RegGRAV

Regional Gravity field modeling as multi-resolution representation estimated from the combination of heterogeneous data sets

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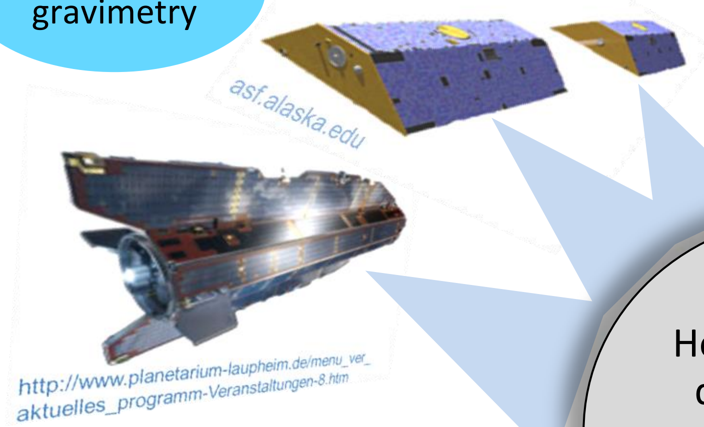
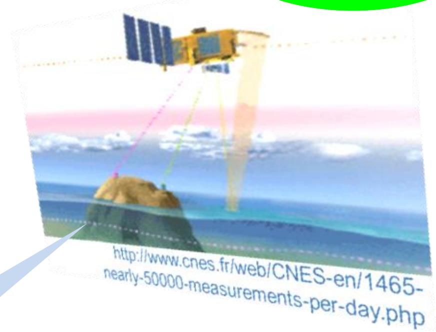
Motivation

Task: Software developement for the flexible computation of regional gravity and geoid models from …

Satellite gravimetry



Altimetry



Difficulty:

How to combine data sets with different distribution, resolution, and accuracy?

http://geograficamente.word-press.com/2009/03/19/1621/

airborne gravimetry

terrestrial gravimetry

… heterogeneous data sets.

Aim: Construction of precise height systems in operational areas.

Approach

Regional gravity modeling

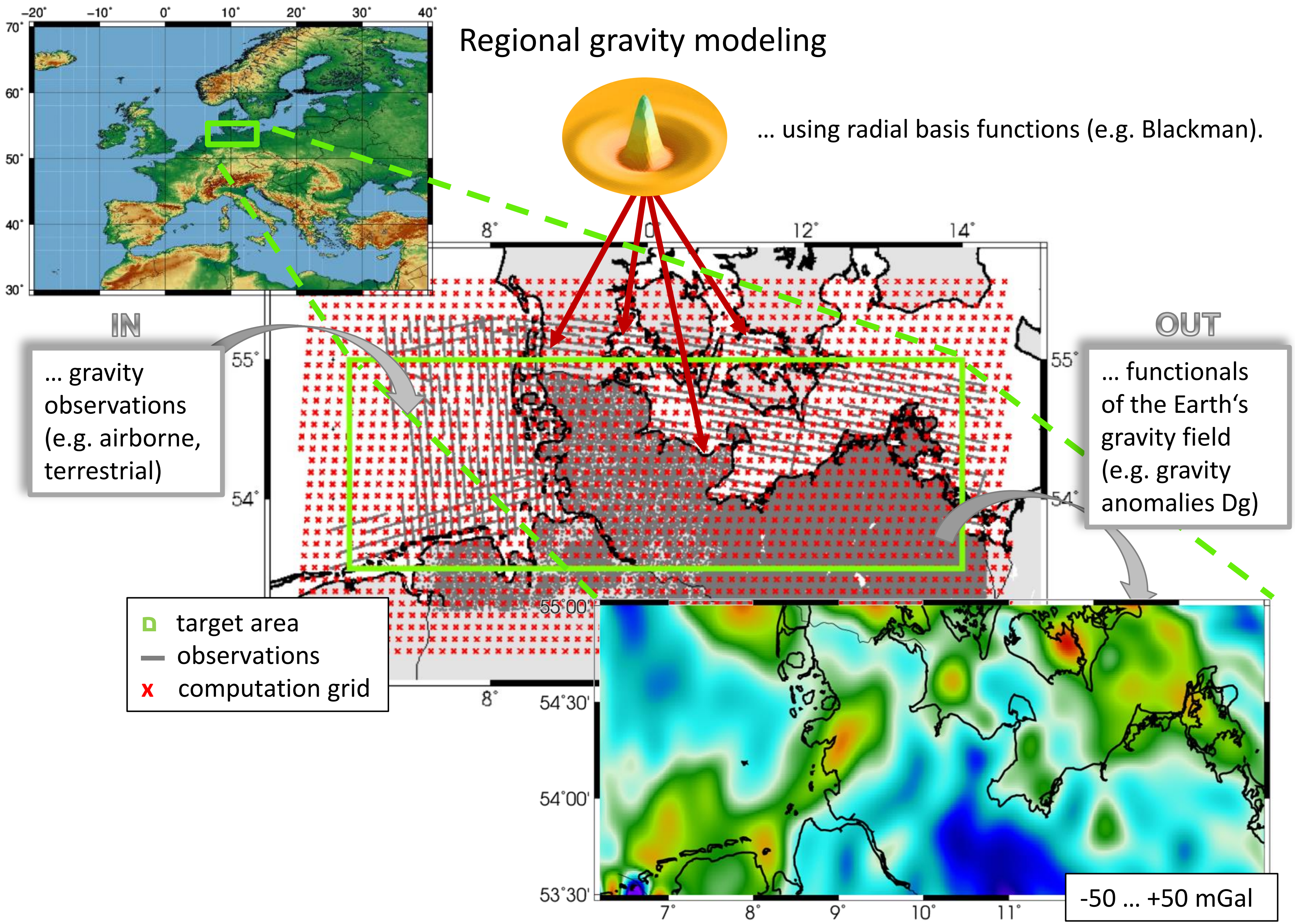
... using radial basis functions (e.g. Blackman).

IN

... gravity observations (e.g. airborne, terrestrial)

OUT

... functionals of the Earth's gravity field (e.g. gravity anomalies Dg)



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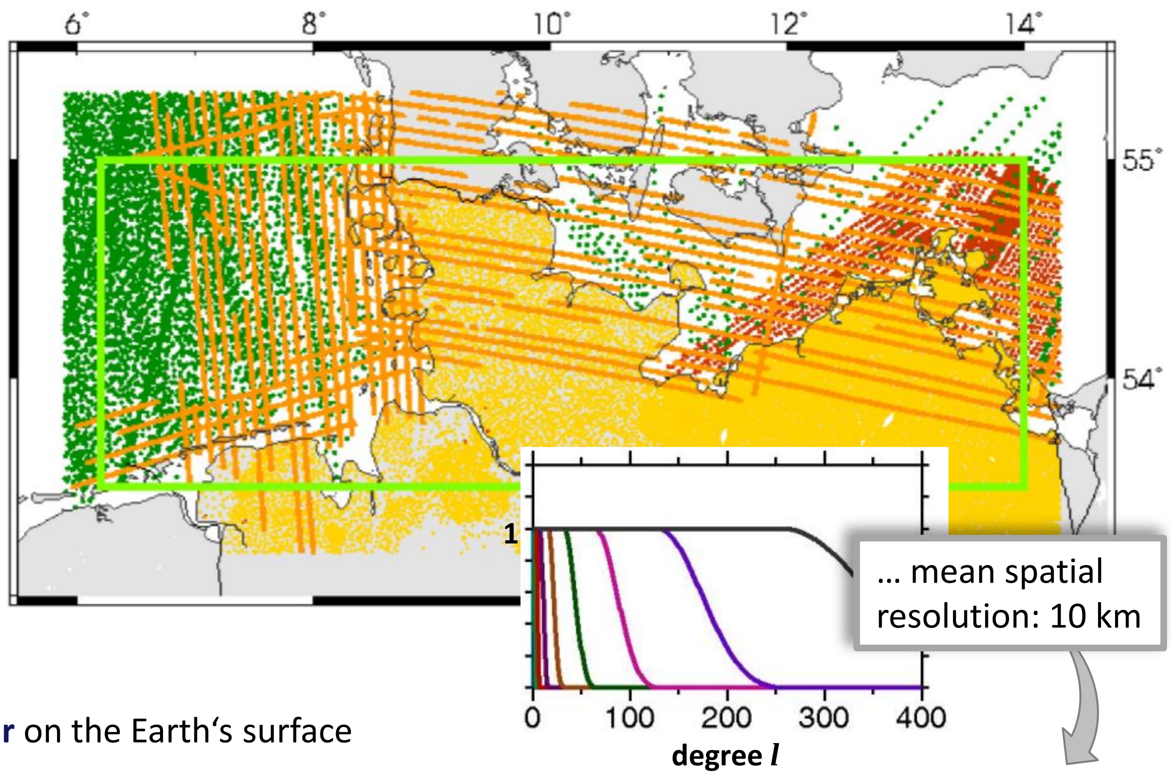
Observations

- GOCE GG
- altimetry,
- airborne,
- sea ground,
- terrestrial measurements



… sensitive to different frequency bands

… defined by **resolution levels j**

… limited by **maximum degree L_j** in a series expansion

… related to the **spatial resolution r** on the Earth's surface

j [level]	1	2	3	4	5	6	7	8	9	10	11	…
L_j [deg] r [km]	1 20000	3 6667	7 2857	15 1333	31 645	63 317	127 157	255 78	511 39	1023 20	2047 10	… …
		satellite gravimetry										
		altimetry										
							airborne + terrestrial gravimetry					

frequency →

Combination of data sets

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$$\Delta\mathcal{F}(\boldsymbol{x}) = \sum_{q=1}^{Q} d_{J,q}\, b_J(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{q=1}^{Q} d_{J,q} \sum_{l=0}^{L_J} \frac{2l+1}{4\pi}\left(\frac{R}{r}\right)^{l+1} B_l P_l(\cos\theta)$$

GOCO03s[1] up to d/o 127
[1]Combination of GRACE, GOCE, SLR, …

technique		Modified basis function $\tilde{b}_J$
GRACE	ΔV	$\tilde{b}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_q) = \sum_{l=0}^{L}\frac{2l+1}{4\pi}\left[\left(\frac{R}{r_1}\right)^{l+1} B_l P_l(cos\theta_1) - \left(\frac{R}{r_2}\right)^{l+1} B_l P_l(cos\theta_2)\right]$
GOCE	V_{zz} (e.g.)	$\tilde{b}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{l=0}^{L}\frac{2l+1}{4\pi}\frac{(l+1)\,(l+2)}{r^2}\left(\frac{R}{r}\right)^{l+1} B_l P_l(cos\theta)$
Altimetry	N = SSH - DOT	$\tilde{b}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{l=0}^{L}\frac{2l+1}{4\pi}\frac{1}{\gamma}\left(\frac{R}{r}\right)^{l+1} B_l\ P_l(cos\theta)$
Terrestrial, air-, shipborne, bathymetry measurements	Dg	$\tilde{b}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{l=0}^{L}\frac{2l+1}{4\pi}\frac{l-1}{r}\left(\frac{R}{r}\right)^{l+1} B_l P_l(cos\theta)$
	δg	$\tilde{b}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{l=0}^{L}\frac{2l+1}{4\pi}\frac{1-l}{r}\left(\frac{R}{r}\right)^{l+1} B_l\ P_l(cos\theta)$

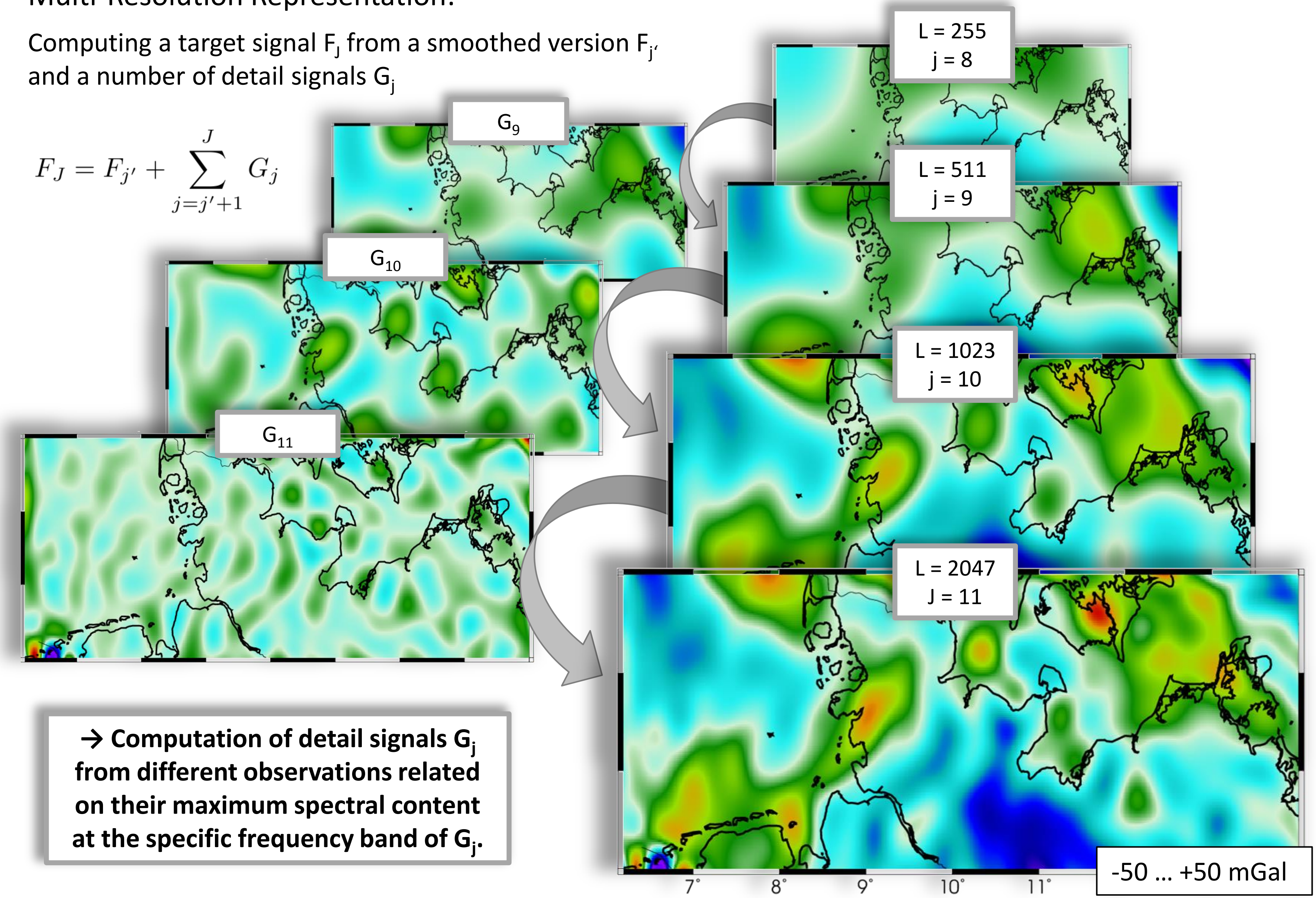
- **Estimation of unknown scaling coefficients d_J**
  … by using an extended Gauß-Markov model and VCE (rigorous combination at one level j).

MRR

Multi-Resolution Representation:

Computing a target signal F_J from a smoothed version $F_{j'}$ and a number of detail signals G_j

$$F_J = F_{j'} + \sum_{j=j'+1}^{J} G_j$$



→ Computation of detail signals G_j from different observations related on their maximum spectral content at the specific frequency band of G_j.

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Relative weighting

Observation	j = 8 (L = 255)	j = 9 (L = 511)	j = 10 (L = 1023)	j = 11 (L = 2047)
GOCE V_{xx}	1	10^{-1}	10^{-3}	10^{-5}
GOCE V_{xy}	10^{-4}	10^{-5}	10^{-7}	10^{-9}
GOCE V_{xz}	10^{-1}	10^{-2}	10^{-4}	10^{-6}
GOCE V_{yy}	1	10^{-1}	10^{-3}	10^{-5}
GOCE V_{yz}	10^{-4}	10^{-5}	10^{-7}	10^{-9}
GOCE V_{zz}	1	10^{-1}	10^{-3}	10^{-5}
ERS-1e	1	1	10^{-2}	10^{-3}
ERS-1f	1	1	10^{-2}	10^{-3}
Jason 1 GM	1	1	10^{-1}	10^{-3}
Envisat EM	1	1	10^{-1}	10^{-3}
Cryosat RADS	1	1	10^{-2}	10^{-3}
Airb. North Sea	10^{-1}	1	1	1
Airb. Baltic Sea	10^{-1}	1	10^{-1}	10^{-2}
Terrestrial Data	10^{-1}	1	1	1
Bathymetry	10^{-2}	10^{-1}	1	10^{-1}
Prior information GOCO03s d/o 127	10^{-3}	10^{-4}	10^{-4}	10^{-5}

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Criteria

● ● ● ● high sensitivity

Relative weighting

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Cryosat RADS		1	10^{-2}	10^{-3}
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Airb. Baltic Sea			10^{-1}	10^{-2}
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Prior information GOCO03s d/o 127	10^{-3}	10^{-4}	10^{-4}	10^{-5}

Criteria

- ● ● ● ● high sensitivity
- ● ● no correlations
- ● ● spatial distribution (prior information not sufficient)

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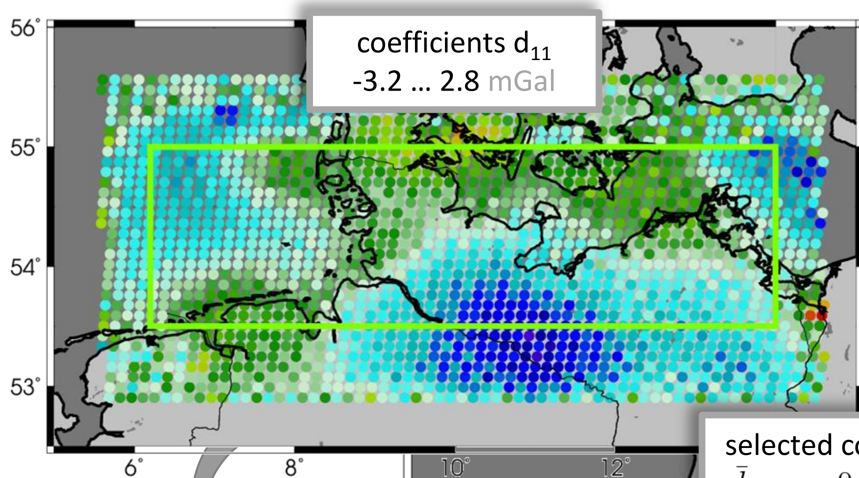
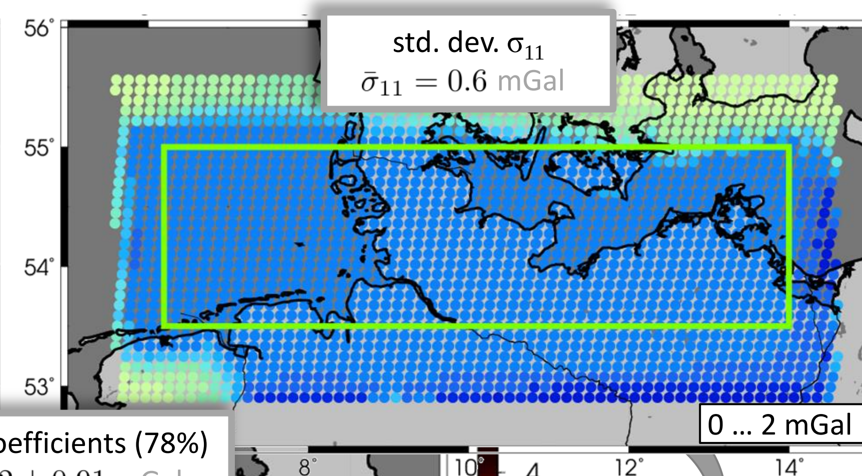
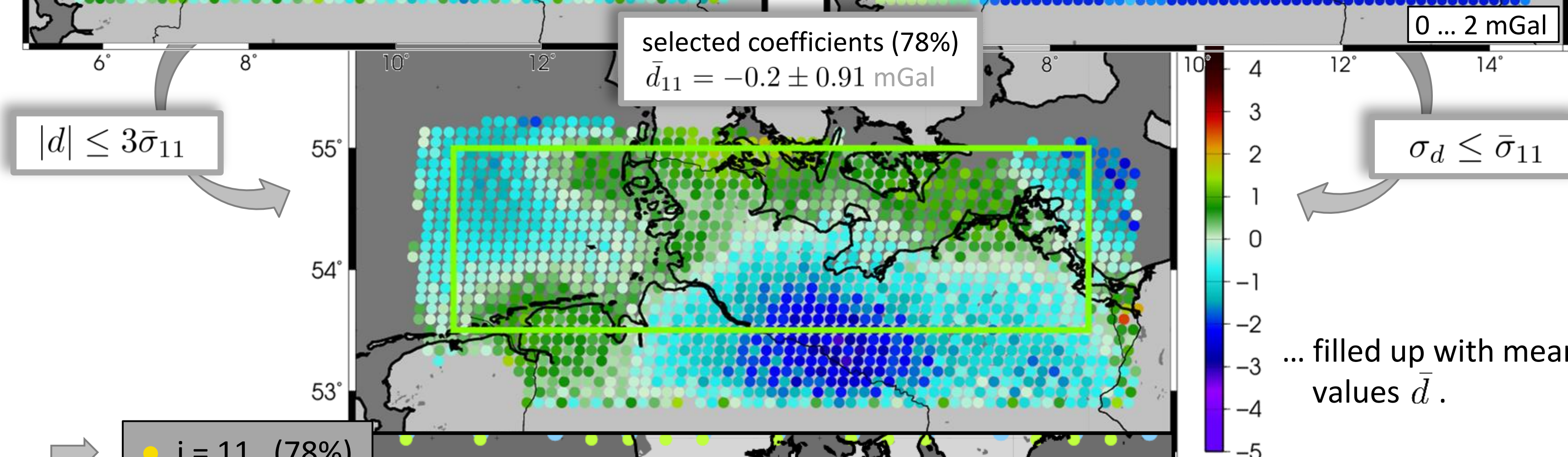
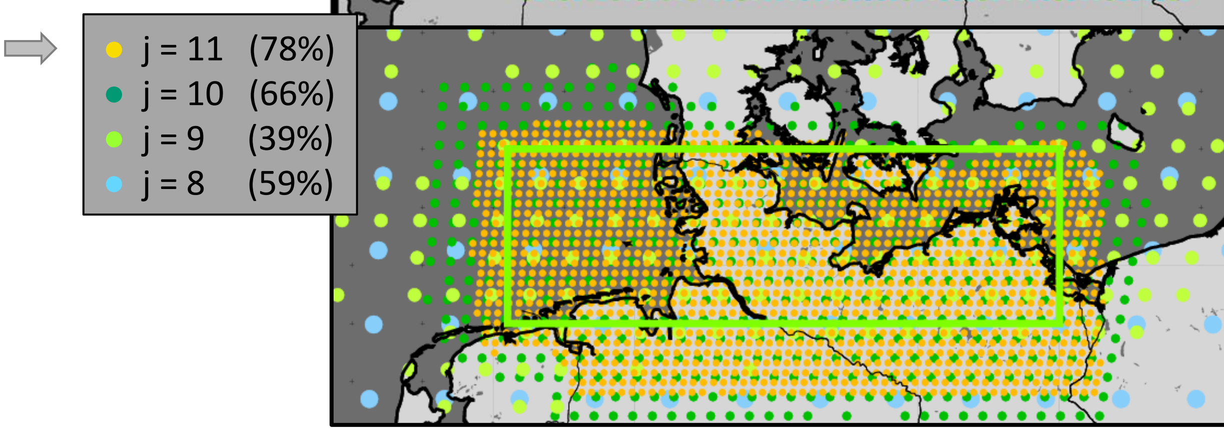
Coefficients







… filled up with mean values $\bar{d}$.



detected outliers depending on:

- observation type -> data gaps
- dimension of computation grid

Summation of detail signals

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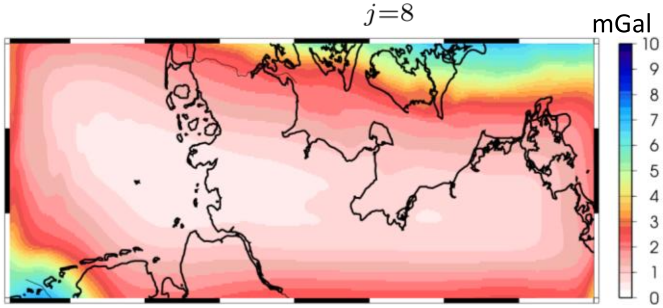
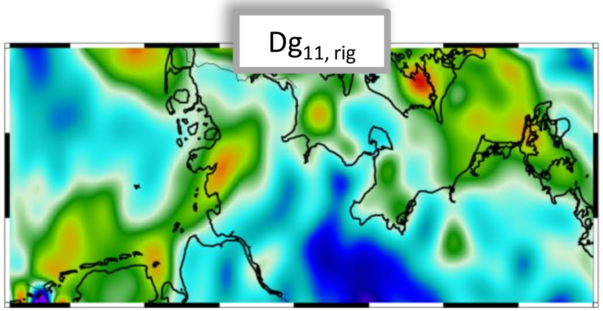
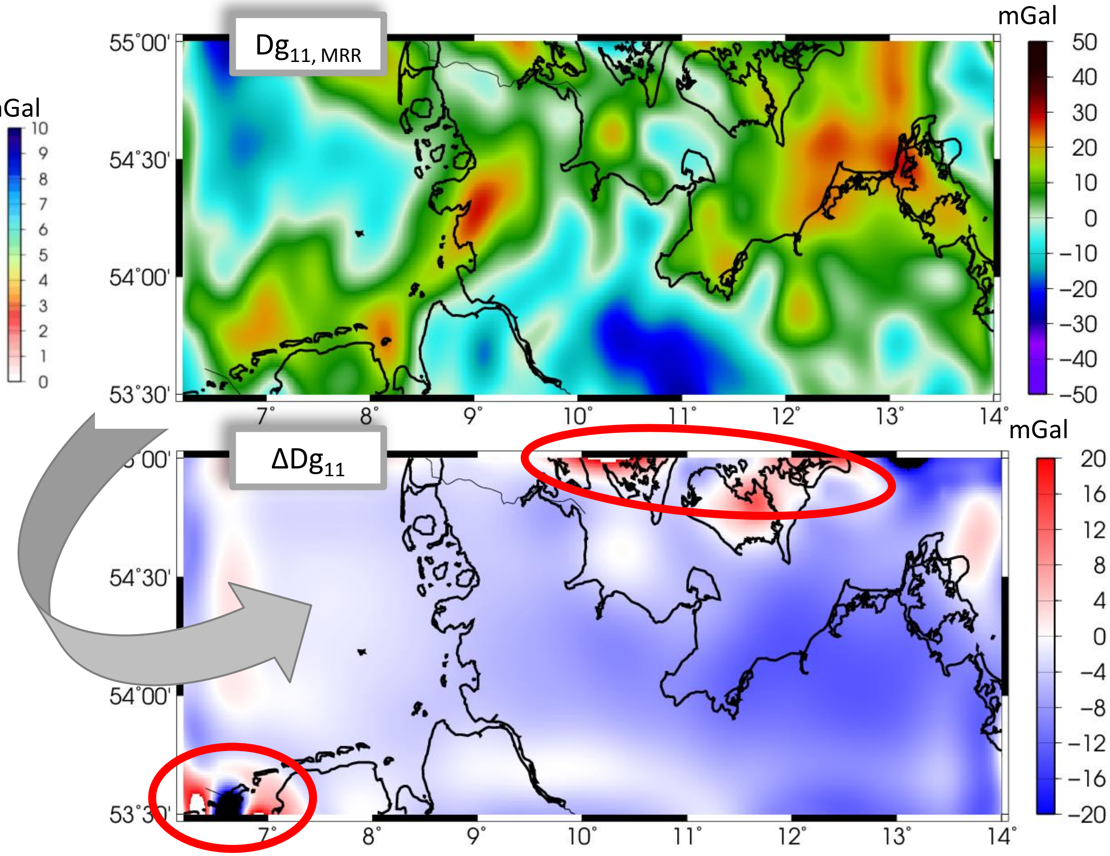
MRR: $$Dg_{11,MRR} = \sum_{j=8}^{11} G_j$$

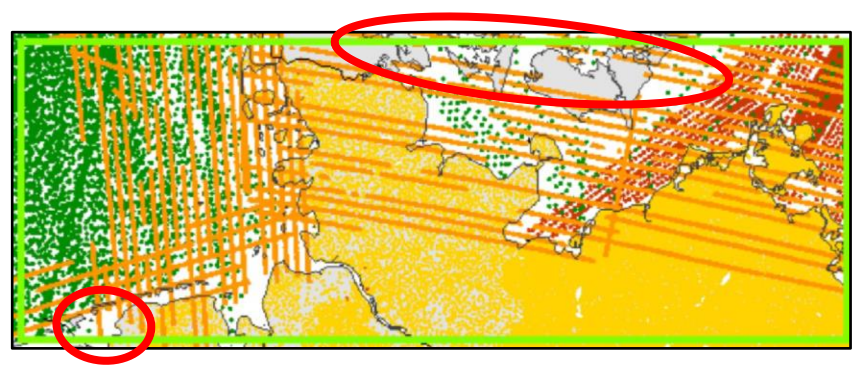


Dg_{11}: -28.41 … 28.47 mGal
σ_{11}: 0.46 … 7.22 mGal

- Largest standard deviations in less observed regions!







ΔDg, mean +/- std: **-4.79 +/- 4.82** mGal

- Largest differences at data gaps close to the borderline.
- **Improvement:** MRR-solution contains optimized spectral information in all frequency domains (contribution of GOCE)!

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Outlook & Summary

Outlook

- improving selection of input data
- choosing prior information with higher spectral content (e.g. topographic models)
- considering correlations between detail signals (e.g. introducing a filter matrix)
- improving outlier detection
- validation with real data
- further study areas
- ...

Criteria

● ● ● ●	high sensitivity
● ●	no correlations
● ●	spatial distribution (prior information not sufficient)

Summary

Rig. combination @ j = 11	MRR combination up to j = 11
+ less unknowns to estimate	- larger number of unknowns
- relative weighting of obs. at highest level	+ relative weighting of obs. at each level + spectral information in all frequency bands + improved handling of data gaps ➢ stabilized solution

➢ **Exploiting the highest degree of information out of each data set.**

Bundesamt für Kartographie und Geodäsie

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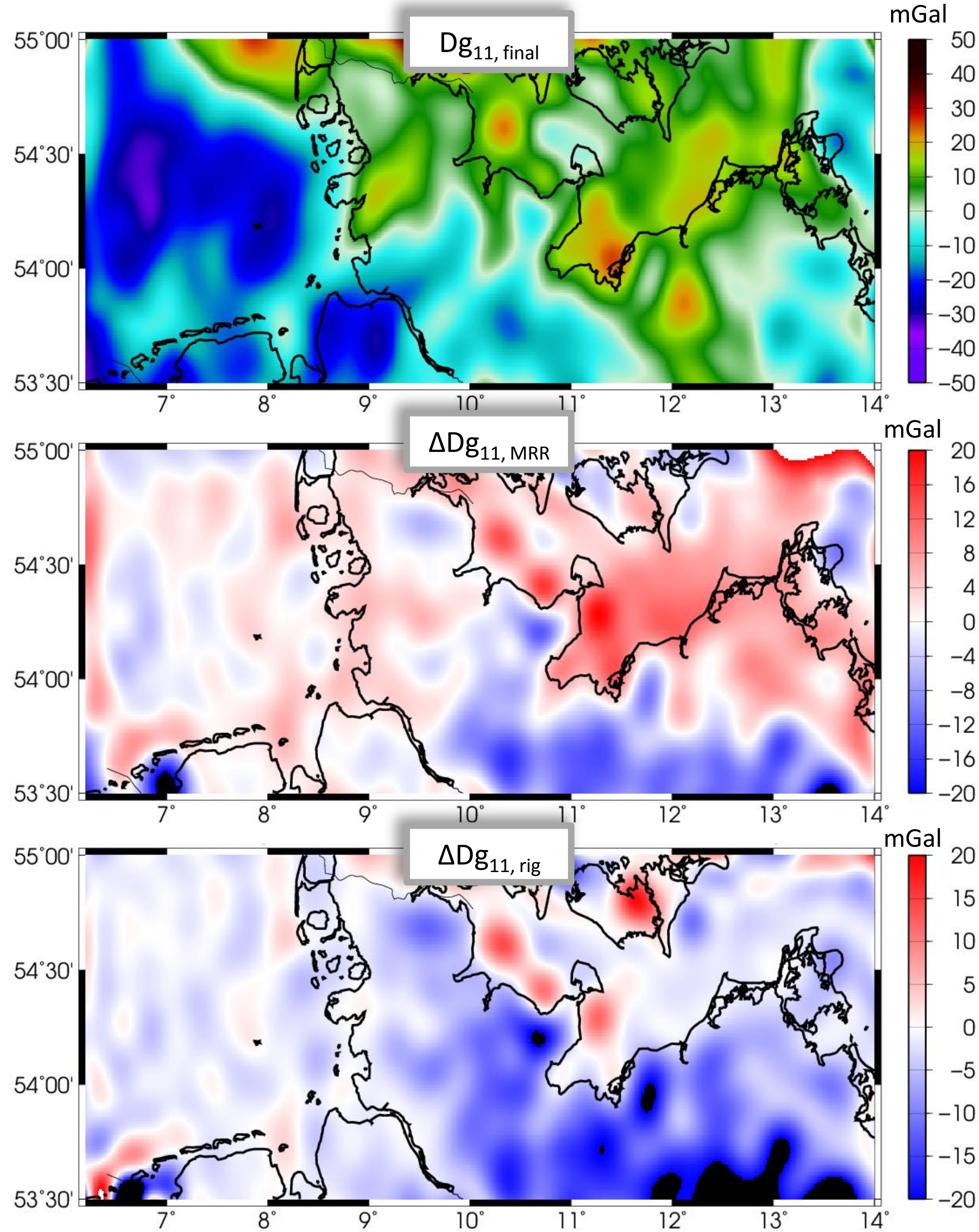
Appendix

Comparison with EGM2008

$$Dg_{11,final} = GOCO03s + \sum_{j=8}^{11} G_j$$



Difference $Dg_{11,\,MRR}$ – EGM2008
(j = 11, l = 2023, Blackman smoothed)

ΔDg, mean +/- std: **0.86 +/- 5.80** mGal

- Largest differences at data gaps.
- Differences up to +/- 10 mGal in western parts (new data set?)
- and in the Baltic Sea (missing airborne data in EGM?).

Difference $Dg_{11,\,rig}$ – EGM2008

ΔDg, mean +/- std: **-3.93 +/- 6.01** mGal

- Larger differences (especially in western parts).
- Missing spectral information in mid and low frequency domains.

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Software – Specifications

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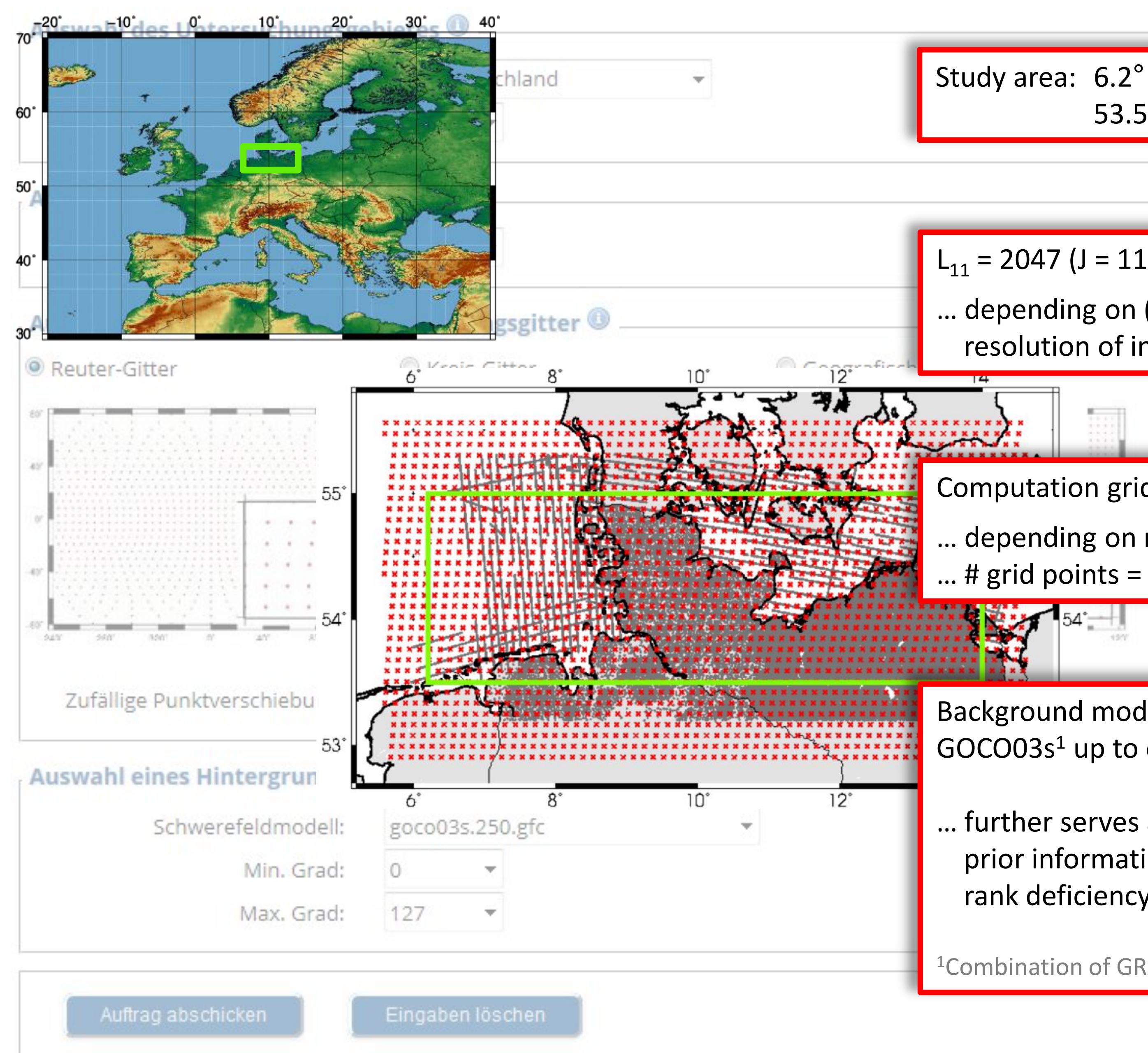


Study area: 6.2° … 14.0° longitude
53.5° … 55.0° latitude

L_{11} = 2047 (J = 11)

… depending on (spectral/spatial) resolution of input data

Computation grid: Reuter

… depending on resolution level J
… # grid points = # unknowns Q

Background model:
GOCO03s[1] up to d/o 127

… further serves as prior information due to rank deficiency problems

[1]Combination of GRACE, GOCE, SLR, …

Software – Output

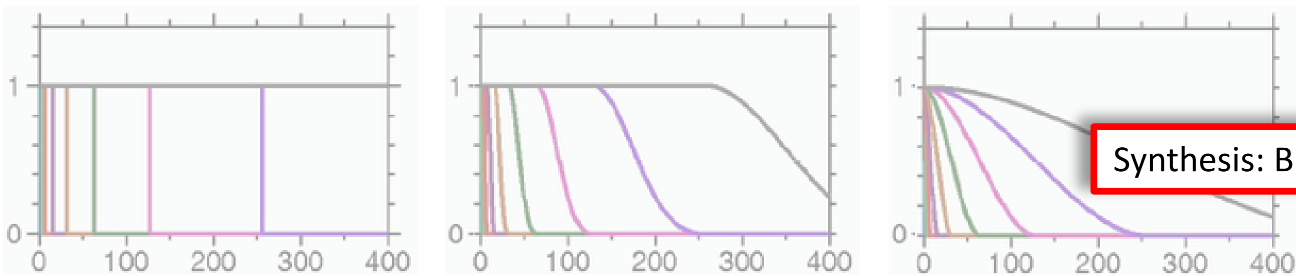
Auswahl der Level

Level: 8 - 11

Computation of the target signal up to max. level J = 11

Auswahl der Wavelet- und Skalierungsfunktion

Shannon | Blackman | Cubic Polynominal



(Darstellung im Freqenzbereich)

Synthesis: Blackman scaling funct.

Auswahl des Ausgabe-Funktionals

N	Geoidhöhen [m]
Dg	Schwereanomalien [mGal]
V	Potential [m²/s²]
T	Störpotential [m²/s²]
Txx	Ableitung ∂²T/∂x² [E]
Txy	Ableitung ∂²T/∂x∂y [E]
Txz	Ableitung ∂²T/∂x∂z [E]
Tyy	Ableitung ∂²T/∂y² [E]
Tyz	Ableitung ∂²T/∂y∂z [E]
Tzz	Ableitung ∂²T/∂z² [E]

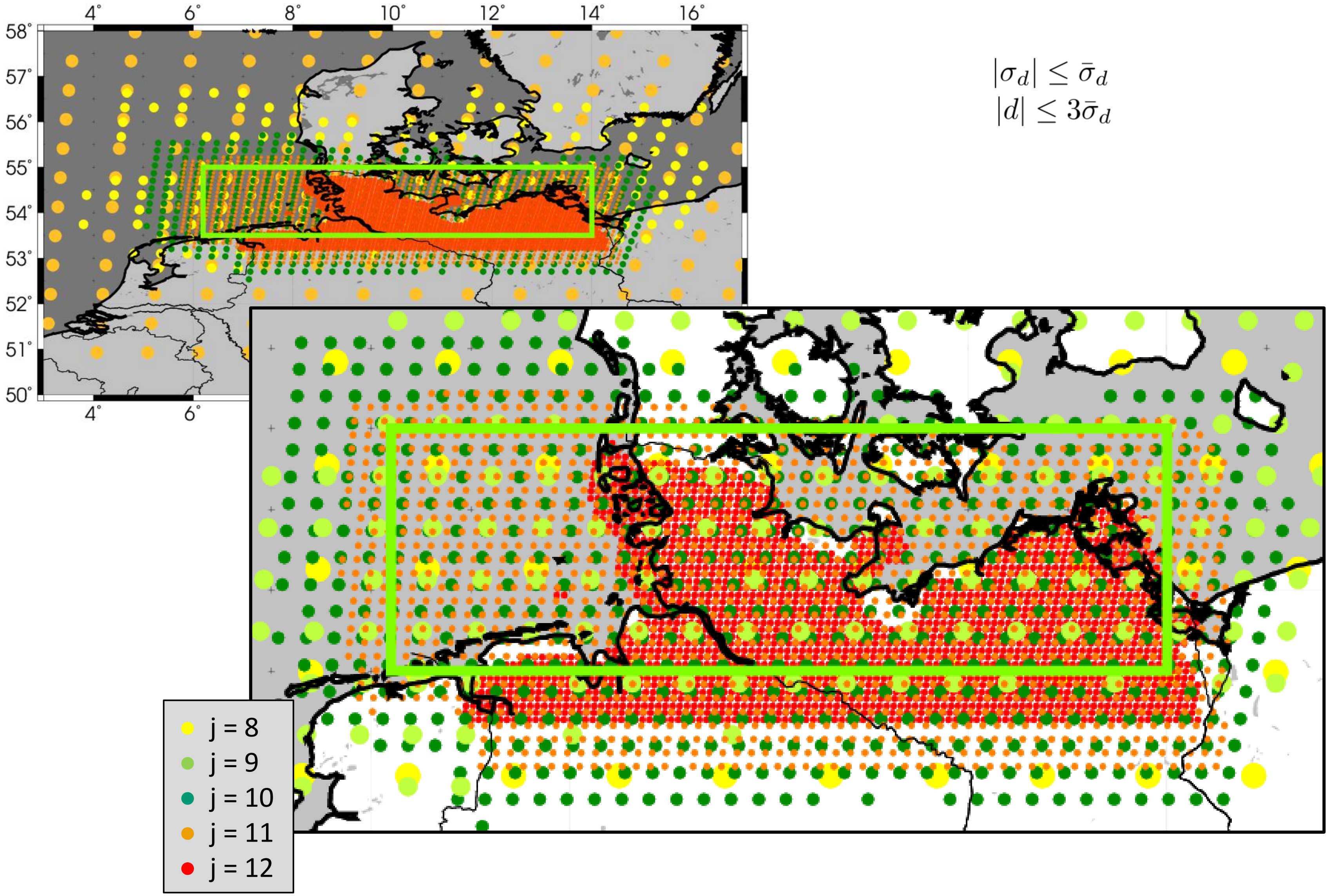
Output:
… different functionals of the Earth's gravity field (e.g. gravity anomalies *Dg*)

Auswahl des Ausgabegebiets

Gitter aus Gebietsdefinition 0 [km]

Punkteliste ---

Coefficients



$$|\sigma_d| \leq \bar{\sigma}_d$$
$$|d| \leq 3\bar{\sigma}_d$$

Analysis

$$\Delta\mathcal{F}(\boldsymbol{x}) = \sum_{q=1}^{N} d_{J,q}\, b_{J+1}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{q=1}^{N}\sum_{l=0}^{L_J} \frac{2l+1}{4\pi} d_{J,q} \Phi_{J+1,l} \left(\frac{R}{r}\right)^{l+1} P_l(\cos\psi)$$

Observation equation for one observation:

Deterministic part

$$\Delta\mathcal{F}(\boldsymbol{x}) + e(\boldsymbol{x}) = \boldsymbol{b}_{J+1}{}^T(\boldsymbol{x},\boldsymbol{x}_q)\, \boldsymbol{d}_J$$

IN:	$\Delta\mathcal{F}$	observation
	e	measurement error
	$\boldsymbol{b}_{J+1}$	(Nx1) vector of basis functions
OUT:	$\hat{\boldsymbol{d}}_J$	(Nx1) vector of scaling coefficients

Stochastic part

$$D(\Delta\boldsymbol{\mathcal{F}}_k) = \sigma_k^2 \boldsymbol{P}_k{}^{-1}$$

IN:	$\Delta\boldsymbol{\mathcal{F}}_k$	vector of observations
	$\boldsymbol{P}_k$	weighting matrix of observations
OUT:	$\hat{\sigma}_k$	variance components (VCs)

Estimation of unknown scaling coefficients $\boldsymbol{d}_J$

Introduction of additional observations $\boldsymbol{\mu}_d$

$$\boldsymbol{\mu}_d + \boldsymbol{e}_d = \boldsymbol{d} \qquad \text{with} \qquad D(\boldsymbol{\mu}_d) = \boldsymbol{\sigma}_d^2 \boldsymbol{P}_d{}^{-1}$$

- $\boldsymbol{\mu}_d$ … prior information
- avoiding singularity problems
- rank deficiencies (in general number of grid points too large)

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Modelling approach – Analysis

Extented Gauß-Markov model for several observation techniques:

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \\ \boldsymbol{\mu}_d \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_1 \\ \vdots \\ \boldsymbol{e}_K \\ \boldsymbol{e}_d \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^T_{J+1,1} \\ \vdots \\ \boldsymbol{b}^T_{J+1,k} \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{d}_J$$

$$\boldsymbol{y} = \begin{bmatrix} \Delta\boldsymbol{V}_1 \\ \vdots \\ \boldsymbol{\mu}_d \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{\mu}_d \end{bmatrix}$$

vector of observations

(Nxn_k)

matrix of scal. functions

(Nx1)

vector of scal. coefficients

$$D\left(\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{\mu}_d \end{bmatrix}\right) = \sigma_k^2 \begin{bmatrix} \boldsymbol{P}_1^{-1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix} + \ldots + \sigma_d^2 \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\Sigma}_d^{-1} \end{bmatrix}$$

k = 1 … K various observation techniques

n_k number of observations from technik k

Note: the measurements $\boldsymbol{y}$ are treated as independent observations, i. e. without correlations.

- Soving the normal equations (by iteratively determined VCs) results in $\hat{\boldsymbol{d}}_J$.
- Extracting the erroneous observations $\boldsymbol{y}$ and applying the law of error propagation then results in the variance covariance matrix $D(\hat{\boldsymbol{d}}_J)$.

Detail signal j = 11

G_{11}	min … max [mGal]	mean [mGal]	+/- std. [mGal]
Dg_{11}	-23.86 … 39.74	0.06	3.58
σ_{11}	0.11 … 18.18	0.86	1.61

large standard deviations in areas of replaced „mean-coefficients“
-> outlier detection $|Dg_{11}| \leq 3\bar{\sigma}_{11}$

$Dg_{11, red}$	-17.44 … 12.64	-0.07	2.96