Consistent estimation of Earth rotation, geometry and gravity with DGFI’s multi-satellite solution

Mathis Blossfeld¹, Stefka V.², Mueller H.¹, Gerstl M.¹
¹DGFI, Munich, Germany
²Astronomical Institute, Academy of Sciences of the Czech Republic
blossfeld@dgfi.badw.de

Abstract. Satellite Laser Ranging (SLR) is the unique technique to determine station coordinates together with Earth Orientation Parameters (EOPs) and Stokes coefficients of the Earth’s gravity field in one common adjustment with very high accuracy. In this study, up to ten satellites (Etalon 1/2, LAGEOS 1/2, Stella, Starlette, Ajisai, Larets, LARES and BLITS) with different orbit characteristics (e.g. inclination and altitude) are combined. Together with the Earth’s gravity field and rotation (terrestrial pole coordinates, UT1-UTC), also the 3D station coordinates are estimated on a weekly and monthly basis. The results are compared to common state-of-the-art products which are provided by the International Earth Rotation and Reference Systems Service (IERS).

Test data and relative weighting

Satellite Laser Ranging (SLR) is able to determine the “three pillars” (Plag & Pearlman, 2009) of the Global Geodetic Observing System (GGOS). These pillars are the Earth’s geometry, rotation and the Earth’s gravity field. In its function as an official analysis center of the International Laser Ranging Service (ILRS, Pearlman et al., 2002), DGFI is developing and maintaining software to process SLR observations called "DGFI Orbit and Geodetic parameter estimation Software" (DOGS). The software is used to analyze SLR observations and to compute multi-satellite solutions.

This study is based on a combined weekly solution of up to ten satellites between January 2000 and May 2013. The satellites are Etalon 1/2, LAGEOS 1/2, Stella, Starlette, Ajisai, Larets, LARES and BLITS. The relative weighting of the satellites is done using variance component estimation (VCE). Since the VCE is based on the main diagonals of the individual and combined normal equations (NEQs), the obtained relative weight per satellite NEQ depends on the included parameters in the adjustment. To evaluate the impact of each satellite on the estimated parameters, we performed three test scenarios:

(i) only station coordinates are estimated,
(ii) station coordinates together with Earth Orientation Parameters (EOPs) are estimated and
(iii) station coordinates, EOPs and gravity field coefficients (GFCs) are estimated.

A weekly NEQ of type (iii) contains orbit parameters (Keplerian elements, empirical accelerations, model scaling factors), EOPs (terrestrial pole coordinates at midnight epochs and UT1-UTC values at midnight epochs extrapolated using Length-Of-Day (LOD)), 3D station coordinates and GFCs up to degree and order six. In addition to the weekly NEQs, a test monthly solution for January 2007 with GFCs up to degree and order 20 is obtained by stacking four weekly solutions.

If only station coordinates are estimated, the largest weights are obtained for both LAGEOS satellites. The Etalons are down-weighted w.r.t. the LAGEOS satellites since their altitude is much larger than the LAGEOS altitudes and since they are observed much less by the global ILRS telescope network. The Low Earth Orbiters (LEOs) with orbit altitudes smaller than 1500 km (Stella, Starlette, Ajisai, Larets, LARES and BLITS) are down-weighted since the drag of, e.g., the high atmosphere is much more difficult to model. This fact causes a deficiency in the functional model of these satellites which results in a smaller relative weight. If EOPs are included in the estimation, the
weights for all satellites remain nearly the same. If, in addition to station coordinates and EOP, the GFCs are estimated, the weight of the LAGEOS and Etalons remains still the same as in solution type (i) and (ii) but the weights of the LEOs are increased. This behavior is caused by the increased sensitivity of LEOs to the Earth’s gravity field. The smaller the orbit altitude of a satellite is, the larger is its sensitivity on the gravity field. The exact numbers are summarized in Bloßfeld et al., 2014b.

**Improvement of station coordinates, EOPs and GFCs**

The diversity of the orbits allows to de-correlate highly correlated parameters such as GFCs and EOPs. Therefore, an estimation of the station coordinates together with the satellite orbits, the EOPs and the GFCs in one common adjustment is performed.

**Station coordinates**

The origin and the scale of an SLR-only derived terrestrial reference frames (SLRF) are realized implicitly by taking advantage of the satellites orbit dynamics (Bloßfeld et al., 2014a). The station coordinates are evaluated by comparing global weekly networks to an updated version of the SLRF2008\(^1\) using a weekly performed 7-parameter similarity transformation. We compared the root mean square (RMS) of the translation and scale parameter time series of three different solutions. This allows us to investigate the datum stability of the estimated SLRFs. For the weekly LAGEOS-only SLRFs, the RMS values are between 0.5 and 0.7 mm for the x-translation, y-translation and the scale. The z-translation RMS is about 1.2 mm. If LARES is included in the TRF estimation, the RMS values in all four datum parameters decrease between 14 % and 22 %. If all satellites are included, the RMS of the y- and z-translation decreases whereas the RMS of the x-translation and the scale slightly increase compared with a LAGEOS-LARES solution. This fact might be caused by the perturbed orbits of some LEOs. If the drag is not modeled perfectly, spurious signals might propagate into the station coordinate estimates.

In addition to the datum parameters, we compared the global mean weighted RMS (WRMS) of the station coordinate residuals from the similarity transformation. Whereas the LAGEOS-only solution shows WRMS values of 5.1 mm, 4.8 mm and 4.8 mm in the North, East and Height component, the LAGEOS-LARES WRMS values decrease by about 14 % in the horizontal components and about 1 % in the height component. Using all satellites, the horizontal WRMS values are further decreased by about 20 % to 3.2 mm and 3.1 mm in the North and East. The Height WRMS decreases only marginally. The reduction of the horizontal WRMS is caused by the improved sky coverage when more satellites are observed by a station. The vertical coverage improves only slightly when using more satellites.

**Earth Orientation Parameters (EOPs)**

In a weekly solution, eight values of a piece-wise linear polygon at midnight epochs are estimated for the terrestrial pole coordinates and UT1-UTC. Thereby, UT1-UTC is extrapolated using the LOD information from SLR. The absolute offset of the UT1-UTC polygon is a singularity in the NEQ which has to be removed by fixing at least one UT1-UTC value to its a priori value (IERS 08 C04 time series\(^2\)). This is done for the mid-arc value in this study. If only observations to LAGEOS 1 are used for a determination of the EOP, the UT1-UTC polygon shows a significant systematic deflection w.r.t. its a priori values (Bloßfeld et al., 2011). Therefore, during 2012, the estimated UT1-UTC values differ up to 25 ms from the IERS 08 C04 time series (at the week boundaries). If at least LAGEOS 2 is combined with LAGEOS 1, the deflection is decreased by a factor of ten. This is caused by the de-correlation of orbit parameters and LOD if different inclinations are com-

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\(^2\) IERS 08 C04 time series: http://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html (2014/02/12)
bined (Bloßfeld et al., 2011). Within the LAGEOS-LARES solution, the systematic deflection is further reduced (differences smaller than 1.0 ms) and if all satellites are used, the UT1-UTC polygons shows nearly no systematics any longer (differences smaller than 0.45 ms). Also the scatter (WRMS) of the terrestrial pole coordinates is reduced from 39.2 μas (LAGEOS 1) to 15.9 μas (LAGEOS-LARES and all satellites) in the x-component. In the y-component, the WRMS is reduced from 50.0 μas (LAGEOS 1) to 26.5 μas (LAGEOS-LARES) and 16.4 (all satellites).

**Gravity field coefficients (GFCs)**

The GFCs are included up to degree and order 20 in every weekly NEQ. Within a monthly NEQ, the GFCs can be estimated up to degree and order 20 due to the improved spatial resolution of the observations. To investigate the benefit of the GFCs, we computed exemplarily a monthly solution for January 2007. Therein, we stacked four weekly NEQs in January 2007 to one monthly NEQ and equalize common parameters such as station coordinates, GFCs and EOPs at the week boundaries. The monthly NEQ contains EOP polygons with 29 values at midnight epochs and monthly station coordinates and GFCs. If only LAGEOS 1/2 are used for the GFC determination, reliable estimates can only be obtained for GFCs with degree and order smaller than five. This fact can be explained with the high orbit altitude (about 6000 km) of the LAGEOS satellites and the resulting insensitivity of the LAGEOS orbits to higher degrees and orders. Using, in addition, Starlette, especially the tesseral GFCs (C_{nm}, S_{nm} with n ≠ m ≤ 20) are improved (smaller standard deviations). If all other LEOs are included in the solution, the higher degrees and orders benefits significantly from the smaller orbit altitudes. Reliable GFC estimates up to degree and order 20 can be obtained except for GFCs with degree 10 ≤ n ≤ 18 and order 0 ≤ n ≤ 5. These near-zonal GFCs have a larger standard deviation than the other GFCs. This fact is caused by the orbit characteristics of the used satellites. Since the Etalon satellites have a resonance of their orbit with degree two of the Earth’s gravity field (Bloßfeld et al., 2014b), they might help to improve these estimates although their orbit altitude is very large. A limitation of this possibility is that they are currently poorly tracked by the ILRS network.

**Further information**

A detailed description of the study is published in the Proceedings of the International Association of Geodesy (IAG) Symposia Series (Bloßfeld et al., 2014b).

**References**


Bloßfeld M., Stefka V., Müller H., Gerstl M., *Satellite Laser Ranging - A tool to realize GGOS?*, International Association of Geodesy Symposia Series (accepted), 2014b
