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On uncertainty quantification in sandwich structures with spatial random damping behavior

K. Sepahvand and S. Marburg
Institute of Mechanics, Universität der Bundeswehr, Munich, Germany
sepahvand@unibw.de

ABSTRACT : Lightweight structures of multi–layer fiber–reinforced polymer composites and hybrid metal–plastic composites offer high damping capabilities and a high stiffness–weight ratio. The damping properties of such components are influenced by interfaces and local inhomogeneities resulting in uncertainties and non–linearities of the characterizing parameters. The investigations are focused on the uncertainty quantification of amplitude depending damping by stochastic finite element procedures. Uncertain and locally varying damping parameters are represented with the help of a stochastic field using Karhunen–Loève expansions. The uncertainty is involved into the damping properties of viscoelastic layers. The complex frequencies are represented using generalized polynomial chaos expansion. To verify the developed and extended simulation method, experimental modal analysis is performed on samples of the polymer reinforced composite plates. All investigated samples show a varying damping ratio depending on the displacement amplitude. This helps to identifying prior information on the damping properties of the viscoelastic layer. Knowing the Karhunen–Loève expansions of the damping properties, a stochastic finite element model is executed to estimate the unknown coefficients of the polynomial chaos expansions representing the frequencies. The results are compared with the experimental data.
Keywords: Composite materials, viscoelastic materials, uncertainty quantification, uncertain damping

1 Introduction

Bending vibrations of thin–walled components contribute significantly to the noise generation of machines, equipment and vehicles. Implementing additional damping materials may help to reduce noise propagation, however, may be in conflict with the component light–weight properties. Such mixed adverse properties are particularly common in wide range of composite materials where the damping behavior is dependent on a number of parameters which are generally characterized by non–linear relationships. Available numerical models are not sufficiently accurate to capture these relationships. For instance, the influence of the deflection on the damping behavior cannot be modeled by using existing material models, even though this influence has been experimentally demonstrated. Furthermore, the impact of uncertainties and material parameter scattering has not been reported. This is much more dealt in multi–layer composites where material parameters describing the structure behavior are usually affected by manufacturing process.
There are numerous physical interdependencies and mathematical models to describe the damping behavior of sandwich composite structures. A key objective here is to reflect as accurately as possible the energy dissipation during dynamic loading in order to predict the dynamic behavior of components realistically. Several analytical and numerical methods have been developed in past decades to predict the damping in such composites. Comprehensive theoretical considerations can be found in relevant literatures where various nonlinear dependence of the damping has been experimentally demonstrated [1–4]. Work like [5] consider nonlinearities with respect to the strain amplitude and frequency of cross–material for metals, ceramics, mineral materials, as well as comparative for thermoplastics ([6]). Especially, for continuous fiber–reinforced multilayer composites having matrix material, the fiber volume content and fiber orientation of the individual layers have a dominant influence on the damping properties while providing a high potential of the targeted property modification, see [7–9]. The works [10, 11] describe in detail a method to identify damping behavior of fiber reinforced polymer (FRP) experimentally and, as well as [12] for linear application of viscoelastic material behavior in the finite element method (FEM).
Demonstration of the damping using generalized rheological serves constitutive laws involving fractional time derivatives and thus take into account the deformation history. In [13] the applicability of such a 3–parameter models for polymers shown by means of damped vibration, but so far not taken into account the deflection dependency. The noise–insulating behavior of hybrid sandwich composites with polymer core layers of a few micrometers thickness has been investigated in [14]. In all those works, it is assumed that all parameters are either constants or at least their corresponding dependencies are known. Nonlinear behavior typically has a high degree of complexity in terms of models and their parameters. In contrast to the mass and stiffness parameters, the reliable quantification of damping is difficult due to the influencing state variables and associated uncertainties. For that reason, the real dynamic of such structures is consequently described incompletely or with limited accuracy. In several publications simple viscose damping model is used to quantify the damping uncertainties where the damping coefficient assumed to be a random

variable, see e.g. [15, 16]. The effects of uncertain or scattering damping on the system response using linear models by means of the perturbation theory, the first and second order reliability method (FORM/SORM) or Monte Carlo (MC) simulation analysis has been reported [17–19]. The influence of this uncertainty and the geometric variability was investigated by possibilistic and probabilistic methods for different FRP structures [20, 21]. They are based primarily on methods of fuzzy sets [22, 23], stochastic FEM (SFEM) and MC simulation [24, 25]. The determination of uncertainties using probabilistic methods is very complex due to the underlying sampling method and high accuracy of the simulation requires a very large number of calculations. For example, MC method converge slowly and their efficiency depends on the standard deviation [26]. Furthermore, the calculations have to be performed again in full even with a slight modification of the frame data. To reduce the high computational cost associated with this, alternative methods have been developed, such as the Karhunen–Loéve the generalized polynomial chaos (gPC) expansions [27, 28] . These expansions in combination with the FEM have been used for the uncertainty analysis of different stochastic problems, see e.g. [29–33], as well as to nonlinear problems [34, 35].

In this paper, we employ the SFEM to analysis of sandwich structures with random geometrical and material parameters. To this end, the KL expansion is used to represent the input random parameters and free vibration responses are approximated using the gPC expansions. Stochastic simulation is carried out using deterministic FE model on sample collocation points in random space. The method is applied to sandwich beams with viscoelastic layer for which the thickness and material parameters of core are considered random. The results show high accuracy of the method with MC simulation of 1000 realizations.

The rest of the paper is organized as follows. Section 2 reviews two important discretization methods for random space. General SFEM formulation of structural free vibration is presented in section 3. The numerical simulations of the method illustrate in section 4. Section 5 discusses the conclusions.

2 Discretization of random quantities

As the complex deterministic problems are discretized by FEM techniques, the discretization of random quantities in stochastic space is required. Namely, the stochastic space has to be discretized for the treatment of randomness in the physical system to be adapted to the implementation of the deterministic FEM model, which can deal with geometrical complexity. In this paper we use spectral methods based on constructing the functional dependence expressed in terms of a series [36, 37]. In these methods the random quantities are represented by spectral decomposition with unknown coefficients and orthogonal polynomial basis. A least–squares fit can be used to determine the coefficients of the expansion. They use commonly the Karhunen–Loève (KL) theorem and polynomial chaos as discussed below.

2.1 Karhunen–Loève expansion

The expansion is a representation of a random quantity as an infinite linear combination of orthogonal functions which are the eigenfunctions of the covariance function. By definition, the covariance function is real, symmetric and positive definite. Thus, all its eigenfunctions are mutually orthogonal and form a complete set spanning the function space. This property is employed to construct the KL expansion of the random field w as

$$w(\mathbf{x}, \boldsymbol{\xi}) = w_0(\boldsymbol{x}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \xi_i f_i(\mathbf{x}) \tag{1}$$

where ξ_i denote independent standard normal random variables spanning the probability space and λ_i and $f_i(\boldsymbol{x})$ are the eigenvalues and eigenfunctions of the covariance function, i.e.

$$\int_D C(\mathbf{x}, \mathbf{x}') f_i(\mathbf{x}') d\mathbf{x}' = \lambda_i f_i(\mathbf{x}) \tag{2}$$

in which D denotes the spatial domain over which the random quantity $w(\mathbf{x}, \boldsymbol{\xi})$ is defined. Usually only a few of the terms with the largest eigenvalues are important assuming the eigenvalues are ordered by decreasing magnitude. For a higher rate of spectral decay, the smaller number of terms is required for approximation. Provided that the exact eigenfunctions of the covariance coefficients are available; the KL method is the most efficient method for discretizing a random quantity. The most important feature of the KL discretization method is that the spatial random fluctuations are decomposed into a set of deterministic functions f_i multiplying random coefficients. However, the application of the KL expansion is limited by the fact that the covariance function has to be known apriori. In particular, it is extremely difficult to determine a covariance function if the KL is employed to represent uncertainty in model responses. In such condition, they can be formally expressed as some nonlinear functional of random vector $\boldsymbol{\xi}$ by using the polynomial chaos to overcome the KL limitations.

2.2 Polynomial chaos expansion

It was first introduced in the form of homogeneous chaos expansion by Wiener [38]. The generalized Polynomial Chaos (gPC) permits the discretization of multi–dimensional non–Gaussian and non–stationary random quantities. Using this approach, the random quantity w is expanded as

$$w(\mathbf{x}, \boldsymbol{\xi}) = \sum_{i=0}^{\infty} w_i(\mathbf{x}) \Psi_i(\boldsymbol{\xi}) \tag{3}$$

where w_i are deterministic unknown functions. The random base functions Ψ_i are a set of complete multidimensional polynomials in terms of the multidimensional random variable $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_n\}^t$ with the orthogonality relation of

$$\mathbb{E}[\Psi_i, \Psi_j] = \mathbb{E}[\Psi_i^2]\delta_{ij} = h_i^2 \delta_{ij} \tag{4}$$

where δ_{ij} represents the Kronecker delta, h_i is the norm of the polynomials. The unknown deterministic functions can be determined using Galerkin projection with the base orthogonal polynomials $\Psi_k(\boldsymbol{\xi})$ as [39]

$$w_k = \frac{\langle w(\mathbf{x}, \boldsymbol{\xi}) \ , \ \Psi_k(\boldsymbol{\xi}) \rangle}{\langle \Psi_k(\boldsymbol{\xi})^2 \rangle} \ , \quad k = 0, 1, 2, \ldots \tag{5}$$

A complete discretization is obtained once the deterministic functions are known. The dimension of Ψ reflects the spatial complexity of the process. It has the same order as the expansion. The first order terms of Ψ represent a Gaussian process and thus the expansion reduces to the KL representation of the same random quantity. We note that the knowledge of the correlation function is not necessary to uniquely determine the set of coefficients $w_i(\mathbf{x})$ appearing in the gPC expansion. Higher order terms in Ψ show the probabilistic complexity and can be applied for representation of non–Gaussian processes. For practical simulation, the series in Eq. (3) is truncated to a finite number of terms.

3 Stochastic FEM on vibration of sandwich beams

In this section, the stochastic FEM (SFEM) for free vibration of sandwich beams is formulated based on spectral expansion. The sandwich beam is comprised of three layers; two elastic face layers denoted as layer 1 and 3 and constrained viscoelastic core layer 2. The beam is made of two thin face sheets with constrained viscoelastic core. It is assumed that the transverse displacement and the rotational deflection are constant across the beam cross–section. Furthermore, the core layer has negligible bending and extensional rigidity and is subjected only to shear deformation. In contrast, the face sheets possess no shear deformation. It is also assumed that no slip occurs between the layers, and there is perfect continuity at the interface. The deflections are defined with respect to the neutral axis of the composite beam. Four degrees of freedom is assumed per section of the finite element as shown in Fig. 1: u_i and u_j denote the axial displacements

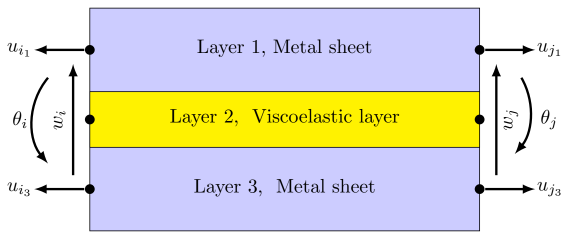

Figure 1: Degrees of freedom for sandwich beam element.

of each face sheet metals, w is the transverse displacement and the rotational degree of freedom of the face sheets at each section is defined as θ. Accordingly, the vector of element displacements for the core is defined as

$$\boldsymbol{u} = \{u_{i_1}, u_{i_3}, w_i, \theta_i, u_{j_1}, u_{j_3}, w_j, \theta_j\}^t \tag{6}$$

The elastic isotropic material model is assumed for face layers and the viscoelastic core is modeled with its complex shear modulus G_c^* as

$$G_c^*(\omega) = G_c^{'}(\omega) + jG_c^{''}(\omega) = G_c^{'}(\omega)\,[1 + j\eta(\omega)] \tag{7}$$

in which $G_c^{'}$ and $G_c^{''}$ are the shear storage modulus and the loss modulus, respectively, and η is the loss factor. Here, and subsequently, f and c denote face and core. The stiffness and mass matrices are estimated from the strain and kinetic energies of the element. These energies are calculated for each layer individually. In particular, the strain energy U of faces is constructed from extensional and bending components. The shear strain in the core is used to develop the core stiffness matrix, i.e.

$$U = \frac{1}{2}\boldsymbol{u}^t\left(\boldsymbol{K}_b^f + \boldsymbol{K}_e^f + \boldsymbol{K}_s^c\right)\boldsymbol{u} \tag{8}$$

In which $\boldsymbol{K}_b^f$, and $\boldsymbol{K}_e^f$ are bending extension stiffness matrices of the faces and $\boldsymbol{K}_s^c$ is the shear stiffness matrix of the core. Due to the complex structure of the core, it is not constant within the element and uncertainties are involved in this layer. The face sheet parameters are considered deterministic. To this end, the stiffness matrix $\boldsymbol{K}_s^c$ is represented as a function of space coordination $\boldsymbol{x}$ and random vector $\boldsymbol{\xi}$, i.e.

$$\boldsymbol{K}_s^c = \boldsymbol{K}_s^c(\boldsymbol{x}, \boldsymbol{\xi}) \tag{9}$$

In this work, we approximate stochastic element stiffness of the core by means of KL expansion as

$$\boldsymbol{K}_s^c(\boldsymbol{x}, \boldsymbol{\xi}) = \boldsymbol{k}_0(\boldsymbol{x}) + \sum_{i=0}^{N}\sqrt{\lambda_i}\boldsymbol{k}_i(\boldsymbol{x})\xi_i f_i(\boldsymbol{x}) \tag{10}$$

where $\boldsymbol{k}_0(\boldsymbol{x})$ is the mean value of the core stiffness and $\boldsymbol{k}_i \in \mathbf{R}$ are deterministic coefficient matrices of the KL expansion. It is written in the compact form of

$$\boldsymbol{K} = K_0 + \sqrt{\Lambda}\mathcal{K}\boldsymbol{\xi}\mathcal{F} \tag{11}$$

The column vectors $\boldsymbol{\xi}$ and $\mathcal{F}$ assume the role of shape functions characterizing the random and spatial variation of material properties. Clearly, terms associated with the small amplitudes tend to have only slight impact on the overall representation of the stiffness. Accordingly, the stochastic elemental strain energy are adopted from Eq. (12) as

$$U(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{u}^t(\boldsymbol{x}, \boldsymbol{\xi})\left[\boldsymbol{K}_b^f + \boldsymbol{K}_e^f + \boldsymbol{K}_s^c(\boldsymbol{x}, \boldsymbol{\xi})\right]\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{\xi}) \tag{12}$$

The element mass matrix $\boldsymbol{M}$ is deduced deterministically for all layers derived from the kinetic energy:

$$T = \frac{1}{2}\dot{\boldsymbol{u}}^t\boldsymbol{M}\dot{\boldsymbol{u}} \tag{13}$$

The damping behavior of faces is ignored owing the fact that it is very small compared with the core damping. The random eigenvalues γ of the beam are complex exhibit as

$$\gamma(\boldsymbol{\xi}) = \omega^2(\boldsymbol{\xi})\,[1 + \mathrm{j}\eta(\boldsymbol{\xi})] \tag{14}$$

Assembly of the element stiffness, mass and damping matrices lead to the SFEM model of the beam vibration in the frequency domain in the following form

$$\gamma(\boldsymbol{\xi})\boldsymbol{M}\boldsymbol{U}(\boldsymbol{x}, \boldsymbol{\xi}) = \boldsymbol{K}(\boldsymbol{x}, \boldsymbol{\xi})\boldsymbol{U}(\boldsymbol{x}, \boldsymbol{\xi}) \tag{15}$$

Due to the lack of information about the covariance function of responses, the model nodal vector, the eigenfrequency and the damping ratio are approximated using the gPC expansion at the same time as

$$\boldsymbol{U}(\boldsymbol{\xi}) = \boldsymbol{u}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{16}$$

$$\omega^2(\boldsymbol{\xi}) = \boldsymbol{a}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{17}$$

$$\eta(\boldsymbol{\xi}) = \boldsymbol{d}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \tag{18}$$

In which $\boldsymbol{u}$ is deterministic unknown functions, $\boldsymbol{a}$ and $\boldsymbol{d}$ are vectors of unknown deterministic coefficients. The basis $\boldsymbol{\Psi}$ are multi–dimensional random orthogonal polynomials of the same random variables used for KL expansion of the material properties. Substitution of the KL and gPC expansions in Eq. (15) yields to

$$\begin{aligned} &\boldsymbol{a}^t\boldsymbol{\Psi}(\boldsymbol{\xi})\left[1+\mathrm{j}\boldsymbol{d}^t\boldsymbol{\Psi}(\boldsymbol{\xi})\right]\boldsymbol{M}\boldsymbol{u}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \\ &=\left[K_0+\sqrt{\Lambda}\mathcal{K}\boldsymbol{\xi}\mathcal{F}\right]\boldsymbol{u}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \end{aligned} \tag{19}$$

This equation is SFEM of free vibration of sandwich beam from which the gPC coefficients of responses are calculated. To this end, the approximation stochastic error $\epsilon(t,\boldsymbol{\xi})$ has to be minimized, i.e.

$$\begin{aligned} \epsilon(t,\boldsymbol{\xi}) = \quad &\boldsymbol{a}^t\boldsymbol{\Psi}(\boldsymbol{\xi})\left[1+\mathrm{j}\boldsymbol{d}^t\boldsymbol{\Psi}(\boldsymbol{\xi})\right]\boldsymbol{M}\boldsymbol{u}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \\ &-\left[K_0+\sqrt{\Lambda}\mathcal{K}\boldsymbol{\xi}\mathcal{F}\right]\boldsymbol{u}^t\boldsymbol{\Psi}(\boldsymbol{\xi}) \end{aligned} \tag{20}$$

Once the stochastic structural responses are approximated by means of the gPC expansion, the non–intrusive SFEM runs the deterministic FE model of the structure as black–box and it is only called as a function for numerical calculation of the unknown coefficients. The method intends to compute the unknown gPC coefficients of random outputs by means of a pseudo–random sampling of the random variables in stochastic space. The sampling space are implicitly prescribed by the selected a few abscissae $\xi_i^n,\ n=1,2,\ldots,,M$. Therefore, in Eq. (20) one has to solve the minimization problem of

$$\int_{\{\xi_i\}}\epsilon(t,\boldsymbol{\xi})\,\delta(\xi_i-p_i)f(\xi_i)d\xi_i=0\,, \tag{21}$$

in which δ is the delta function and p_i denotes the set of specific collocation points. The support space can be simply chosen from the roots of random orthogonal polynomial Ψ. The method provide this major facility to use third–party commercial or free license deterministic FE codes as a black–box to get the system responses associated with each realization of random vector. These characteristics make the method very attractive for parametric quantification in complex models and industrial applications where the deterministic FE model has been already developed. The key idea is to use deterministic FE codes to perform spatial discretization to arrive at a system of random algebraic equations to employ in the SFEM.

4 Numerical results

In this section, we apply the above formulation to a cantilever sandwich beam with random core properties, see Fig. 2. The dimension and material parameters are given in Table .1. All

Table 1: Nominal dimensions and material properties of investigated sandwich beam, cf. Fig. 2.

Dimensions [mm]	$l=180$ $h_f=0.45$	$h_c=0.05$
Material: faces	$E_f=200$ Gpa	$\nu_f=0.3$
core	$G_c=1.73$ Mpa	$\nu_c=0.48$

topological and material parameters of face layers are considered

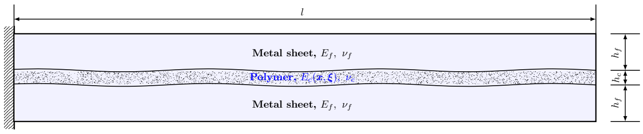

Figure 2: Cantilever sandwich beam with random constrained layer properties. Face layers possess deterministic parameters.

deterministic. The thickness h_c and the shear modulus G_c of the core layer are treated as random parameters. They are approximated locally be means of the KL expansion with covariance function of the spatial homogeneous as a function of the distances between two points on the beam core. The squared–exponential covariance function $C(x_1,x_2)$ which reflects the correlation at any two points (x_1,x_2) in the beam domain defined as

$$C(x_1,x_2)=\sigma^2\mathrm{e}^{-\frac{(x_1-x_2)^2}{l_c^2}} \tag{22}$$

The standard deviations are assumed to be $\sigma_G=0.35$ Mpa and $\sigma_h=0.01$ mm. The correlation length is assumed to be $l_c=l$. Large correlation length, $l_c>>L$, means small variations within the beam length. In such a case, the spatial dependency of the random parameters can be ignored and only σ–values play major role and the uncertain parameters can be represented as a random variable depending only to random vector $\boldsymbol{\xi}$. An important issue facing the KL approximation of the uncertain parameters is selecting the dimension of random vector $\boldsymbol{\xi}$. In other words, the optimal representation depends to the number of terms in the KL expansions which directly limits the random space dimensions. For that we calculate the relative weight $\mathcal{W}$ of i^{th}–eigenvalue of covariance function as

$$\mathcal{W}_i=\frac{\lambda_i^2}{\sum_{i=1}^n\lambda_i^2}\times 100 \tag{23}$$

which yields to $\mathcal{W}=\{69.01,\ 21.53,\ 6.17,\ 2.07,\ 0.83\}$ for the first 5 eigenvalues. For a good approximation we used the first 2 terms of the KL expansion to estimation the spatial variations of the core thickness and the shear modulus. This implies that the dimension of random space is reduced to four, i.e. $\boldsymbol{\xi}=\{\xi_i\}_{i=1,2}$ with $\xi_i\in N(0,1)$. The mean values and some realizations of the uncertain parameters are shown in Fig. 3. A robust SFEM procedure must capture the structure behavior for all realizations represented as the KL expansion. Third order gPC expansions employing 2–dimensional Hermite polynomials are used to approximate the uncertain natural frequencies ω^2 and damping ratios η. A deterministic FE model with 10 SHELL281 elements in ANSYS is considered as a black–box solver. The model is executed on 17 sample collocation points produced from the roots of fourth order Hermite polynomials. The mean values and the standard deviations of output parameters are given in Table 2. The

Table 2: The mean values and standard deviations of the modal parameters, frequencies f and damping ratios η.

	(f_1,η_1)	(f_2,η_2)	(f_3,η_3)	(f_4,η_4)
μ	$(63,0.37)$	$(303,0.26)$	$(361,0.29)$	$(617,0.48)$
σ	$(1.04,0.005)$	$(15.17,0.0065)$	$(28.05,0.015)$	$(33.50,0.033)$

probability density function (PDF) of the natural frequencies f [Hz], $\omega=2\pi f$, and damping are shown in Fig. 4(a) and (b) in

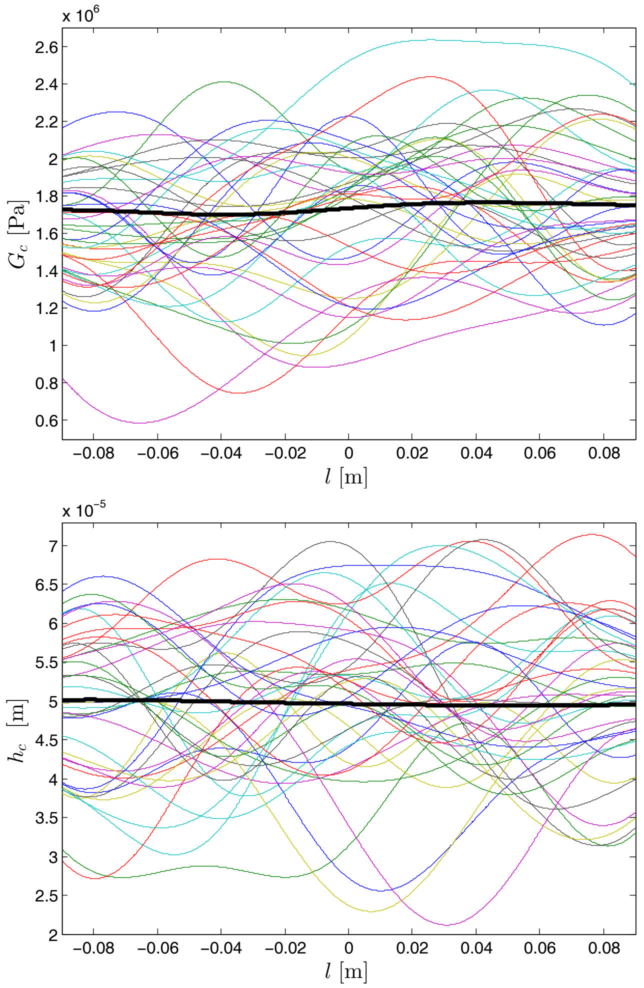

Figure 3: Some realization samples of the random core parameters, G_c and h_c. The bold lines show the mean values.

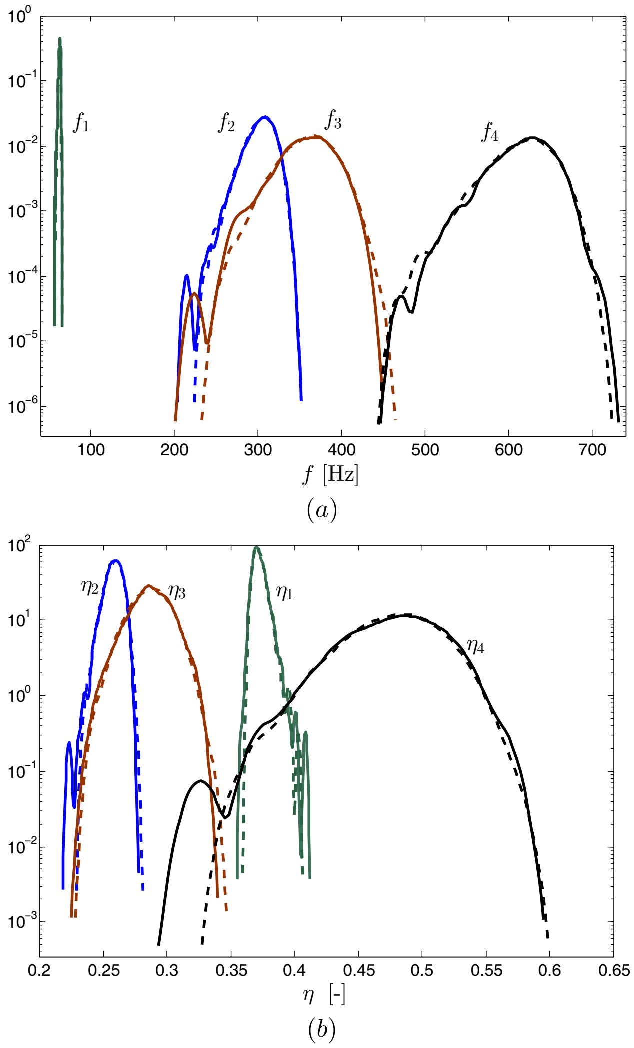

Figure 4: Probability density functions of : (a) the first 4 natural frequencies and (b) damping ratios of the cantilever sandwich beam, constructed from the third order gPC (bold lines) and 1000 realization from the MC simulation (dashed lines).

comparison with 1000 realizations form the MC simulations. As shown, the results from the third order gPC with 17 collocation samples are in high agreement with the MC simulations. The parameter uncertainties had different impact on the natural frequencies and damping ratios. While the first mode frequency is least affected from these uncertainties, other modes show large variations. This is, particularly, distinguished for the third and fourth modes. The parameters display considerable overlap among the intervals, with a great deal for the third mode frequency. Third mode is torsion mode which is most sensitive to uncertainty in shear modulus. This is a result of the greater uncertainty in the latter mode. The critical issue relating to the mode overlap comes out when this overlapping causes the mode shape change. This, however, requires to investigate the mode shape spatial variations.

5 Conclusions

In this paper, we presented and discussed the application of stochastic FEM to free vibration of sandwich composite beams with random viscoelastic parameters. The locally and random variations of the core layer represented by means of KL expansions and the gPC expansions used to approximated the modal outputs, i.e. natural frequencies and damping frequencies. A deterministic FE model employed as a solver to evaluate the modal outputs at some collocation points. The results were in high agreement with MC simulations of 1000 realizations. Furthermore, modal parameters are influenced by the input parameter uncertainty in different levels. More especially, the natural frequency of the first torsion mode is mostly affected from the randomness of core layer. The output parameters also demonstrated some interval overlap. This may be more critical when this overlapping results to the mode change, an issue which cannot be followed from the modal data and a spatial variations in mode shape in required.

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