Discrete Signaling for Non-Coherent, Single-Antenna, Rayleigh Block-Fading Channels

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Rayleigh Fading Channel (RBFC)
- Used in modeling of wireless channels — it captures the uncertainty of wireless channels and fading correlation-in-time
- The fading coefficient $H$ is constant within a block of $T$ symbols and changes independently between blocks
- The parameter $T$ represents the channel coherence time
- The fading coefficient $H$ is unknown to the transmitter and receiver
- The receiver has to estimate the channel state and the data from the received symbols

$$H \sim \mathcal{N}(0,1)$$

$$[Y_1, ..., Y_T] = H \sqrt{X_1, ..., X_T}^2 + [N_1, ..., N_T]$$

$\sum H \sqrt{X} \sim \mathcal{N}(0,1)$

The Capacity-Achieving Coding Scheme
The capacity-achieving signaling over non-coherent RBFC (also called product form) reads as

$$V \sim \mathcal{V}$$

- $V$ and $\mathcal{V}$ are independent RVs
- $\mathcal{V}$ is uniformly distributed on the complex $T$-dimensional sphere with radius $\sqrt{T}$
- $V$ is discrete real non-negative RV

$$\mathcal{V}[i] = \mathcal{V}_{[i]} \{ \Theta \} \sim \mathcal{U}([0,1])$$

Computing $h[Y]$:
- The following formula can be used to compute $h[Y]$

$$h[Y] = E\left[\log_2 |1 + \gamma Y|^2\right]$$

- The knowledge of $p(y)$ is needed to evaluate (9). However, a direct computation fails due to exponential complexity growth in block length $T$

$$p(y) = \sum_{x=0}^{N} p(x)p(y|x)$$

- Instead, we use the fact that $Y$ conditioned on $V$ and $H$ is a vector of IID RVs. First expand

$$p(y|v) = \frac{1}{2\pi} \int p(y|v,h) \, dh$$

- Then the second term

$$p(y|v) = \int_{-\infty}^{\infty} p(y|v,h) \, dh$$

- Both are on-off schemes so the gain is due to the on-off surface of the RBST on-set during the on-cycle. For low rates, also discrete Product Form (light green) is better than On-Off Gaussian for the same reason.

Results: Rate vs $E_b/N_0$

Computing the Mutual Information
- The following expansion will be used to compute the mutual information

$$\mathcal{V}[i] = \mathcal{V}_{[i]} \{ \Theta \} \sim \mathcal{U}([0,1])$$

- $\mathcal{V}$ is a zero-mean circular-symmetric Gaussian RV
- The covariance matrix equals to

$$\mathcal{V}[i] = \mathcal{V}_{[i]} \{ \Theta \} \sim \mathcal{U}([0,1])$$

Using the log-det formula, the entropy $h[\mathcal{V}]$ is

$$h[\mathcal{V}] = E_\mathcal{V} \left[ \log_2 \det \mathcal{C} \right]$$

- The complexity of computing $h[\mathcal{V}]$ grows linearly with respect to the block size $T$. To evaluate the expectation in (12) we apply MC averaging.

References