

Multi-scale model of the ionosphere from the combination of modern space-geodetic satellite techniques

Urs Hugentobler¹, Michael Schmidt², Norbert Jakowski³, Denise Dettmering², M. Mainul Hoque³, Marco Limberger¹, Wenjing Liang², Volker Wilken³

¹ Technische Universität München (TUM), Institut für Astronomische und Physikalische Geodäsie (IAPG), Munich, Germany, urs.hugentobler@bv.tu-muenchen.de

² Deutsches Geodätisches Forschungsinstitut (DGFI), Munich, Germany, schmidt@dgfi.badw.de

³ Deutsches Zentrum für Luft- und Raumfahrt (DLR), Institut für Kommunikation und Navigation, Neustrelitz, Germany, Norbert.Jakowki@dlr.de

Project overview

Near real-time high resolution and high precision ionosphere models are used for a large number of applications e.g. in navigation, positioning, telecommunications or astronautics. Today, these ionosphere models are mostly empirical, relying on extensive pure mathematical approaches. However, the complex phenomena within the ionosphere can only be understood and modeled when taking into account the physics governing the phenomena.

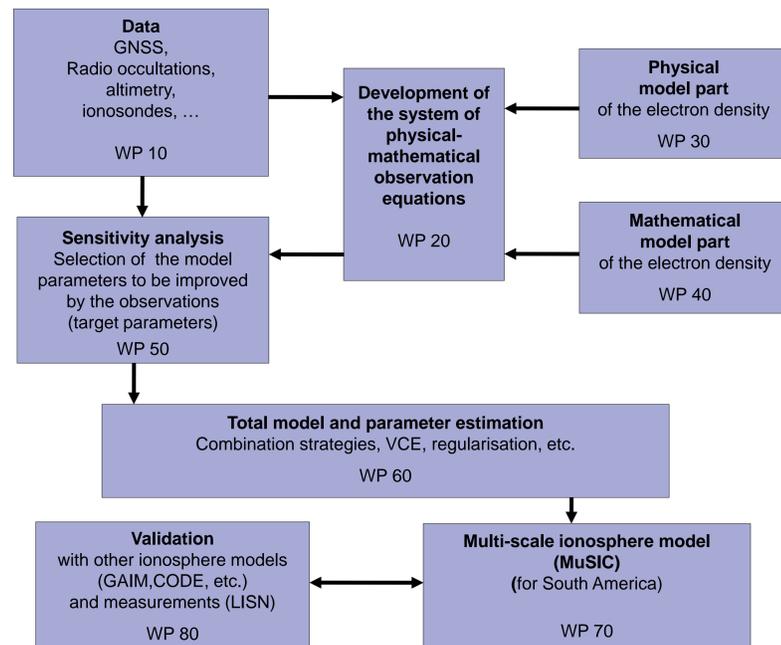
Here we present the basic structure of a model for the electron density of the ionosphere, which will be developed by a cooperation of the German Geodetic Research Institute (DGFI), the Institute of Astronomical and Physical Geodesy (IAPG) of the Technical University Munich (TUM) and the German Aerospace Center (DLR), Neustrelitz.

The main features of the project are

1. the consideration of physics-motivated modeling approaches, which are introduced in the multi-dimensional ionosphere model by means of appropriate mathematical base functions,
2. the estimation of the model parameters from the combination of various space-geodetic techniques, such as terrestrial and space-based GPS observations, altimetry and/or VLBI as well as
3. the transformation of the results into a multi-scale representation, which allows both an effective data compression necessary for handling the huge ionosphere data sets and near real-time applications as well as the identification of physical phenomena at different spatial and temporal scales.

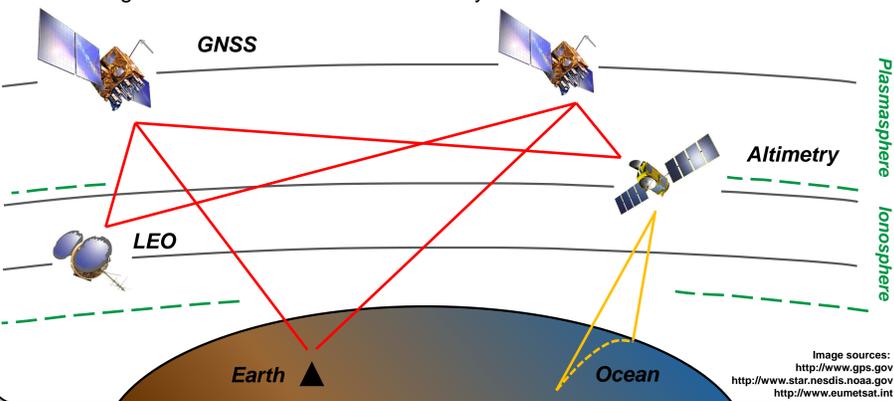
For testing the procedure, the model will be applied to an appropriate region in South America, which covers relevant ionospheric processes and phenomena such as the Equatorial Anomaly.

The project structure is given by the following flowchart where the boxes symbolize the workpackages (WP). More information on the selected WP is given on the right.



Data (WP 10)

For this project, a database consisting of observations from various techniques will be established. Beside GNSS data of the IGS and SIRGAS network stations, **dual-frequency altimetry** observations, e.g., derived from Jason-1 and Jason-2, are included to close the measurement gaps over the oceans. To overcome the insensitivity of **ground-based GNSS** to the radial geometry, **radio occultation** measurements to Low-Earth-Orbiting (LEO) satellites like CHAMP, GRACE and COSMIC are considered. The suitability of further techniques such as **VLBI**, **DORIS** or the usage of **ionosonde** data is to be analyzed.



Physical model part (WP 30)

The Chapman layer function is very efficient for describing the vertical structure of the electron density. It is a physics-motivated function depending on height h which means that its parameters have a physical meaning. In this project, the height dependency of the electron density will be modeled by combining a F2-Chapman layer with a plasmasphere profile, i.e.

$$N_e(h) = N_e^{F2} + N_e^p \quad (1)$$

where the Chapman layer for the F2 layer reads

$$N_e^{F2}(h) = N_0 \exp[0.5(1 - z - \exp(-z))] \quad (2)$$

$$\text{with } z = (h - h_{\max}^{F2}) / H. \quad (3)$$

The plasmasphere profile is given by

$$N_e^p(h) = N_0^p \exp(-|h - h_{\max}^{F2}| / H^p) \quad (4)$$

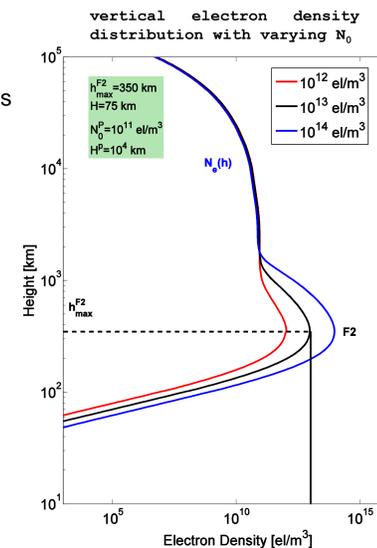
$$\text{for } h < h_{\max}^{F2}: H^p = 10 \text{ km}$$

wherein N_0 and h_{\max}^{F2} are the F2-peak electron density and peak height, H is the F2-scale height, N_0^p and H^p are the plasmasphere basis density and scale height.

Each of the five unknown target parameters

$$N_0, N_0^p, h_{\max}^{F2}, H, H^p$$

can be modeled in three-dimensional (3-D) series expansions depending on the spatial position and time (see WP 40).



Physical-mathematical modeling (WP 20, WP 40)

The space-geodetic techniques (WP 10) provide information on ionospheric parameters, e.g. a geometry-free GNSS observation yields the Slant Total Electron Content (STEC). The STEC is defined as the integral of the space- and time-dependent 4-D electron density $N_e(h, \lambda, \varphi, t)$ along the ray path between the satellite S and the receiver R , i.e.

$$\text{STEC} = \int_R^S N_e(h, \lambda, \varphi, t) ds \quad (5)$$

where λ , φ and t denote longitude, latitude and time.

Inserting Eq. (2) into Eq. (1) yields

$$N_e(h) = N_0 \exp[0.5(1 - z - \exp(-z))] + N_e^p. \quad (6)$$

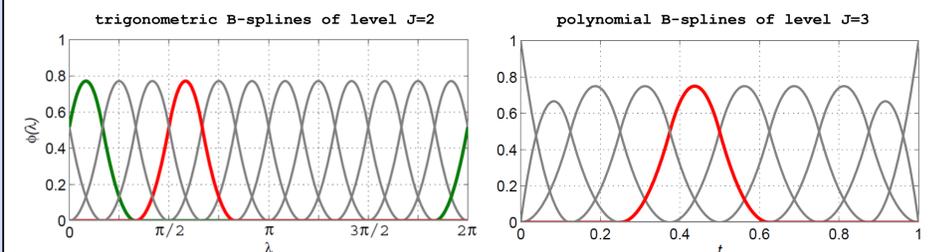
The representation of the unknown Chapman parameter N_0 reads

$$N_0 = N_0^{\text{back}} + \Delta N_0. \quad (7)$$

As background model N_0^{back} we can choose e.g. IRI-2007. The correction part ΔN_0 is modeled by a series expansion

$$\Delta N_0(\lambda, \varphi, t) = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} \sum_{k_3=0}^{K_3-1} d_{k_1, k_2, k_3} \phi_{k_1}^{j_1}(\lambda) \phi_{k_2}^{j_2}(\varphi) \phi_{k_3}^{j_3}(t) \quad (8)$$

in **tensor products** of three 1-D scaling functions $\phi_k^j(\cdot)$ with initial **unknown scaling coefficients** d_{k_1, k_2, k_3} . As $\phi_k^j(\cdot)$, we choose **trigonometric B-spline functions** for λ and **polynomial B-spline functions** for φ and t . Considering Eqs. (6), (7) and (8), we can model the 4-D electron density $N_e(h, \lambda, \varphi, t)$.



Similar to Eq. (8), 3-D B-spline expansions will be introduced for the other four unknown target parameters (WP 30) into Eq. (6).

For each observation technique (GNSS, altimetry, radio occultation, etc.) or mission (JASON-1/2, Envisat, etc.), resp., denoted as group i , we derive an **observation equation** for estimating the coefficients d_{k_1, k_2, k_3} of the B-spline model within an adjustment process (Gauss-Markov Model), i.e.

$$y_i + e_i = A_i \beta \quad \text{with} \quad D(y_i) = \sigma_{y,i}^2 P_{y,i}^{-1}. \quad (9)$$

The observation vector y_i of an observation group i collects ΔN_0 values, e_i represents the observation errors, $\sigma_{y,i}^2$ and $P_{y,i}$ are the unknown variance factor and the given positive definite weight matrix. The vector β consists of the unknown coefficients d_{k_1, k_2, k_3} and other auxiliary parameters (DCBs, bias, etc.).

Acknowledgement: This work is funded by the Deutsche Forschungsgemeinschaft (DFG), Bonn, Germany, under the grant HU 1558/3-1, JA 640/8-1, and SCHM 2433/3-1.