RESEARCH NOTE

• *ABSTRACT*

The dynamics of scientific specialization are investigated in the field of mathematical logic — a major subdiscipline of mathematics, embracing some 15,000 authors from 1874 through 1990. The following dimensions of specialization are described quantitatively, using a comprehensive bibliography: the number of areas of this subdiscipline in relation to the number of contributors; the frequency distribution of the number of areas within logic that those contributors deal with; the analogous frequency distribution of the most prolific logicians; and the degree of division of labour between these prolific logicians. The salient characteristics of these distributions is their skewness, pointing to 'Lotka's Law' and other similar distributions, which are discussed as quantitative indicators of scientific self-organization.

Self-Organization of Scientific Specialization and Diversification: A Quantitative Case Study

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Over-specialization is often complained about in science. The worries mainly concern the narrowness of an expert's knowledge, with a fragmentation and disintegration of the stock of scientific findings as a first, and poor utilization as a second, consequence. The number of such complaints, however, seems to be inversely proportional to the number of concrete empirical investigations dealing with specialization and scientific division of labour. And since little is known about specialization, not much more will be known about over-specialization. In what follows, patterns of specialization will be studied in a subdiscipline of modern mathematics — namely, mathematical logic.

By 'specialization' one could mean the quantitative relationship between (in this case) the number of logicians and the number of logic fields. Therefore, the first aspect to consider will be that relationship from the beginning of logic until the mid-1980s. The

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next aspect will be the degree of specialization inside the community of logicians: the question of how many logicians work in how many areas. The notions of 'specialization' and 'diversification' are treated as two sides of the same coin. Both will be described with the help of a single frequency distribution of the number of different occupations a logician deals with during a certain period in the development of logic. The specialization of a group of logicians is low if many logicians work in many fields of logic; it is high if many logicians work only in one or a few fields.

The third aspect concerns the specialization of a single logician. It is low if he or she works evenly in many fields and does not concentrate in only one field; it is high if there is a concentration in only one or a few fields. This question will be examined in a group of the most prolific logicians.

But before I proceed, I would like to explain why mathematics is a most attractive subject in science studies, as a paradigm of self-organization. Mathematics — and especially mathematical logic — is more suitable for such an analysis than many other types of science or scholarship, since one is dealing here with activities of 'pure reason'. The content of those activities does not depend on machines (with the partial exception of computers), not on experiments, not on labs and assistants, and not on knowledge of things beyond its formal realms. But, despite its 'pureness', there is a steady flow of mathematical knowledge into other domains of science. The utilization of mathematical knowledge is well known in physics, for example; conversely, the application of physical knowledge to mathematical theorems is not possible. There is an inflow of ideas and stimuli from physics into mathematics, of course; nevertheless, there is not any kind of direct application of a theorem concerning the external physical world in a purely mathematical context. All this implies an independence of this formal science, on the one hand, and applicability, on the other hand, which makes studies of cognitive endeavours in mathematical fields more attractive than studies of cognitive efforts in other fields of science and scholarship. A further advantage is the existence of elaborate and consistent classification schemes in mathematics.

The importance of mathematical logic as a subdiscipline of modern mathematics lies not only in its influence on other areas of mathematics and on philosophical foundations of abstract reasoning. Formal logic was also involved in the development of the theoretical foundations of a device which has become part of our daily lives, namely the computer. But there was no typical client–supplier relationship; many logicians had been dead for a long time before the fruits of their work influenced computer science. This may make even an ethnographic observer (and user of computers, as a rule) concede that the logicians' job might include a bit more than telling 'good stories' to other logicians; and that there are more important functions and tasks for logicians than to strive for consensus with scientific fellows, or for power over them, with the help of their work.

Thus, the edifice of mathematical logic is set up by men and women who presumably were working, interacting and sometimes collaborating in a highly autonomous manner. In many instances, nobody would ever have been able to foresee that the outcome would be something that now is recognized in hindsight as a coherent system of fruitful logic areas.

The roots of mathematical logic can be found in the middle of the nineteenth century, in a 'philosophical' and esoteric little sub-area of the rapidly expanding pure mathematics of those times. In its first decades, the formation of that sub-area went on only slowly, and interest in it was moderate; there was neither a visible nor an invisible college. This amorphous condition began to change at the turn of the nineteenth century. Now the new field also reached such a scientific profile as to evoke major opposition. The opposition found its strongest expression in Germany, where a mélange of romanticism, anti-Anglicism and misunderstanding (apart from profound critique) was brewed. Mathematical logic was a threat to metaphysical and philosophical logic, just as experimental psychology was a threat to philosophical psychology, some years before.[1] Logic survived, however. In the middle of our century, its mathematical branch already involved roughly 300 participants. In 1965 it surpassed 1000 participants, and at the end of the 1970s, 3000. Logicians are counted as 'participants' from the first until (and including) the last year of their publications in logic. From 1874 through 1990, about 15,000 logicians had written some 50,000 books and articles.[2] For the purpose of this investigation, these publications are seen as a reflection of logicians' cognitive activities, and they can be studied with the help of a comprehensive bibliography. Some features of the pertinent *Ω-Bibliography of Mathematical Logic* will be reported on now.[3]

The Data-Source

The electronic version of the bibliography was at the disposal of the Institute for Philosophy of the Technical University in Munich, thanks to the courtesy of the *Forschungsstelle Mathematische Logik* of the Heidelberg Academy of Sciences in Germany, and of the Springer publishing house. The main purpose of the bibliography is a coherent bibliographic presentation of the research literature from the beginning of mathematical logic to the present day.

To prepare the bibliography for our purposes, all entries under editors were eliminated, and only contributions of authors or co-authors of books or papers were considered. A co-author's contribution was treated in the same manner as a contribution of one author only. I give an example of an entry in our working database:

> Goedel K. Die Vollständigkeit . . . [rest of the title in the printed version] 124 [= code number of publication source] 1930 [= publication year] B28 B30 D20 D35 F25 F30 F35 [= classification codes]

The bibliography conceives of logic in a mathematical sense, embracing formal, symbolic and algebraic logic as well as proof theory and metamathematics; not included is philosophical logic (included, however: philosophical questions of mathematical logic) or general philosophy of mathematics. The first entry of the (slightly more comprehensive) electronic edition is a famous article by Georg Cantor in 1874, entitled *Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen*; the first entry of the printed version is dated 1878. Of course, Cantor's article is not considered to be apt as a starting point for mathematical logic by all experts; but the identification of a starting point in many a new field of science is controversial to some degree.

The last entries of the electronic version of the bibliography are dated in 1990; since the finer classification is not done for entries after the publication year 1985, due to a lack of external support, our analysis will leave out the years afterwards. All entries of the bibliography were classified by a team of internationally acknowledged contemporary specialists in the respective main areas of logic — namely, classical and non-classical logic, set theory, model theory, recursion theory and proof theory. These broad divisions are subdivided into finer classification units. On the whole, the classification is an adapted and refined version of the classification scheme of the *Mathematical Reviews* and the *Zentralblatt für Mathematik* *und ihre Grenzgebiete*. Leaving out the broad divisions and considering only the finer classification units, 113 research areas are found in the bibliography, not considering categories having only a documentary function — for example, 'Collected Works' and 'Uncertain content, not identifiable'. Examples of research areas are: fragments of classical logic, many-valued logic, fuzzy logic, Turing machines, and so on.

The classification system is of crucial importance for the following analysis. It reflects mathematical logic from a modern viewpoint; sometimes, it was difficult for the classifiers not to do injustice to older works in logic, for which modern classification categories were used, but for which older categories seemed to be at least equally adequate. A classification system has the advantage that it is consistent and systematic; its disadvantage, compared with uncontrolled descriptors, is that it is sluggish and coarse. Apart from this, a classification decision inevitably includes subjective preferences of a classifier, even if competent classifiers will agree in the majority of cases. That a logic classification scheme can be fairly different from the *Omega* scheme becomes obvious by the inspection of a bibliography compiled by Alonzo Church for symbolic logic in 1936.[4] It would be almost impossible to convert some of the categories used by Church into categories used in the *Ω-Bibliography*.

But an act of classifying will not only betray the scientific preferences of the deciding expert; in addition, some mathematicians (as well as some sociologists) may disagree about whether a classification unit reflects the objective existence of a logic area or whether it is a social construct. I do not feel competent to comment upon this question. Rather, I restrict myself to the following two claims: If one prefers to consider logic areas (as classified in the bibliography) as social constructs, then my analysis is a description of the outcomes of such social constructs and a description of the interactions of those constructs with logicians. If, on the other hand, the logic areas are supposed to reflect the objective existence of logic entities, then I describe how logicians deal with that reality.

When one goes back to the roots of the logical sub-areas in terms of the number of participants, a kind of 'epidemic' curve is seen in many cases. The epidemic process often follows a threefold scheme of inception, expansion and stagnancy or (moderate) decline in terms of the number of participating scientists.[5]

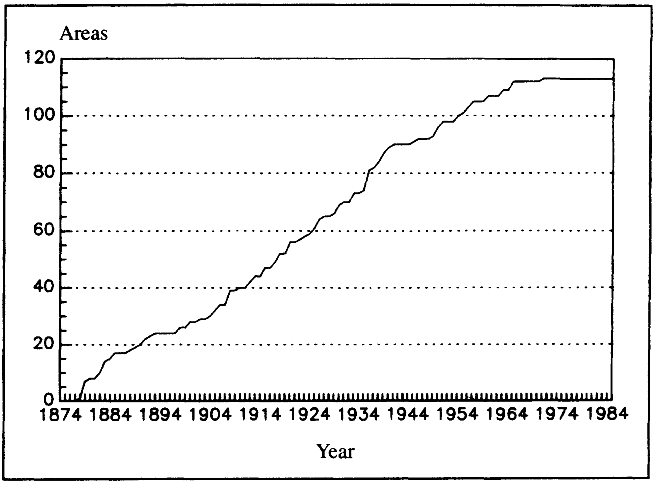
Results

The Ratio between the Number of Logic Areas and the Number of Contributors to Logic in the Last Hundred Years

In Figure 1, the cumulative number of new logic areas is shown. As its initial year, I took its first appearance in the *Ω-Bibliography*.[6] A marked linear growth is obvious from the figure; in the late 1960s, the growth fades out. With hindsight, however, it could turn out that, after some ten or twenty years, the growth continued linearily. It should be noted here that not a single one of the new-born areas died out. All 113 areas were traced until 1985 (with just one until 1984) — with differing strength, of course. Rescher has claimed that:

> At all of the lower taxonomic levels — problem-areas, subspecialties, branches, etc. — the subdivisions of science have been increasing in a geometric progression, doubling with inexorable regularity during every repetition of a fixed-size period.[7]

FIGURE 1
Cumulative Number of New Research Areas in Logic



Source: areas dated by their first appearance in the Ω-Bibliography, op.cit. note 3.

Obviously, this is not true in the case of logic. However, the total number of scientists participating in logic grew exponentially for a long time.[8]

Because the number of new logic areas did not grow as fast as the number of participating scientists (and the number of their publications), the ratio between the total number of areas and the number of contributors can be expected to decrease in the development of logic. In fact, the annual number of different logic areas treated by all logicians together divided by the number of participants in a year (with 'participant' defined in the above-mentioned sense), has fallen steadily since the first years of our century.[9]

Logicians appear to have been increasingly less able or willing to work on new logic areas, as far as can be seen through the classification scheme applied here. But there is no sign that the average intensity of commitment to logic changed greatly in that time, if one takes the average amount of publication output as a statistical indicator of scientists' occupation with their discipline. The average productivity did not change considerably, with some reservations to be made for the time before World War II. Hence, in a flourishing field of scientific activity (flourishing in terms of the increasing number of contributors and of scientific areas), one has to expect a process in which the rapidly increasing number of participants is occupied with a number of specialties increasing at a slower pace. And since the average number of contributions per author fluctuated for some time in logic, but did not show a markedly increasing or decreasing trend from 1874 through 1990,[10] I assume that the average strength of engagement of a scientist neither decreased, nor increased during this time.[11]

The emergence of a fruitful specialty can be understood as a scientific innovation process. Because the 'terms of trade' between the number of participants in logic and the returns (as measured in new specialties) are getting worse, one can speak of a 'principle of diminishing returns'.[12] It has to be stressed here again that the principle works only with respect to scientific activities as they become visible by means of the bibliographic classification scheme described above. Activities which were not yet sufficiently successful, or are too recent to establish a new area in logic, fall through the raster. Uncontrolled subject descriptors are surely a more flexible indicator of subject content; it is an open question whether the use of such descriptors would change the picture essentially.

Nevertheless, a principle of diminishing returns also proved to work in terms of the proportion of important contributions to all contributions in logic from 1847 to 1936. The evaluation of the literature was undertaken by Alonzo Church, one of the leading logicians of our century.[13]

The decreasing number of new areas (compared to the growth of contributors) can be interpreted as an increasing elaboration of known ideas. An increasing refinement and elaboration of existing ideas is a characteristic concomitant of major and complex innovations. If such a process is meant by the notion of 'specialization', then specialization undoubtedly increased in logic.

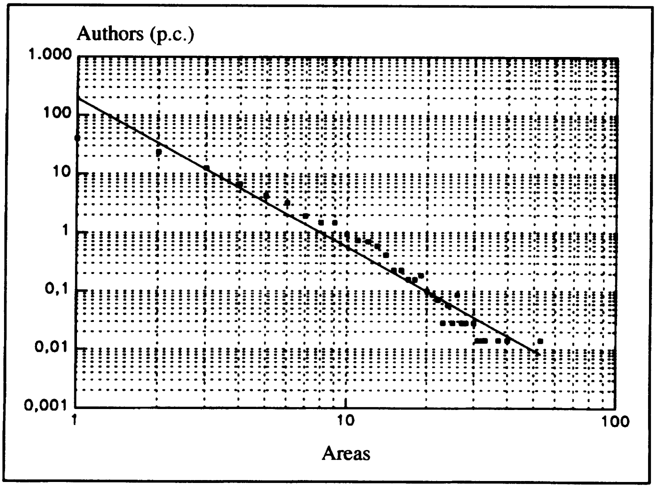
The Frequency Distribution of the Number of Areas in which a Logician Works

So far, only averages have been tackled — for example, the average number of logic areas a logician is confronted with. But skew distributions of scientists' capacity, possibility or desire to cope with different scientific fields were to be expected, conforming with well-known skew distributions in economic or scientific contexts. This means, concretely, that only a few logicians are supposed to work in many areas of logic; the majority of logicians are expected to work only in one or a few areas. A previous analysis resulted in an approximately hyperbolic frequency distribution for all logicians from 1874 until 1985;[14] in that analysis, however, only one (the first) classification code of a publication was taken into account.

Now each area touched by a logician's contribution is drawn upon for a more comprehensive computation of the frequency distribution of scientific diversification. I took into consideration the publications of the 7094 authors in the decade from 1976 to 1985. In Figure 2, the frequency distribution is shown on a double-logarithmic scale, with y denoting the number of logicians (in per cent of all logicians of the period) working in the number of logic areas denoted by the x-axis. One can see that 39.9% of the logicians worked only in one area; 23.5% worked in two areas; 12.6% worked in three; and only about 10% worked in more than six areas, in the ten years under consideration.

The regression curve is computed according to the inverse power function $f(x) = c/x^{\gamma}$ (method of least squares). The power function is known as 'Lotka's law' for the distribution of the scientific output in terms of books and articles; in Lotka's law, f(x) denotes the number of people with x contributions. However, the fit of the regression of the distribution is not good. The points of the first part of the distribution deviate heavily in the direction of a minor γ, and the points of the second part in the direction of a major γ — both of these behaviours being frequent occurrences in distributions of scientific productivity. The similarity is no surprise: if a publication were to be classified only with a single classification code (for example, the most important subject of a publication), and if no author were to publish in an area more than once, the frequency distribution of specialization would then be identical with 'Lotka's law'. Of interest here is not so much the regularity as such, but the causes of deviations from it. One may speculate, for example, that deviations in the tail of the curve indicate innovative activities of the group of the most prolific scientists, whose scientific inventiveness is not fully reflected by a conventional classification scheme used in a bibliography. Analogous deviations are well known in economics, where Pareto income distributions are often not valid

FIGURE 2
Frequency Distribution of the Number of Different Logic Areas which Authors in Logic Dealt with from 1976 to 1985



Number of 7,094 authors in per cent (y-axis) which treated x areas (Lotka distribution). Regression line (least squares method): γ = 2.536; s.d. = 0.105; R^2 = 0.95.

TABLE 1
Frequency Distributions of the Number of Different Logic Areas Treated by an Author in Different Time Spans

	Time interval									
	1976	1976–1977	1976–1978	1976–1979	1976–1980	1976–1981	1976–1982	1976–1983	1976–1984	1976–1985
Authors	1355	2211	2946	3715	4382	5080	5737	6226	6774	7094
Areas	113	111	112	113	113	113	113	113	113	113
γ	2.669	2.645	2.649	2.592	2.596	2.511	2.527	2.551	2.553	2.536
s.d.	0.255	0.183	0.181	0.171	0.159	0.161	0.121	0.126	0.121	0.105
R^2	0.91	0.93	0.91	0.91	0.92	0.91	0.94	0.93	0.94	0.95

Regressions according to Lotka's distribution (cf. Figure 2).

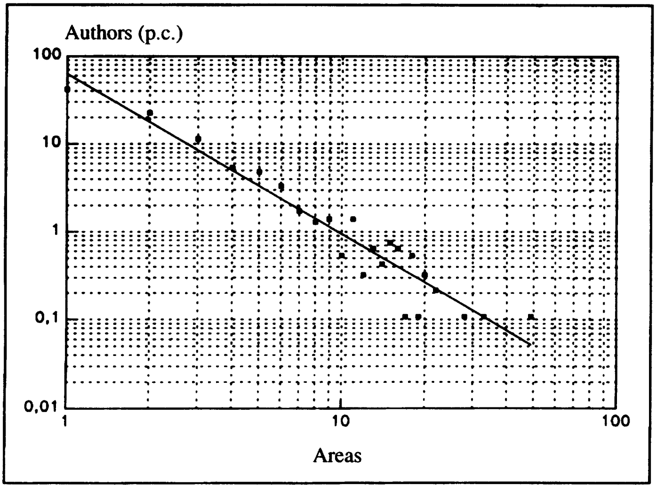
for very low and very high incomes, and where often the whole curvature is concave.[15]

The skewness of the distribution is almost independent of the time-window, over a range of ten years (see Table 1); but it is not independent of the *historical phase* in the development of a discipline, as will be shown in the next section.[16] A similar shape of the distribution, on the other hand, can also be found for publications of nineteenth century mathematical authors, and it seems to be independent of historical circumstances concerning (for example) the organization of the research system.[17]

As Figure 3 shows, the frequency distribution for the period ending twenty-five years ago (1951–60) has the same character, but with a slightly lower exponent. The lower the exponent, the higher is the concentration — underlining the role of the more prolific logicians in the process of occupying new territories in logic. The upper part and the tail deviate from the regression, but less so than in Figure 2.

Because both the *Ω-Bibliography* and Church's bibliography cover the years from 1876 to 1935, the effect of different classification systems can be scrutinized. Table 2 shows a comparison of three ten-year intervals (considering all classification codes in Church's bibliography). It makes clear that, according to both sources, the concentration increases retrospectively, back to the pioneering times. Church's classification system is, however, not so broadly differentiated as the modern classification system, which has had to deal with the full vast amount of recent literature. Furthermore, the Lotka-exponent grounded on Church's biblio-

FIGURE 3
Frequency Distribution of the Number of Different Logic Areas which Authors in Logic Dealt with from 1951 to 1960



Number of 931 authors in per cent (y-axis) which treated x areas (Lotka distribution). Regression: $\gamma = 1.858$; s.d. $= 0.115$; $R^2 = 0.92$.

TABLE 2
Frequency Distributions of the Number of Different Logic Areas Treated by an Author for Different Phases of the Development of Logic and in Different Bibliographies and Classifications of the Same Subject Area

	Time periods of logic									
	Database: Ω-Bibliography					Database: Church-Bibliography				
	Authors	Areas	γ	s.d.	R^2	Authors	Areas	γ	s.d.	R^2
1926–1935	212	70	1.607	0.179	0.85	380	15	1.965	0.165	0.95
1901–1910	68	29	[1.236]	[0.272]	[0.75]	88	13	1.884	0.343	0.86
1876–1885	11	17	[0.760]	[0.211]	[0.76]	20	9	1.744	0.310	0.91

Regressions according to Figure 1. Brackets: The distribution loses its Lotkean character.

graphy is generally higher than the exponent grounded on the modern bibliography for the same periods. This is in agreement with a plausible rule (derived by analogy with, for example, income distributions in populations): the less differentiation, the higher is the 'equality' of a population. This can also be shown in the *Ω-Bibliography* itself. I computed the frequency distribution of specialization for all 7094 authors (1976–85), considering only the following *broad* subject divisions in the *Ω-Bibliography*: A, general logic; B, classical and non-classical logic; C, model theory; D, recursion theory; E, logical set theory; F, proof theory; G, universal algebra; H, non-standard analysis. The exponent of the Lotka distribution was 3.334 — considerably higher than the exponent of the finer classified distribution. The convex deviations (visible also in a high standard deviation of 0.572 of the exponent) were even higher than in the other examples; obviously, the asymmetric character of the Pareto- or Lotka-like distributions is disappearing, the less a classification system discriminates. If only the sole criterion of 'having worked in logic anyhow' were to be applied, all logicians would, of course, be totally equal.

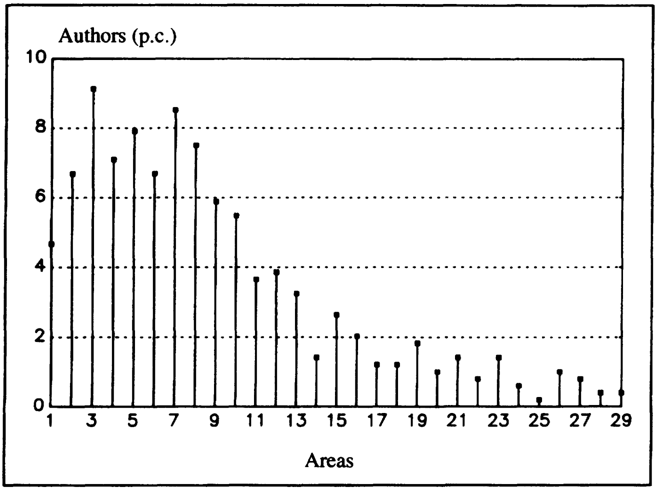
A further example of a less sharply differentiated population are logicians active for a longer time period. Figure 4 shows the frequency distribution of specialization for the 493 authors active in logic for twenty to twenty-five years (again, the period of activity is defined as the year of the first until, and including, the year of the last publication). This is a distribution which can be seen roughly as the right three-quarters of a normal frequency distribution. The left quarter of the distribution is hidden, to some extent, by authors who contributed twice or less, but were nevertheless active for twenty to twenty-five years. (The contributorship could be computed only for authors publishing at least twice, due to the working definition of 'active participation'.)[18] One could hypothesize that a skew distribution of the total output of the different logic areas is responsible for the skew frequency distribution of specialization. However, a computation of that distribution resulted only in a minimal skewness.[19]

Among the most productive logicians, in terms of the number of special areas dealt with, there are also many of the most productive logicians, in terms of the number of publications.[20] Taking the number of publications as a rough indicator of scientific standing, one can suppose that the often-recommended specialization is not the best route to scientific achievement. On the contrary, most successful logicians are obviously the least specialized, in terms of the number of areas they have worked in. Later, the composition of this versatility will be investigated.

The logicians surely did not work on all areas simultaneously, but moved step-by-step from one area to the next; on average, they published roughly once every 1.3 years.[21] Obviously, the frequency distribution of special areas dealt with also reflects a similar frequency distribution of cognitive mobility. By 'cognitive mobility', I understand the operation by which a scientist works in field A and then turns to field B. A raw indicator of this mobility is the succession of publications when a scientist publishes first in A, and next in B. It is supposed that this process is connected with a certain type of information transfer from A to B.[22] Very rarely will scientists steadily publishing in one discipline burn their bridges behind them if they change areas. Not very much is known, however, about the more precise contours of the kind of information transfer connected with cognitive mobility. In any case, as statistics show, the capacity to make many of those steps seems to be

FIGURE 4

Frequency Distribution of the Number of Different Areas Treated by the 493 Authors Active for 20–25 Years in Logic



Number of authors in per cent (y-axis) which treated x areas. This excludes four authors with 30, 32, 34 and 35 areas, and two with 53 areas.

connected with major scientific success. That the most versatile scientists are in most cases the most productive, and the supposition that high cognitive mobility is correlated with high scientific productivity, is confirmed on a more aggregated level than in this study, in an investigation of the mobility of physicists.[23] In the philosophy of mathematics, the suggestion has been made that one should take as a criterion for the fruitfulness of mathematical ideas, the application in realms of mathematics other than the one the ideas originated in.[24] This suggestion is supported by evidence from empirical indicators — namely, the convergence of scientific achievement (as far as it is indicated by publication output) and of cognitive mobility.

These results should not be misinterpreted as suggesting an asymmetric distribution of scientific capabilities as such. The skew distributions are a result of a coming and going in logic — of intensive personal fluctuations between logic areas and between logic and other areas of mathematics, with the second type of fluctuation falling out of the range of this investigation. Some of the participants played their role in some areas for the whole period of ten years; other scientists exhibited only a guest performance, and then returned to their scientific domiciles. Some of them may turn their backs on matters of logic forever.

Up to this point, the structure of specialization in logic for a certain period of time has been analyzed in terms of the frequency distribution of special areas logicians have dealt with. This can be conceived of as a structure of specialization on the level of individual scientists within a scientific community. But so far nothing has been said about the structure of specialization 'inside' a single scientist. Scientists may devote their energies to the whole range of their interests in a quite even manner; or they may exercise a hierarchy of preoccupations.

Because it is not possible to study this question with less prolific logicians, I have to focus on the most productive men and women in logic, where the magnitude of publication output will guarantee enough data. Previously we hypothesized that these productive logicians are a mirror of their whole discipline; in a certain time interval, the discipline typically consists of many logicians occupied only with one or two areas, and a few logicians publishing in twenty, thirty or more areas of logic.[25] Transferring this structure of specialization to the case of a single productive logician means that many areas of logic may deserve his or her interest (and energy to work in) only once; and that some few areas will absorb most of the work of a logician. One can imagine that a 'single' logician consists of a 'team' of fractional, part-time logicians, most of them engaged in an area only one or a few times. (In the case of an author like N. Bourbaki,[26] this analogy has to be taken quite seriously.)

The Frequency Distribution of Special Areas dealt with by Individual Scientists

In our previous study, the frequency distributions of only three logicians were investigated; in two cases they seemed to be approximately Lotka-like. Now I would like to analyze the frequency distributions of the fifteen most prolific logicians — 'prolific', that is, either in terms of the number of publications, or in terms of the number of special areas dealt with. By applying these criteria, a list of twenty-three different logicians was obtained, among them, for example, Alfred Tarksi, Solomon Feferman, Abraham Robinson, Hao Wang, Jaakko Hintikka, and others. Most of them were active participators for twenty-five or more years up to 1985.

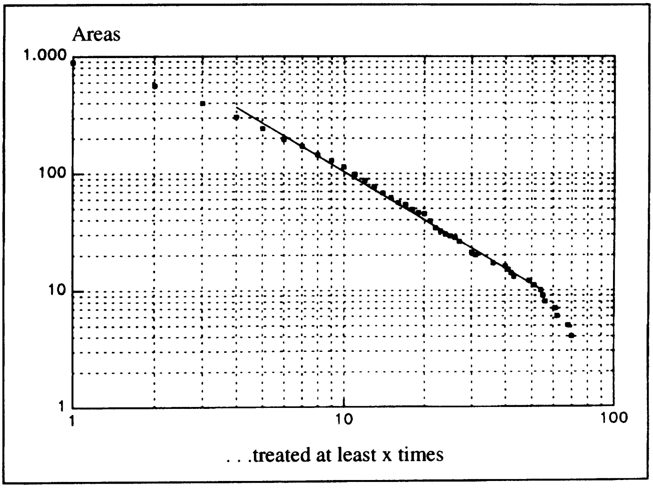
In order to detect statistical patterns, all twenty-three frequency distributions of occupations with different logic areas were added. For example, Alfred Tarski treated 12 areas of logic one time, 8 two times, 4 three times, and so on; Hao Wang treated 22 areas of logic one time, 6 two times, 5 three times, and so on; the frequency distributions of the authors were added. Figure 5 shows the curve of the added data in the form of a Pareto distribution. The y-axis denotes the frequency that an area was treated by a logician *at least* x times (x-axis). The regression line for the major part of the data (computed with the method of least squares) fits the Pareto power function reasonably well. Obviously, the amount of energy devoted to the various areas of logic is distributed unevenly. There also seems to be a Lotka-law-like regularity in the distribution of individual scientific energy.

It is noticeable that the succession of various occupations of individuals leads in the end to this distribution. And it seems to be quite improbable that this is an outcome of conscious long-range planning by any of the logicians under consideration. Logicians may have their own individual research programmes, but they will proceed, step by step, from one logical result or insight to the next.

One could hypothesize that a continuous process of concentration is going on in the course of these logicians' lives resulting, at the end, in the skew Lotka distribution. I therefore examined the distribution in different phases of the logicians' lives. Would they tend, in the first years of their careers, to publish evenly in the different fields of logic, or would they follow the perhaps problematic advice to specialize fast in order to achieve scientific success?

The exponent was computed for the respective distributions for six periods of five years each in the careers of the logicians, the periods beginning with the first contribution of each logician. The statistical material for the single periods is not very prolific;[27] computing the exponents of the six distributions would result in the following power regression exponents (in brackets are the values for truncated calculated distributions excluding empirical output numbers, which cease to continue as a consecutive series of natural numbers): (1) 1.91 (2.11); (2) 1.82 (1.75); (3) 1.79 (1.99); (4) 1.84 (2.03); (5) 1.57 (2.12); (6) 1.44 (1.81). The values range roughly

FIGURE 5
Added Frequency Distribution of the Most Prolific 23 Logicians



4,641 activities (contribution to a logic area during the whole span of the career). Read as: Number of areas (y-axis) treated by a logician at least x times (Pareto distribution). Not shown: the two highest x-values. Regression: γ = 1.369, s.d. = 0.017, R^2 = 0.99.

between 1.5 and 2.0, and seem to indicate no clear trend in the course of the logicians' lives. Obviously, there were many different areas nesting at the periphery of these logicians' minds during their whole careers. Those areas might be understood as pools of ideas, not very active, but susceptible at the moment when a stimulating 'infection' is going on. The data are too scarce, however, to permit definite conclusions.

Divide et Impera: The Division of Labour between Outstanding Logicians

So far, I have shown that the amount of labour logicians invested in different areas of their interest was distributed unevenly according to an inverse power function. We know nothing, however, about whether the most prolific logicians tend to concentrate on the same or on different areas — in other words, about the extent of the intersection between their cognitive interests and investments. To obtain an answer, the area of strongest activity was determined for each of the 23 logicians, giving the following results:

> C55, set-theoretic model theory (2x); B50, many-valued logic; E75, applications of set theory; A05, philosophical aspects (2x); B30, foundations of classical logical theories; C60, model-theoretic algebra (2x); D05, logical automata (2x); E05, combinatorial set theory (2x); C62, models of arithmetic and set theory (2x); B46, relevance logic; E70, nonclassical set theories; E07, relations and orders; F35, higher-order arithmetic (3x); B40, combinatory logic and lambda-calculus; F50, metamathematics of constructive systems.

One can conclude that many of these logicians are the masters in their own reigns. But six of the first-place fifteen areas were the main occupations of two logicians, and one area has three proponents. However, the last word has not been said about the whole extent of the division of labour.

To complete the picture, the difference between the logicians had to be computed with respect to all the areas involved. To do this, the 'occupational distances' between each of the logicians were calculated. As an expression of distance, I took the absolute value of the difference between the number of times author A treated universal algebra, for example, and the number of times author B treated it. I considered all 83 areas which were touched by the twenty-three logicians at least fifteen times altogether in their publications.

The differences in publication habits, however, matter in the cases of Waclav Sierpinski and Shelah Saharon. With their prolific and wideranging publications, they are automatically placed far beyond their peers in logic; 'distances' to them would appear very high as a consequence of their atypical publication behaviour alone. Therefore, they are excluded from the calculation. For the remaining twenty-one logicians, the differences with respect to each of the 83 areas of logic were computed, resulting in a single number which expresses the (Euclidean) distance to a fellow logician.

Table 3 is a distance matrix with 210 positions. Since every author contributed on average 155 times to the 83 areas, the average

TABLE 3
Distance Matrix of the 21 Most Prolific Logicians

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1																					
2	155																				
3	187	218																			
4	272	215	329																		
5	227	188	290	257																	
6	173	184	260	259	140																
7	178	111	235	274	243	237															
8	167	186	270	281	196	148	233														
9	187	230	56	335	288	250	255	252													
10	238	197	297	312	251	257	242	223	295												
11	224	201	285	282	255	253	268	241	293	266											
12	333	306	406	355	208	236	363	290	406	275	365										
13	192	157	249	256	207	205	194	197	257	192	240	317									
14	212	233	323	304	213	155	306	175	297	288	276	287	256								
15	162	137	219	214	125	107	170	149	219	190	226	231	132	202							
16	294	225	365	252	313	315	300	313	371	310	250	397	288	330	276						
17	224	201	287	302	265	261	226	241	293	244	286	375	192	318	188	324					
18	166	159	247	232	181	159	200	165	245	220	196	263	174	204	124	256	224				
19	207	226	268	321	150	162	263	210	262	279	283	274	243	235	171	349	295	203			
20	230	245	315	314	297	297	320	313	323	306	288	393	302	294	264	330	352	262	297		
21	226	215	317	322	219	209	256	183	311	264	316	269	236	238	180	338	276	208	239	286	

1. Alexander Abian; 2. Andrzej Ehrenfeucht; 3. Paul Erdös; 4. Y.L. Ershov; 5. Solomon Feferman; 6. Harvey M. Friedman; 7. Seymour Ginsberg; 8. Petr Hajek; 9. Andras Hajnal; 10. K. Jaakko Hintikka; 11. H. Jerome Keisler; 12. G. Kreisel; 13. Robert K. Meyer; 14. Andrzej Mostowski; 15. John R. Myhill; 16. Abraham Robinson; 17. A. Rose; 18. Dana S. Scott; 19. Gaisi Takeuti; 20. Alfred Tarski; 21. Hao Wang.

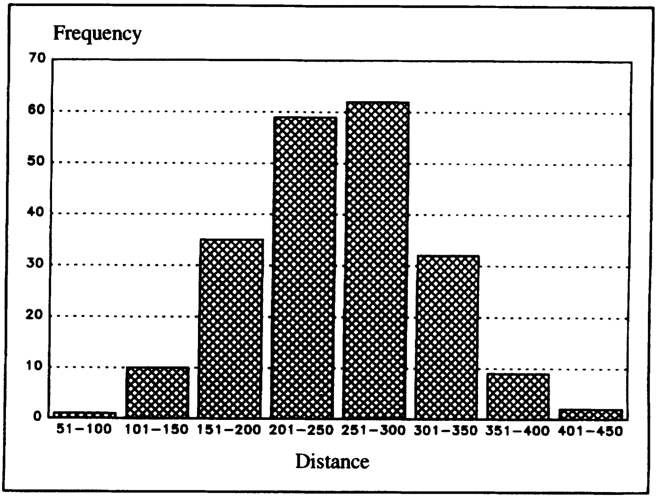
Examples: Most similar are P. Erdös and A. Hajnal; most different are G. Kreisel and P. Erdös, and G. Kreisel and A. Hajnal.

maximum distance would be 310 if there were no intersections between the authors; the average distance would be 64 points if the only differences between the authors lay in the number of contributions, and not in the areas of contribution.[28]

The computed average distance between the authors, however, is 249.3, with a standard deviation of 61.7. The size distribution of the 210 distances is almost perfectly gaussian (see Figure 6). The impression is reinforced that each of the logicians has a unique profile, building individual empires of their own — or finding unique niches. Such unique profiles may be a necessary, but not a sufficient, presupposition to establish what is called a 'scientific school'.

At the same time, however, the size of the average distance allows us to conclude that there must be a zone of intersection between the logicians; in other words, we are dealing with an interplay of concentration and dispersion, appearing as two sides of the same coin. Most logicians seem to be involved, at the periphery of their interests, with areas which are in the centre of interest of

FIGURE 6
Size Distribution of the 210 Distances of Table 3



The distribution is gaussian (mean 249.3; σ = 61.7; 66.7% of all 210 values are within $\pm$ 1σ of mean; 95.2 within $\pm$ 2σ; 99.5 within $\pm$ 3σ).

other logicians. One may assume that the weak ties of logicians to some of the areas they deal with are important ties for the cohesion of logic as a whole; without them the discipline would be fragmented into unconnected and disintegrated parts. Granovetter's much cited statement about the importance of weak ties for the linkage of networks can be applied directly here. Granovetter pointed out the importance of peripheral figures at the edges of networks which are, however, in or connected intensively with, the centres of others.[29] The peripheral occupation of many logicians with parts of their discipline can be interpreted as an important integrative mechanism for that discipline. Or, to consider it the other way round, from single eminent logicians to the whole discipline: the whole discipline's 'superfluous' scientific workers, dealing only with one or a few areas of logic, are important elements in a system of information transfer, which may transcend the realm of pure logic and go into other regions of science and technology.

Concluding Considerations

Is it too pretentious to call the process we have observed 'self-organization'? Under self-organization, the development of a system is understood as moving from a less structured to a more structured state without the intervention of external actors.[30] What is called the 'metabolism' of such a system and its environment in the theory of self-organization is reflected here by the inflow and outflow of manpower, and by inflow of unformed, and outflow of structured, logic ideas. Typically, the laws governing such a self-organizing system have the same form on all levels: on the level of the individual, on the level of certain groups or networks of individuals, and on the level of groups of groups.

Certainly, bibliometric indicators as used here yield only a pale reflection of scientific processes. But that reflection seems to me promising enough to apply scientific theories of self-organization, not only verbally or symbolically, to science itself.

The power laws described above gave us some footprints of a process of scientific self-organization that could lead, in the end, to a better quantitative understanding. After pioneering works (for example, by H.A. Simon) we know that the self-organization of complex systems is connected with special types of hierarchization and stratification.[31] In this context, of course, nothing bureaucratic or peremptory is understood by 'hierarchization'. Rather, this feature is conceived of as a complex and organized, but not commanded, interplay of the parts of a system. This interplay may yield a distribution of regularities on a personal level similar to those on the level of a whole discipline — in which case, the activity distribution of scientific communities, on the one hand, and of single scientists, on the other, would be related on a single fractional dimension.

Anthony van Raan has made similar reflections in his study of the frequency distributions of the size of clusters of co-citing scientists. These clusters can be constructed on different threshold values of co-citation strength: if one chooses a high threshold value, the result is many small clusters; if one chooses a lower value, the result is a few large clusters. One can construe these clusters as communities of scientists connected through a common basis of cited work, and also partly reflecting a common informational foundation. The middle parts of the rank-size distribution of these clusters follow an inverse power function of the Zipf type. Van Raan has shown that the exponents of the curves of different levels of co-citation strengths are in a fractional proportion.[32] He interpreted these size distributions as distributions of (scientific) energy (in terms of the number of scientists connected in co-citation clusters). Many years ago, George K. Zipf elaborated this approach with the help of a host of empirical examples: he called it 'human ecology', with a crucial role played by a 'principle of least effort'.[33] Obviously, scientific manpower organizes itself in such a way that there are many small cognitive enterprises; some enterprises, however, attract high amounts of energy and interest. Exactly this proved also to be true on the level of single scientists. Thus, the self-similarity of the scientific system becomes visible from power laws valid on several levels — just as is claimed for in the theory of self-organization: on the level of single scientists, on the level of a group of scientists, and on the level of groups of groups.

One may understand such distribution regularities, so widespread also in economics, as a quantitative expression of a 'Wille zur Macht', or something similar. Beyond these metaphysical associations, however, another aspect seems to me to be more crucial: the creation and diffusion of human knowledge, with logic being just one example among many others.

• NOTES

I am grateful to Professor Dr Jan Berg for enlightening talks and for his linguistic consulting. I owe much to the referees for suggestions and critical remarks.

1. See Jarmo Pulkkinen, *The Threat of Logical Mathematism: A Study on the Critique of Mathematical Logic in Germany at the Turn of the 20th Century* (Frankfurt a.M.: Lang, 1994). That mélange never disappeared completely, but it changed its geographical centres and its political affinities, *inter alia*.

2. All global descriptions are from: Roland Wagner-Döbler and Jan Berg, *Mathematische Logik von 1847 bis zur Gegenwart, Eine bibliometrische Untersuchung* (Berlin & New York: de Gruyter, 1993).

3. Gert H. Müller, in collaboration with Wolfgang Lenski (ed.), *Ω-Bibliography of Mathematical Logic*, Vols 1–6 (Berlin, Heidelberg & New York: Springer, 1987).

4. Alonzo Church, 'A Bibliography of Symbolic Logic', *The Journal of Symbolic Logic*, Vol. 1 (1936), 121–216; 'Additions and Corrections', ibid., Vol. 3 (1938), 178–92. For a quantitative comparison with the *Ω-Bibliography*, see Wagner-Döbler & Berg, op. cit. note 2, 60–63.

5. Wagner-Döbler & Berg, op. cit. note 2.

6. In ibid., 116, a number of documentary categories were not excluded, and only the first classification codes of an entry were considered.

7. Nicholas Rescher, *Cognitive Systematization* (Oxford: Blackwell, 1979), 190. Rescher is one of the few philosophers of science dealing with specialization, classification, and taxonomy of science.

8. Wagner-Döbler & Berg, op. cit. note 2, 60ff.

9. Ibid., 118–19. This phenomenon has also been observed in biology: Petr B. Shelishch, 'A Quantitative Study of Biologists in the 18th and 19th Centuries', *Scientometrics*, Vol. 4 (1982), 317–29.

10. Wagner-Döbler & Berg, op. cit. note 2, 65ff.

11. These reductions in the radius of attention to other areas may be the main mechanism by which scientists come to cope with the 'information explosion', as Bar-Hillel observed many years ago: see Yehoshua Bar-Hillel, 'Is Information Retrieval Approaching a Crisis?', *American Documentation*, Vol. 14 (1963), 95–98.

12. Other modes of calculation would not touch this outcome. For example, I considered all authors and *all* (not only the first one) of the publications' classification codes for the periods 1926–35 (1), 1951–60 (2), and 1976–85 (3), with the following results, corresponding with the data exhibited in Figure 54 in Wagner-Döbler & Berg, op. cit. note 2: (1) 70 areas/212 authors = 0.330; (2) 106 areas/931 authors = 0.114; (3) 113 areas/7094 authors = 0.016.

13. In his bibliography: see Church, op. cit. note 4; Wagner-Döbler & Berg, op. cit. note 2.

14. Ibid., 82.

15. A Pareto distribution has the form $f(x) = c/x^{\gamma}$. Here $f(x)$ denotes the number of people with *at least* x contributions. With concern to firm sizes, see Yuji Ijiri and Herbert A. Simon, 'Interpretations of Departures from the Pareto Curve Firm-Size Distributions', *Journal of Political Economy*, Vol. 82 (1974), 315–31.

16. The same is valid for Lotka distributions of publication output: see Roland

Wagner-Döbler and Jan Berg, 'The Dependence of Lotka's Law on the Selection of Time Periods in the Development of Scientific Areas and Authors', *Journal of Documentation*, Vol. 51 (1995), 28–43.

17. Roland Wagner-Döbler and Jan Berg, 'Nineteenth-Century Mathematics in the Mirror of Its Literature: A Quantitative Approach', *Historia Mathematica*, Vol. 23 (1996, 288–318). Because here the publications are indexed only with one classification code, the conformity with Lotka's law is far better.

18. With respect to the frequency distribution of scientific output in terms of publications see R. Wagner-Döbler, 'Where has the Cumulative Advantage Gone? Some Observations about the Frequency Distributions of Scientific Productivity, of Duration of Scientific Participation, and of Speed of Publication', *Scientometrics*, Vol. 32 (1995), 123–32.

19. Wagner-Döbler & Berg, op. cit. note 2, 131. The result did not change when I then computed the distribution for a decade, namely 1961–70.

20. Wagner-Döbler & Berg, op. cit. note 2.

21. Ibid., 66.

22. For an early attempt to explore this topic, see Michael Mulkay, 'Conceptual Displacement and Migration in Science: A Preparatory Paper', *Science Studies*, Vol. 4 (1974), 205–34.

23. For an institutional level of analysis see: Arie van Heeringen and Pieter A. Dijkwel, 'The Relationship between Age, Mobility, and Scientific Productivity, Part 1: Effect of Mobility on Productivity', *Scientometrics*, Vol. 11 (1987), 267–80. Productivity (not in terms of papers, but in terms of citations) tended to increase if institutional mobility was connected with the move from one area to another. Without this, it did not increase. Nothing can be said so far about the causal relation between cognitive mobility and productivity.

24. Michael Hallet, 'Towards a Theory of Mathematical Research Programmes', *British Journal for the Philosophy of Science*, Vol. 30 (1979), 1–25 and 135–59.

25. Wagner-Döbler & Berg, op. cit. note 2, 83–84.

26. 'Nicholas Bourbaki' is a pseudonym for a distinguished group of (mostly anonymous and mostly French) mathematicians. Under that pseudonym, mathematical monographs appear continuously which are written by members of the group and passed jointly by the whole group. The composition of the group changes by means of informal mechanisms. See Jean A. Dieudonné, 'The Work of Nicolas Bourbaki', *The American Mathematical Monthly*, Vol. 77 (1970), 134–45.

27. Three authors had only a full fifth period; three had only the fourth completely; two authors had only the third period completely. Twice an author's last period comprised four years only.

28. I computed the average distance of a distance matrix considering only the differences between the twenty-one logicians with respect to the sheer number of their contributions. The average distance is 64.29 (s.d. 46.63).

29. Mark S. Granovetter, 'The Strength of Weak Ties', *American Journal of Sociology*, Vol. 78 (1973), 1360–80.

30. This working definition is widespread. See, for example, contributions in Milan Zeleny (ed.), *Autopoiesis, Dissipative Structures, and Spontaneous Social Orders* (Boulder, CO: Westview Press, 1980).

31. Herbert A. Simon, *The Science of the Artificial* (Cambridge, MA: The MIT Press, 1969), 84–118. See also: Dieter Germert, 'The Formation of Hierarchical

Structure as a Key to Self-Organization', in G.J. Dalemoort (ed.), *The Paradigm of Self-Organization*. (New York: Gordon & Breach, 1989) 60–72.

32. A.F.J. van Raan, 'Fractal Dimension of Co-Citations', *Nature*, Vol. 347 (18 October 1990), 626. George Zipf found similar distributions in linguistics, economics, biology, and other fields: see G.K. Zipf, *Human Behavior and the Principle of Least Effort* (New York & London: Hafner, 1949; repr. 1965).

33. See Zipf, op. cit. note 32.

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