Efficient Limited Feedback for MIMO-Relay Systems

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***Abstract*—In this letter, the limited feedback scheme in MIMO-Relay systems is considered. We propose a new strategy that optimizes the allocation of feedback bits between channel direction information (CDI) and phase information (PI) for multi-relay systems under the total number of feedback bits constraint. The proposed scheme eventually trades off some CDI bits for PI feedback to diminish the phase ambiguity. Simulation results show that compared to the conventional and the 1-bit PI feedback methods, the proposed scheme remarkably improves the system performance without increasing the feedback overhead.**

***Index Terms*—MIMO, relay, feedback, phase.**

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) is a promising technology that can tremendously improve the system capacity [1]-[2]. Recently, the relay technique has also attracted much attention, and the MIMO-Relay, which combines MIMO with relay, becomes a hotspot both in fields of literature and industry [3]-[4]. In [3], the authors proposed to use matched filtering at the relay nodes to forward multiple data streams, which assumed that each of the multiple relays had the perfect channel state information (CSI) between itself and users. In [4], the feedback strategy for multi-relay system was investigated, and it was found that the phase ambiguity, caused by channel quantization, resulted in tremendous throughput loss if the destination received signals from different relays. To tackle the problem, a new strategy combining CDI and PI feedback was given, and the simulation results showed that with proper allocation of the feedback bits between CDI and PI, the scheme can achieve further system performance improvement without increasing the feedback overhead. However, the authors did not give any detailed solutions.

In this letter, the MIMO-Relay system based on the half-duplex amplified-and-forward (AF) relay protocol [5]-[6] and zero-forcing beamforming (ZFBF) technology [7]-[8] is considered. We characterize the quantization error with limited feedback, and optimize the allocation of feedback bits between CDI and PI.

As for notation, we use uppercase boldface letters for matrices and lowercase boldface for vectors, and $\mathbf{A}^T$, $\mathbf{A}^*$ and $\mathbf{A}^+$ refer to the transpose, conjugate and Moore-Penros pseudo inverse of $\mathbf{A}$. $\mathbf{A}(i,:)$ and $\mathbf{A}(:,j)$ represent the i-th row andthe j-th column of matrix $\mathbf{A}$, respectively. $\mathrm{E}\{\cdot\}$ stands for the expectation operator, and $\|\mathbf{x}\|$ refers to Euclidean norm of vector $\mathbf{x}$.

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II. SYSTEM MODEL

We consider the MIMO-Relay system where one base station (BS) and R relays all have M antennas, and each of K users is equipped with only one antenna (N=1). The transmit power at the BS and relays are P_1 and P_2, respectively. The relays employ ZFBF to transmit different users' data streams simultaneously.

In the first hop, the BS transmits signals to relays. The signal received at the r-th relay can be written as

$$\mathbf{y}_{r,1} = \sqrt{P_1}\sum_{k=1}^{K}\mathbf{h}_{kr,1}s_k + \mathbf{n}_{r,1},\ r = 1,\ldots,R \tag{1}$$

where $\mathbf{h}_{kr,1} \in \mathbb{C}^{M\times1}$ denotes the channel from the BS to relay r transmitting the k-th user's signal, and $\mathbf{n}_{r,1} \in \mathbb{C}^{M\times1}$ is the noise vector of the first hop at relay r. Assume ZF receivers are equipped at relays, the receive matrix at r-th relay can be designed as

$$\mathbf{W}_{r,1} = \mathbf{D}_{r,1}\mathbf{F}_{r,1} \tag{2}$$

where $\mathbf{F}_{r,1} = [\mathbf{h}_{r1,1}, \mathbf{h}_{r2,1}\cdots\mathbf{h}_{rK,1}]^+$, and $\mathbf{D}_{r,1}$ is a diagonal matrix with the i-th element: $\alpha_{ri,1} = 1/\sqrt{P_1 + \|\mathbf{F}_{r,1}(i,:)\|^2}$, $i = 1,2\ldots K$.

It is not feasible for relays to get the full CSI of the second hop (from relay to user), especially in the FDD system. The transmit preceding matrix is usually designed based on the effective CDI $\breve{\mathbf{h}}_{kr,2}$ fed back from users, i.e.,

$$\mathbf{W}_{r,2} = \mathbf{F}_{r,2}\mathbf{D}_{r,2} \tag{3}$$

where $\mathbf{F}_{r,2} = [(\breve{\mathbf{h}}_{1r,2})^T, (\breve{\mathbf{h}}_{2r,2})^T\ldots(\breve{\mathbf{h}}_{Kr,2})^T]^+$, and $\mathbf{D}_{r,2}$ is a diagonal matrix with the i-th element: $\alpha_{ri,2} = 1/\|\mathbf{F}_{r,2}(i,:)\|$, $i = 1,2\ldots K$.

In the second hop, the relays transmit the processed signals. The signal received at k-th user is given by

$$\begin{aligned}\mathbf{y}_{k,2} &= \sqrt{\frac{P_2}{M}}\sum_{r=1}^{R}\mathbf{h}_{kr,2}\mathbf{W}_{r,2}\mathbf{W}_{r,1}\mathbf{y}_{r,1} + n_{k,2}\\ &= \sqrt{\frac{P_1P_2}{M}}\sum_{r=1}^{R}\mathbf{h}_{kr,2}\mathbf{W}_{r,2}\mathbf{D}_{r,1}\mathbf{s}\\ &\quad+ \sqrt{\frac{P_2}{M}}\sum_{r=1}^{R}\mathbf{h}_{kr,2}\mathbf{W}_{r,2}\mathbf{W}_{r,1}\mathbf{D}_{r,1} + n_{k,2}\end{aligned} \tag{4}$$

where $n_{k,2}$ is the noise of second hop at user k, and $\mathbf{s} = [s_1, s_2\ldots s_K]^T$ is the signal vector for all users. We assume the entries of channel matrix are independent and identically distributed (i.i.d) complex Gaussian with zero mean and unit variance, and the estimation of CSI is real-time and accurate.

III. Limited Feedback Optimization

In this section, we will first describe the phase feedback scheme in [4], which is proved to obtain high system throughput with only one additional feedback bit for PI, and then propose a feedback scheme that allocates feedback bits between CDI and PI to optimize the system performance.

A. Phase feedback in [4]

The key idea of phase feedback is to quantize the phase dispersion between real CDI and the quantized CDI to a preconfigured phase codebook. Here, we use B_1 and B_2 to represent the bits for CDI and PI feedback, respectively. The algorithm is summarized as follows:

- Step 1: User k estimates the CSI of second hop and quantizes its channel vector as

$$\hat{\mathbf{h}}_{kr,2} = \arg \max_{1 \leqslant j \leqslant 2^{B_1}} \left| \tilde{\mathbf{h}}_{kr,2} \mathbf{c}_j{}^* \right| \tag{5}$$

  where $\tilde{\mathbf{h}}_{kr,2} = \mathbf{h}_{kr,2} / \|\mathbf{h}_{kr,2}\|$ is real CDI, and $\mathbf{c}$ is the codeword of the CDI codebook $\mathcal{C}_{\mathrm{CDI}}$, which is preset and known to both the relay and user k. For simplicity, we employ random vector quantization (RVQ) based codebook [9] in the following.
- Step 2: Define PI as $\{\phi_{kr} : \tilde{\mathbf{h}}_{kr,2} = \hat{\mathbf{h}}_{kr,2} e^{j\phi_{kr}}\}$, and quantize ϕ_{kr} according to

$$\hat{\phi}_{kr} = \arg \min_{\theta_i \in \mathcal{C}_{\mathrm{PI}}} |\phi_{kr} - \theta_i| \tag{6}$$

  where $\mathcal{C}_{\mathrm{PI}} = \{\theta_1 \ldots \theta_{2^{B_2}+1}\}$ is the phase codebook with $\theta_i = -\pi + \frac{2\pi}{2^{B_2}}(i-1)$. After getting the index of $\hat{\phi}_{kr}$, user k feeds it back to relay r.
- Step 3: When relay r acquires both indices of CDI and PI, it can reconstruct the effective CDI as

$$\breve{\mathbf{h}}_{kr,2} = \hat{\mathbf{h}}_{kr,2} e^{j\hat{\phi}_{kr}} \tag{7}$$

It is shown that adding only 1 bit for PI feedback will greatly increase system throughput, and by properly allocating feedback bits between B_1 and B_2, the performance can be further improved. Unfortunately, [4] did not go deeper in this issue.

B. Proposed feedback scheme

In this part, we characterize the quantization error with limited feedback based on half-duplex AF relay mode and ZFBF technology, and derive the optimal scheme that allocates feedback bits between CDI and PI to maximize user's rate. With the PI feedback introduced into the system, we have

$$u_{kr}^i = \tilde{\mathbf{h}}_{kr,2} \left(\mathbf{W}_{r,2}(:,i) \right) = \left| u_{kr}^i \right| e^{j(\varphi_{kr}^i - \hat{\varphi}_{kr}^i)} \tag{8}$$

From (4) and (8), the SINR at user k can be written as

$$\begin{aligned} \gamma_k = & \frac{P_1 P_2}{M} \Big| \sum_{r=1}^{R} \|\mathbf{h}_{kr,2}\| \, u_{kr}^k \alpha_{rr,1} \Big|^2 \\ & \times \Big(\frac{P_1 P_2}{M} \Big| \sum_{r=1}^{R} \sum_{i=1, i \neq k}^{K} \|\mathbf{h}_{kr,2}\| \, u_{kr}^i \alpha_{ri,1} \Big|^2 \\ & + \frac{P_2}{M} \sum_{r=1}^{R} \sum_{i=1}^{K} \left| \|\mathbf{h}_{kr,2}\| \, u_{kr}^i \alpha_{ri,1} \right|^2 \|\mathbf{F}_{r,1}(i,:)\|^2 + 1 \Big)^{-1} \end{aligned} \tag{9}$$

Let $S_a = \mathrm{E}\left\{\|\mathbf{h}_{kr,2}\|\right\} \mathrm{E}\left\{\alpha_{ri,1}\right\}$ and $S_b = \mathrm{E}\{\|\mathbf{F}_{r,1}(i,:)\|^2\}$, then the mean SINR is given by

$$\begin{aligned} \mathrm{E}\{\gamma_k\} \approx & \tfrac{P_1 P_2}{M} \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} S_a u_{kr}^k \Big|^2 \Big\} \\ & \times \Big(\tfrac{P_1 P_2}{M} \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} \sum_{i=1, i\neq k}^{K} S_a u_{kr}^i \Big|^2 \Big\} \\ & + \tfrac{P_2}{M} \mathrm{E}\Big\{ \sum_{r=1}^{R} \sum_{i=1}^{K} \left| S_a u_{kr}^i \right|^2 \Big\} S_b + 1 \Big)^{-1} \end{aligned} \tag{10}$$

Since we employed the Jensen's inequality to both the numerator and the denominator of SINR, (10) achieves only an approximation of the expected sum rates. According to [7], when we quantize CDI based on the RVQ and employ ZFBF technique, it satisfies

$$\mathrm{E}\Big\{ \Big| \sum_{i=1, i\neq k}^{K} u_{kr}^i \Big|^2 \Big\} = 2^{-\frac{B_1}{M-1}} \tag{11}$$

$$\mathrm{E}\left\{ \left| u_{kr}^k \right|^2 \right\} = 1 - 2^{-\frac{B_1}{M-1}} \tag{12}$$

and

$$\begin{aligned} & \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} u_{kr}^k \Big|^2 \Big\} \\ & = \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} |u_{kr}^k| e^{j(\varphi_{kr}^k - \hat{\varphi}_{kr}^k)} \Big|^2 \Big\} \\ & \approx \Big(1 - 2^{-\frac{B_1}{M-1}} \Big) \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} e^{j(\varphi_{kr}^k - \hat{\varphi}_{kr}^k)} \Big|^2 \Big\} \end{aligned} \tag{13}$$

Let $\varphi_{kr} = \varphi_{kr}^k$, $\hat{\varphi}_{kr} = \hat{\varphi}_{kr}^k$ and $\Delta\varphi_{kr} = \varphi_{kr} - \hat{\varphi}_{kr}$, then we can get

$$\begin{aligned} & \mathrm{E}\Big\{ \Big| \sum_{r=1}^{R} e^{j\Delta\varphi_{kr}} \Big|^2 \Big\} \\ & = \mathrm{E}\Big\{ \Big(\sum_{i=1}^{R} e^{j\Delta\varphi_{ki}} \Big) \Big(\sum_{j=1}^{R} e^{j\Delta\varphi_{kj}} \Big)^* \Big\} \\ & = R + R\,(R-1)\, \mathrm{E}\left\{ \cos\left(\Delta\varphi_{ki} - \Delta\varphi_{kj} \right) \right\} \end{aligned} \tag{14}$$

Because φ_{kr} satisfies the uniform distribution, and is quantized uniformly, $\Delta\varphi_{ki}$ will satisfie a uniform distribution between $\left[-\frac{\pi}{2^{B_2}}, \frac{\pi}{2^{B_2}} \right]$. So, we can calculate the expectation

$$E\{\cos(\Delta\varphi_{ki} - \Delta\varphi_{kj})\} = \frac{2^{2B_2}}{4\pi^2} \left(1 - \cos\left(\frac{2\pi}{2^{B_2}} \right) \right) \tag{15}$$

Substituting (11)-(15) into (10), we get

$$\mathrm{E}\{\gamma_k\} \approx \frac{\left(1 - 2^{\frac{-B_1}{M-1}} \right) \left(R + R\,(R-1) \frac{2^{2B_2}}{4\pi^2} \left(1 - \cos\left(\frac{2\pi}{2^{B_2}} \right) \right) \right)}{2^{-\frac{B_1}{M-1}} R + R S_b / P_1 + M / (S_a^2 P_1 P_2)} \tag{16}$$

Hence, the problem of allocating feedback bits between CDI and PI to maximize user's rate can be formulated as

$$\begin{aligned} \max \; & \frac{\left(1 - 2^{\frac{-B_1}{M-1}} \right) \left(R + R\,(R-1) \frac{2^{2B_2}}{4\pi^2} \left(1 - \cos\left(\frac{2\pi}{2^{B_2}} \right) \right) \right)}{2^{-\frac{B_1}{M-1}} R + R S_b / P_1 + M / (S_a^2 P_1 P_2)} \\ \text{s.t.} \; & B_1 + B_2 = B \end{aligned} \tag{17}$$

where B is the feedback bits used per user feeding back effective CDI, which consists of the conventional CDI and PI. Since variables B_1 and B_2 are integers, and $B_1 + B_2 = B$, it's not hard to get the optimal solution by exhaustive search of all possible compositions of B. After getting the optimal

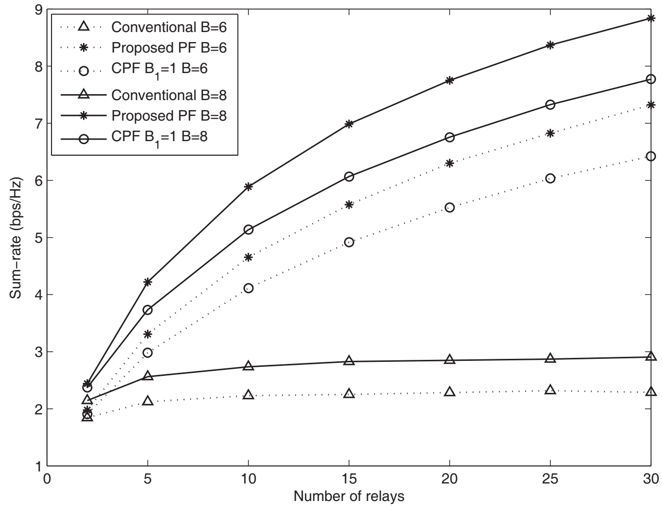

Fig. 1. Throughput versus number of relays when M=4, N=1, K=4.

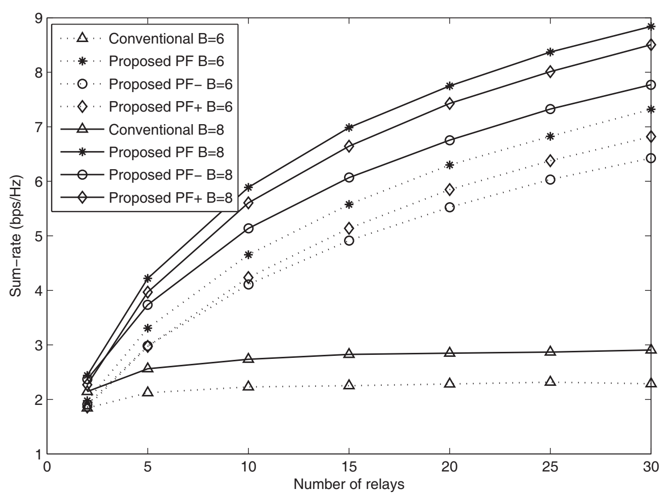

Fig. 2. Throughput versus number of relays to prove the optimality (M=4, N=1, K=4).

B_1 and B_2, the user quantizes CDI and PI according to the procedure shown in Section III-A.

In this proposed feedback scheme, users should know the statistical information of S_a and S_b, which are applied to calculate the optimal solution to (17). This might be realized by broadcast from BS or relays.

IV. Simulation Results

Simulation results are provided to show the throughput improvement achieved by the proposed scheme. In the simulation, we assume a quasi-static flat fading channel model, and the transmit power at the BS and each relay node is set to $P_1 = P_2 = 10$dB.

In Fig. 1, we compare several schemes including the conventional feedback scheme (conventional), the 1-bit phase feedback in [4] (CPF $B_2 = 1$), and our proposed scheme (proposed PF). We set $M = 4$, $K = 4$, and show the results for $B = 6$ and 8. It can be seen that the conventional scheme, which does not take phase feedback into account, leads to a very poor performance that leaves a flat trail, which is in accordance with the analysis in [4]. Meanwhile, the CPF, with only 1 bit for PI feedback ($B_2 = 1$), can greatly increase the system throughput, and our proposed allocation scheme provides even better performance, especially when the number of relays is large.

In order to illustrate the optimality of our proposed scheme, we also simulate the cases of respectively increasing ("proposed PF+") and decreasing ("proposed PF-") 1 bit of B_2, for comparison with our proposed allocation scheme, but still with total B bits, and the results are shown in Fig. 2. Obviously, the proposed scheme achieves the best performance, even 1 bit of fluctuation will notably degrade the system throughput. We also simulate the rest $2B-3$ allocations (for each B), and their performance are much lower than the three strategies above. Thus, These all together show the optimality of our proposed scheme.

V. Conclusion

In this letter, we study the limited feedback in the MIMO-Relay system, where phase information is critical for users to be served by multi-relay. By analyzing the relation between quantization error and feedback bits, we propose a new strategy based on AF mode relay and ZFBF technology, which adaptively allocates feedback bits between channel direction information and phase information. It is shown through simulation that compared with the conventional scheme, even the 1-bit PI feedback can greatly increase the system throughput, while our proposed scheme achieves further improvement, and is also numerically shown to be optimal.

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References

[1] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691-1706, July 2003.
[2] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. *Cambridge University Press*, 2003.
[3] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. Wireless Commun*, vol. 5, no. 6, pp. 1433-1444, June 2006.
[4] E. Zeng, S. Zhu, and M. Xu, "Impact of limited feedback on multiple relay zero-forcing precoding systems," in *Proc. IEEE ICC '08*, pp. 4992-4997.
[5] J. Guo, H.-W. Luo, and H.-X. Li, "Opportunistic relaying in cooperative OFDM networks for throughput and fairness improvement," in *Proc. IEEE Global Telecom. Conf.*, 2008, pp. 1-5.
[6] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocol and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 7, pp. 3062-3080, 2004.
[7] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 8, pp. 1478-1491, Sep. 2007.
[8] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, pp. 5045-5060, Nov. 2006.
[9] W. Santipach and M. Honig, "Signature optimization for CDMA with limited feedback," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3475-3492, Oct. 2005.