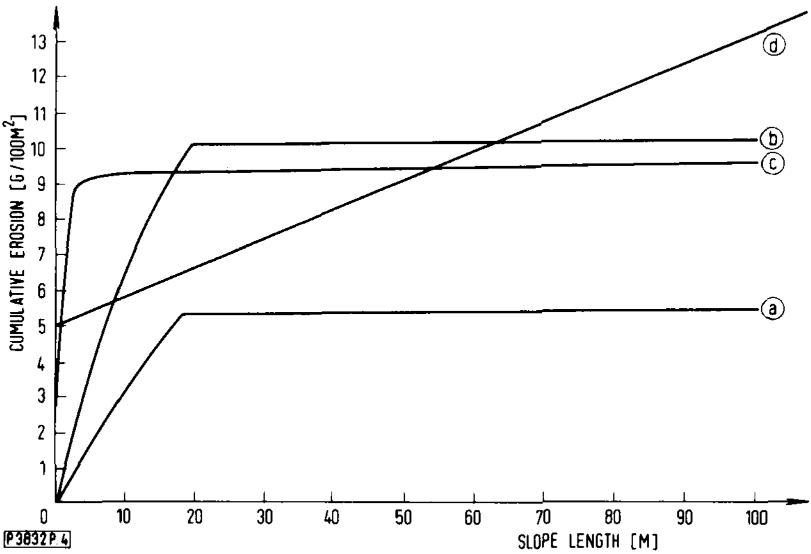

Figure 4: Effect of rain regime on a 10° slope; (a) and (b) represent soil eroded after 12 and 24 hours, respectively under invariable rain intensity (c) and (d) under variable rain intensity

Wirkung des Regens an einem Hang mit 10° Neigung; (a) und (b) geben den nach 12 bzw. 24 Stunden erodierten Boden an bei unveränderter, (c) und (d) bei wechselnder Regenintensität

erosion was dobled, an increase in lateral water flow was registered with a decrease in the underground level.

Runs 5 and 7 test the sensitivity of the model to the depth of the upper porous soil layer. The change in depth from 10 cm to 30 cm influences the appearance of runoff. The shallow soil layer gets saturated after some time and thus runoff appears, whereas the deeper soil layer does not reach saturation in 24 hours of simulation.

The effective porosity of the upper soil layer is another parameter that may influence soil erosion. In run 2 the effective porosity was decreased relative to run 1 from 43% to 20%. This produced a rapid saturation of the layer in run 2 and runoff appeared after 12 hours from start of simulation. No great differences in amount of soil eroded were produced in the two runs because of the relatively low steepness of the slopes and small quantitites of rainfall after the runoff appearance.

The influence of changing of the hydraulic coefficient of the sublayer was tested in runs 4 and 10. An increase in the hydraulic coefficient induces faster drainage of the upper layer and thus prevents saturation of the upper layer and runoff (Figures 6 and 7).

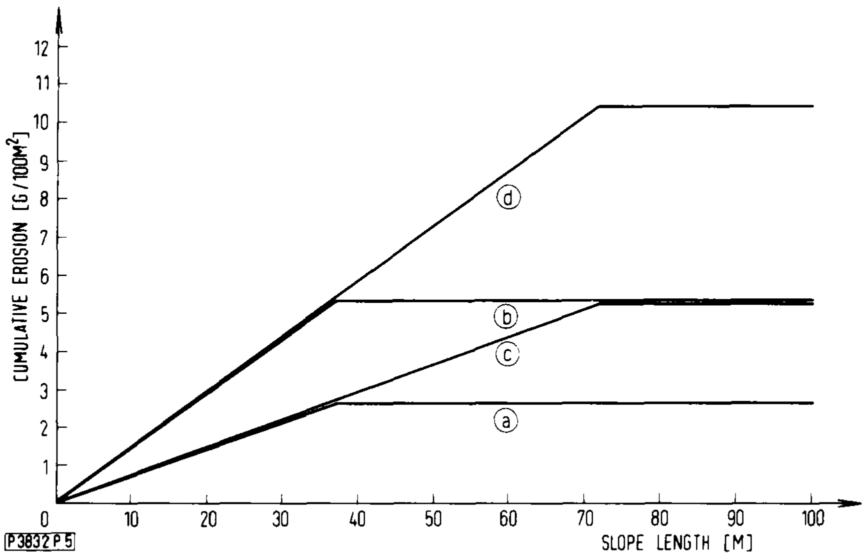

Figure 5: Effect of slope angle on soil erosion; (a) and (b) erosion on a 10° slope after 12 and 24 hours respectively (c) and (d) on a 30° slope

Einfluß des Hangwinkels auf die Bodenerosion, (a) und (b) bei 10° Hangneigung, (c) und (d) bei 30° Hangneigung jeweils nach 12 bzw. 24 Stunden

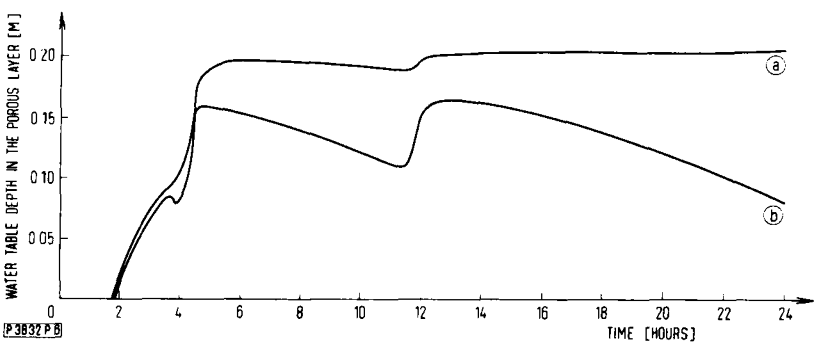

Figure 6: Effect of water conductivity of soil underlayer on water table; (a) water table in a 10° soil slope with water conductivity of 0.00036 m/hour (b) with 0.0036 m/hour

Einfluß des Wasserleitungsvermögens der unteren Bodenschicht auf den Wasserstand (a) in einem Hang mit 10° Neigung und einem Wasserleitungsvermögen von 0.00036 m/Stunde bzw. (b) 0.0036 m/Stunde

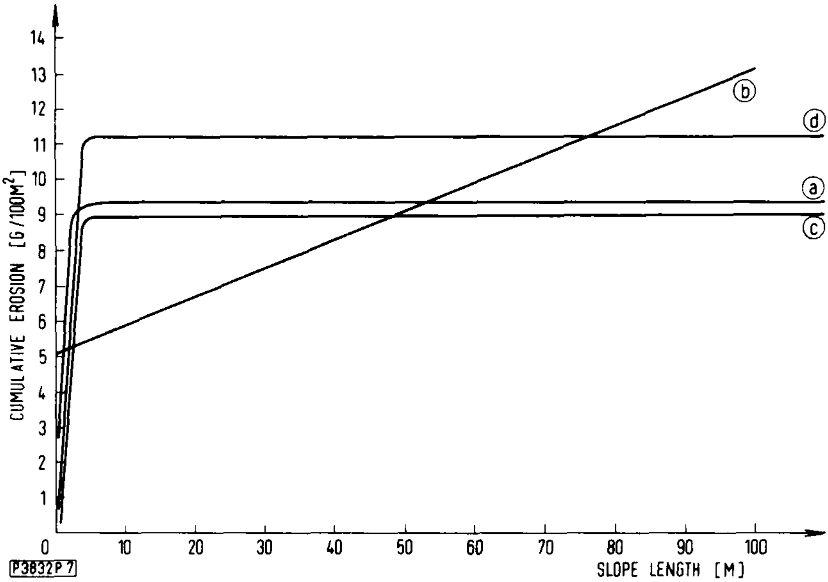

Figure 7: Runoff effect on soil erosion: (a) and (b) erosion with impermeable underlayer permitting run-off appearance after 12 and 24 hours respectively, (c) and (d) with high permeability of underlayer and without run-off after 12 and 24 hours

Wirkung des Oberflächenabflusses auf die Bodenerosion: (a) und (b) bei undurchlässiger Unterbodenschicht und Runoff-Auftreten, (c) und (d) bei sehr durchlässiger Unterbodenschicht ohne Oberflächenabfluß jeweils nach 12 bzw. 24 Stunden

Summary

A model has been designed to describe the process of soil erosion by rain and runoff. The mathematical representation has been derived from the works of *Meyer* and *Wischmeier* (1969) and *Ishihara* (1967). Modifications to those were induced to enable wider application. The model computes the amount of soil and nutrients carried away from field surfaces by the erosion process. In the first stage, the model computes the discharge rate of water on the soil surface and the upper layer of the soil. In the second stage, the contribution of the various erosion mechanisms to the loss of phosphorus, nitrogen and soil are calculated.

A number of runs with changing parameters were presented. They illustrate the sensitivity of the model to changes in soil properties and characteristics and rain pattern.

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Computersimulationsmodell des Phosphat- und Stickstofftransportes durch Wassererosion

Von *G. Kruh, E. Segall, A. Amberger* und *J. Hagin*

Anhand der Ausarbeitungen von *Meyer* und *Wischmeier* (1969) und *Ishihara* (1967) wurde mit einigen Abänderungen ein Computermodell entwickelt, das den Prozeß der Bodenerosion durch Regen und Wasserabfluß beschreibt. Das Modell berechnet die Pflanzennährstoffmengen, die während des Erosionsprozesses von der Oberfläche weggeschwemmt werden. In der ersten Phase berechnet das Modell die Wasserabflußrate auf der Bodenoberfläche und in der obersten Bodenschicht und danach den Anteil der verschiedenen Erosionsmechanismen am Verlust von Phosphor, Stickstoff und Bodenmaterial.

Eine Anzahl von Computerdurchläufen mit verschiedenen Parametern zeigt die Empfindlichkeit des Modells für Änderungen der Bodeneigenschaften und Regenverteilung.

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