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Comparison of different combination strategies applied for the computation of terrestrial reference frames and geodetic parameter series

Manuela Seitz

Abstract The combination of space geodetic techniques is today and becomes in future more and more important for the computation of Earth system parameters as well as for the realization of reference systems. Precision, accuracy, long-term stability and reliability of the products can be improved by the combination of different observation techniques, which provide an individual sensitivity with respect to several parameters. The estimation of geodetic parameters from observations is mostly done by least squares adjustment within a Gauß-Markov model. The combination of different techniques can be done at three different levels: at the level of observations, at the level of normal equations and at the level of parameters. The paper discusses the differences between the approaches from a theoretical point of view. The combination at observation level is the most rigorous approach since all observations are processed together ab initio, including all pre-processing steps, like e.g. outlier detection. The combination at normal equation level is an approximation of the combination at observation level. The only difference is, that pre-processing steps including an editing of the observations are done technique-wise. The combination at the parameter level is more different: Technique-individual solutions are computed and the solved parameters are combined within a second least squares adjustment process. Reliable pseudo-observations (constraints) have to be applied to generate the input solutions. In order to realize the geodetic datum of the combined solution independently from the datum of the input solutions, parameters of a similarity transformation have to be set up for each input solution within the combination. Due to imperfect network geometries, the transformation parameters can absorb also non-datum effects. The multiple parameter solution of the combination process leads to a stronger dependency of the combined solution on operator decisions and on numerical aspects.

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1 Introduction

The combination of different space geodetic techniques is a common procedure in order to compute precise geodetic products today. Combining different observation types, the individual potentials of the different techniques with respect to the determination of certain geodetic parameters can be exploided for the combined product. But also the increased redundancy due to the larger number of observations leads to a higher precision of the parameters. Examples for combinations are the (i) realization of the International Terrestrial Reference System (ITRS) which is computed by combining data of Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS), Satellite Laser Ranging (SLR) and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) (e.g. *Seitz et al.* (2012)), (ii) the computation of gravity field products by combining satellite and terrestrial data and (e.g. *Kern et al.* (2003)) (iii) ionosphere models and troposphere parameters derived from multi-technique data (e.g.

Dettmering et al. (2011), *Krügel et al.* (2007)). The integration of different space geodetic techniques in order to ensure a long-term and precise monitoring of the geodetic parameters is also the goal of the Global Geodetic Observing System (GGOS) a component of the International Association of Geodesy (IAG) (*Plag and Pearlman*, 2009). The derivation of geodetic parameters from the observations is usually done by using the linear Gauß-Markov model. The combination of different techniques can be performed at the three levels within a least squares adjustment: the observation, the normal equation and the parameter level. This paper describes the mathematical fundamentals of the different approaches and discusses the pros and cons with regard to the ITRS realization from a theoretical point of view.

The ITRS is a global Earth related cartesian coordinate system which is well defined (*IERS*, 2010). It is realized by the International Terrestrial Reference Frame (ITRF) which consists of station positions and velocities of global distributed GNSS, VLBI, SLR and DORIS observing stations. It is computed from long-term observation time series of the four techniques, which span between 15 to 25 years. Fig. (1) shows the horizontal velocity field of the ITRS realization DTRF2008 (*Seitz et al.*, 2012). Consistently to the station coordinates time series of Earth Orientation Parameters (EOP) are estimated. DTRF2008 is computed by combining the four space geodetic techniques which contribute to the determination of station coordinates and EOP according to they individual potentials.

2 Least squares adjustment by Gauß-Markov model

In general the linearized observation equation describes the expectation values of the observation vector $\mathbf{b}$ as a linear combination of known coefficients and the unknown parameters $\mathbf{p}$ (*Koch*, 1999)

$$E(\mathbf{b}) = \mathbf{A}\mathbf{x} \tag{1}$$

with

$\mathbf{b}$	$n \times 1$ vector of observations (measurement minus „computed with a priori values”)
$E(\mathbf{b})$	$n \times 1$ vector of the expectation values of observations
$\mathbf{A}$	$n \times u$ coefficient matrix
$\mathbf{x} = \mathbf{p} - \mathbf{p_0}$	$u \times 1$ vector of unknowns (vector of corrections of $\mathbf{p_0}$)
$\mathbf{p}$	$u \times 1$ parameter vector
$\mathbf{p_0}$	$u \times 1$ vector of a priori values of $\mathbf{p}$.

Eq. (1) is the deterministic part of the Gauß-Markov model. The relation between observations and parameters are given by physical or mathematical principles. Linearity is, if necessary, achieved by a linearization of the original observation equations. Therefore, the observations equations are expanded into Taylor series. Usually, the number of observation n is larger than the number of unknowns u in order to reduce the impact of one observation on the estimates (*Koch*, 1999). For $n > u$ the equation system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is generally not consistent. From the addition of the vector of observation errors $\mathbf{v}$, the consistent observation equation is obtained

$$\mathbf{A}\mathbf{x} = \mathbf{b} + \mathbf{v} \tag{2}$$

with $E(\mathbf{b}+\mathbf{v}) = \mathrm{E}(\mathbf{b})$ because of $E(\mathbf{v}) = \mathbf{0}$.

It is expected that the observations are random values. The variance-covariance matrix of the observations is assumed to be known, except of the variance factor σ_0^2

$$\mathbf{C_{bb}} = \sigma_0^2 \mathbf{P}^{-1} \tag{3}$$

with

$\mathbf{C_{bb}}$	$n \times n$ variance-covariance matrix of observations
$\mathbf{P}$	$n \times n$ positive definite weight matrix of observations.

Eq. (3) is the stochastic part of the Gauß-Markov model.

For $n > u$ the solution of Eq. (2) is not unique. A solution can be get, if the squared sum of observation residuals $\mathbf{v} = \mathbf{A}\mathbf{x} - \mathbf{b}$ is minimized (least squares adjustment). The solution means the best linear unbiased estimation. The corresponding normal equation reads (e.g. *Schmidt et al.* (2011))

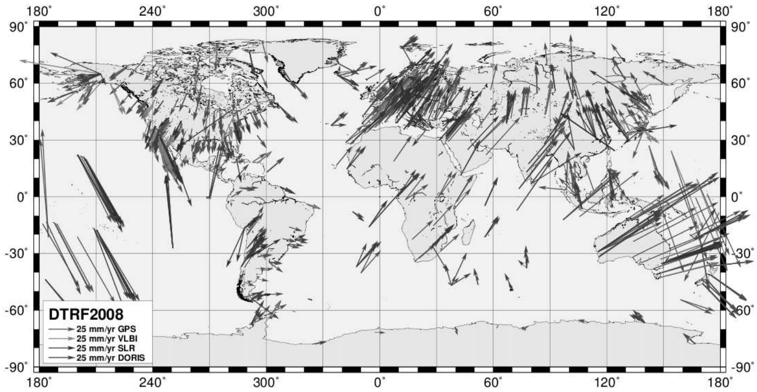

Fig. 1 Horizontal station velocity field of DTRF2008.

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\hat{\mathbf{x}} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{b}. \quad (4)$$

The solution of the normal equation is given by

$$\hat{\mathbf{x}} = (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{b}. \quad (5)$$

With the normal equation matrix $\mathbf{N} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}$ (which has to be regular, see below) and the right hand side $\mathbf{y} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{b}$ the equation can be written in a compact form

$$\hat{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{y}. \quad (6)$$

For the variance also an unbiased estimation is required. That means, the equation $E(\hat{\sigma}_0^2) = \sigma_0^2$ must be fulfilled, which can be reached by the estimation

$$\hat{\sigma}_0^2 = \frac{\hat{\mathbf{v}}^{\mathrm{T}}\mathbf{P}\hat{\mathbf{v}}}{n-u}. \quad (7)$$

The relation between the squared sum of observations and the squared sum of residuals is expressed as

$$\mathbf{b}^{\mathrm{T}}\mathbf{P}\mathbf{b} - \hat{\mathbf{v}}^{\mathrm{T}}\mathbf{P}\hat{\mathbf{v}} = \mathbf{y}^{\mathrm{T}}\hat{\mathbf{x}}. \quad (8)$$

The variance-covariance matrix of the estimated parameters is obtained from the variance-covariance matrix of observations $\hat{\mathbf{C}}_{\mathbf{bb}} = \hat{\sigma}_0^2\mathbf{P}^{-1}$, with the estimated variance factor $\hat{\sigma}_0^2$ by error propagation

$$\begin{aligned}\hat{\mathbf{C}}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} &= (\mathbf{N}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P})\hat{\sigma}_0^2\mathbf{P}^{-1}(\mathbf{N}^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P})^{\mathrm{T}} \quad (9)\\ &= \hat{\sigma}_0^2(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}.\end{aligned}$$

Solving the normal equation with Eq. (6) it is required that the matrix $\mathbf{N}$ is of full rank. This is usually not the case. For example in ITRS realization the matrix $\mathbf{N}$ has a rank deficiency with respect to datum parameters, namely with respect to the orientation of the frame. In order to achieve a regular normal equation matrix, pseudo-observations are added to the normal equation system

$$\hat{\mathbf{x}} = (\mathbf{N}+\mathbf{D})^{-\mathbf{1}}(\mathbf{y}+\mathbf{d}) \quad (10)$$

wherein, $\mathbf{D}$ is the normal equation matrix of pseudo-observations and $\mathbf{d}$ is the right hand side of the normal equation of pseudo-observations (according to Eqs.(4) to (6)). In order to remove the rank deficiency with respect to the orientation of the frame, no-net-rotation conditions in form of pseudo-observations are added (*Angermann et al.*, 2004). In the following, it is assumed, that the initial normal equation system is free of pseudo-observations and all constraints are added at the normal equation level.

3 Combination strategies

Different space geodetic techniques are sensitive to common geodetic parameters (e.g. station coordinates and EOP). Performing technique-individual least squares adjustments, independent solutions of the same parameters are obtained as it is graphically shown in Fig. (2).

The combination can be performed on the different levels of least squares adjustment as it is shown in Fig. (3). If the processing could consistently be done for the techniques (including also the data editing) all three approaches would lead to the same results. But this is usually, and in particular for ITRF computation,

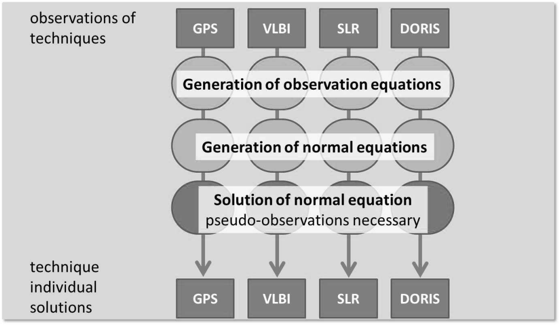

Fig. 2 Individual estimation of geodetic parameters from the observations of different techniques.

not the case. Thus, the three combination approaches provide solutions which differ slightly, illustrated by three individual solutions in Fig. (3). In the following paragraphs, the combination procedures are discussed in more detail.

3.1 Combination at observation level

According to Eqs. (2) and (3) the observation equation system for each of the m techniques k reads

$$\mathbf{A}_k \mathbf{x}_k = \mathbf{b}_k + \mathbf{v}_k \tag{11}$$

$$\mathbf{C}_{\mathbf{b}_k\mathbf{b}_k} = \sigma^2 \mathbf{P}_k^{-1} \tag{12}$$

with k=1,..,m. Preassumptions for a combination are (1) that the equations are related to the same parameters and (2) that the same a priori reduction models are used for modelling the same phenomena (e.g. solid Earth tides, ocean loading, troposphere refraction, ...). Then the individual observation equations can be composed to one system

$$\begin{bmatrix} \mathbf{A}_1 \\ \dots \\ \mathbf{A}_m \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{b}_1 \\ \dots \\ \mathbf{b}_m \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1 \\ \dots \\ \mathbf{v}_m \end{bmatrix} \tag{13}$$

$$\mathbf{C}_{\mathbf{bb}} = \sigma^2 \begin{bmatrix} \mathbf{P}_1^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_m^{-1} \end{bmatrix}. \tag{14}$$

Pre-processing procedures, which includes also editing of the observation data, are performed. Afterwards the least squares adjustment is done according to Eqs. (5) to (9) considering Eq. (10) if necessary.

3.2 Combination at normal equation level

The basic observation equations which are written for each of the techniques are identical to those of the combination on observation level (see Eq. (11)), considering the requirement of using the same standards for parametrization and a priori models. The data editing is done separately for each technique. Before the combination is done, the observation equations are transformed to normal equations applying the condition that the squared sum of residuals is minimized.

The normal equation for the technique k reads

$$\mathbf{N}_k \hat{\mathbf{x}}_k = \mathbf{y}_k. \tag{15}$$

The normal equations of all techniques are combined by adding those elements of the normal equations, that are related to the same parameters. Thus, if the normal equations do not have the same size $u \times u$, the individual equations must be expanded to the full amount of all parameters which shall be solved. This is done by adding zero lines and columns to the normal equation matrix $\mathbf{N}_k$ and zero elements to the right hand side of normal equation $\mathbf{y}_k$. Then all the normal equation systems are sorted by the same order of parameters. The combined normal equation system is written as (*Gerstl et al.*, 2001)

$$\mathbf{N} = \frac{1}{\sigma_1^2}\mathbf{N}_1 + \dots + \frac{1}{\sigma_m^2}\mathbf{N}_m, \quad \mathbf{N} \in u \times u \tag{16}$$

$$\mathbf{y} = \frac{1}{\sigma_1^2}\mathbf{y}_1 + \dots + \frac{1}{\sigma_m^2}\mathbf{y}_m, \quad \mathbf{y} \in u \times 1 \tag{17}$$

$$\mathbf{b}^T\mathbf{P}\mathbf{b} = \frac{1}{\sigma_1^2}\mathbf{b}_1^T\mathbf{P}\mathbf{b}_1 + \dots + \frac{1}{\sigma_m^2}\mathbf{b}_m^T\mathbf{P}\mathbf{b}_m, \tag{18}$$

considering the estimated variance factors σ_k^2. The combined system $\mathbf{N}\hat{\mathbf{x}} = \mathbf{y}$ is solved by using the Eq. (5) to (9) considering Eq. (10) if necessary.

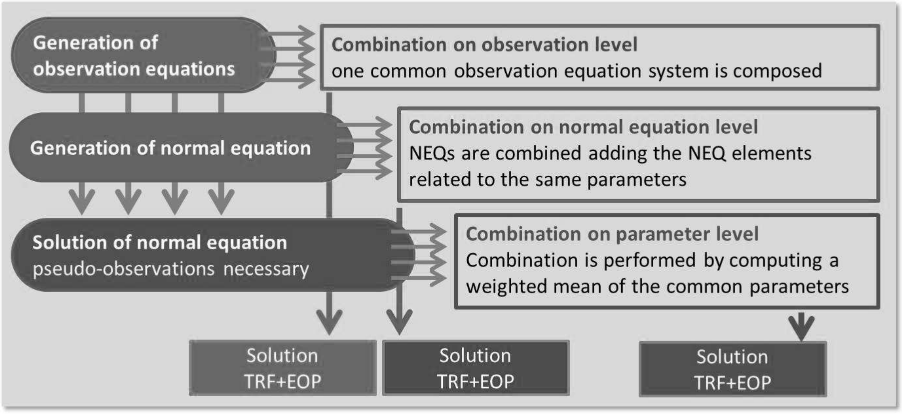

Fig. 3 Combination methods based on different levels of least squares adjustment process shown for the example of the consistent computation of the Terrestrial Reference Frame (TRF) and the EOP.

3.3 Combination at parameter level

Combining techniques at the parameter level individual technique solutions are performed initially, like it is illustrated in Fig. (2). The adjusted parameters are then combined in a second least squares adjustment process by considering the full variance-covariance matrices (see Fig. (3)). Consequently, the observation equation reads different from the combination at observation and normal equation level

$$\mathbf{I}\mathbf{x}_k = \hat{\mathbf{x}} + \bar{\mathbf{v}}_k \tag{19}$$

$$\mathbf{C}_{\hat{\mathbf{x}}_k\hat{\mathbf{x}}_k} = \sigma^2(\mathbf{N}_k + \mathbf{D}_k)^{-1} = \sigma^2\mathbf{P}_k^{-1}, \tag{20}$$

wherein $\mathbf{D}_k$ is the normal equation matrix of pseudo-observations (see also Eq. (10)). That means, in case of a rank deficiency of the matrix $\mathbf{N}_k$ pseudo-observations are added in order to generate the individual technique solutions. The combined system is obtained by composing the observation equations to one equation system

$$\mathbf{I}\begin{bmatrix}\mathbf{x}_1\\ \cdots\\ \mathbf{x}_m\end{bmatrix} = \begin{bmatrix}\hat{\mathbf{x}}_1\\ \cdots\\ \hat{\mathbf{x}}_m\end{bmatrix} + \begin{bmatrix}\bar{\mathbf{v}}_1\\ \cdots\\ \bar{\mathbf{v}}_m\end{bmatrix} \tag{21}$$

$$\mathbf{C}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \sigma^2\begin{bmatrix}\mathbf{P}_1^{-1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \ddots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{P}_m^{-1}\end{bmatrix}. \tag{22}$$

The equation system is solved by a least squares adjustment (see Eq. (5) to (9) and Eq. (10) if necessary).

In ITRF computation, the input solutions are solved by applying pseudo-observations in form of minimum conditions in order to reduce the rank deficiency of the individual techniques with respect to parameters of the geodetic datum. Thus, in case of GPS, SLR and DORIS no-net-rotation conditions (see e.g. *Seitz* (2009)) are used. In case of VLBI, which is not related to the orbit dynamics of a satellite and thus not sensitive to the Earth's centre of mass, additionally no-net-translation conditions are needed to realize the origin of the frame.

Two different aspects, concerning the application of pseudo-observations in case of combination at parameter level, have to be discussed. (1) Applying pseudo-observations it must be considered, that the combined solution depends on the variances and co-variances obtained from the individual technique solutions (see Eq. (20)). Thus, reliable pseudo-observations must be applied in order to provide variances that reflect only the uncertainty of the deterministic part of the Gauß-Markov model and not an uncertainty with respect to the geodetic datum. That means, loose constraint solutions (see e.g. *Angermann et al.* (2004)) cannot be used for a combination at parameter level, but they have to be preliminary resolved applying minimum datum conditions with suitable standard deviations (in ITRF computation a few millimetres or less). (2) To be able to realize the datum of the combined solution independently from the input solutions, parameters of a similarity transformation have to be set up for each individual input solution in order to restore the rank deficiency of the combined equation system with respect to geodetic datum parameters (see Fig. (4)). The singularity can then be removed by arbitrary conditions.

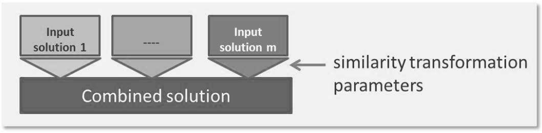

Fig. 4 Combination at parameter level: parameters of similarity transformation are set up per input solution.

4 Comparison of the different combination strategies

The combination at observation level is the most rigorous combination model. All observation types are processed together starting with the generation of observation equations using the same parameterizations and reduction models. Preprocessing steps as the detection of outliers and the editing of the observation data can be done using the full amount of available observations. In an optimal case, the data analysis is done by one single software, which can handle all the observation types together. In case of ITRF computation, such a software, which is able to process VLBI, SLR, GNSS and DORIS data on highest standard, is not available up to now.

The combination at normal equation level is - under certain conditions - a good approximation of the combination at observation level. If the observation equations are generated using the same parametrization and reduction models and no constraints are added before the normal equations are combined, the combination at normal equation level is comparable to the combination at observation level. The only difference is, that the detection of outliers and the data editing is done technique-wise. However, the effect on the solution is, particularly in the case of ITRF computation, assumed to be unverifiable.

A further aspect has to be discussed for the example of ITRS realization. In order to be able to handle the large normal equation matrices, parameters which are not of very direct interest are reduced before the combination. Parameters which cannot be estimated very stable, as e.g. clock parameters, are slightly constrained before reduction. These constraints cannot be removed anymore in the combination, even if the parameter would indirectly benefit from the combination to such an extent, that the constraint would be unnecessary, e.g. if correlations to other parameters are reduced. Thus, in order to avoid deformations of the solution, the a priori constraints on the reduced parameters must be introduced very carefully. A more rigorous way is not to constrain and reduce parameters before combination.

The combination at parameter level shows clear differences with respect to the combination at observation and normal equation level. Individual technique solutions are performed adding the necessary datum conditions in form of pseudo-observations. Only a minimum number of pseudo-observations is allowed to avoid any over-constraining and hence the deformation of the solution. Furthermore, it must be considered, that the combined solution depends on the variances and co-variances obtained from the technique solutions, so that reliable pseudo-observations must be applied while generating the input solutions.

In order to ensure, that the datum of the combined solution can be realized independently, it is necessary to set up parameters of a similarity transformation for each of the input solutions. The estimated similarity transformation parameters and consequently the combined solution depend on the set of stations used for the parameter set up. Due to the inhomogeneous global distribution of stations the transformation parameters are correlated and can absorb also non-datum effects, which becomes particularly critical in case of a poor network geometry.

The combination at solution level is not a straightforward approach due to the multiple application of pseudo-observations and the subsequent removement of datum information by the set up of transformation parameters. Added to that is the fact, that these steps are not independent from operator decisions (Which stations are used for the set up of transformation parameters?).

Table (1) summarizes the most important characteristics of the combination methods.

Table 1 Comparison of the different combination methods.

	observation level	normal equation level	parameter level
common pre-analysis of observations	yes	no	no
pseudo-observations needed for combination	no	no	yes
additional transformation parameters needed	no	no	yes
rigorous method	yes	widely	no

5 Combination in the geodetic practice: the realization of ITRS

The ITRS realization computed at DGFI is one example for the combination at normal equation level (*Seitz et al.*, 2012). Fig. (5) shows the flowchart of the combination procedure. The input normal equations are extracted from files in SINEX (Solution INdependent EXchange) format. The format description is available under http://www.iers.org (2011-05-02). SINEX allows for the storage of normal equations as well as of solutions. For the latest ITRS realization (DTRF2008) GNSS, SLR and DORIS input data are provided as solutions and the normal equation has to be reconstructed using the information about the applied constraints given in the SINEX files. The formulas used for the reconstruction of the normal equation from SINEX can be inter alia found in *Seitz et al.* (2012). The VLBI contribution was provided in form of normal equations and could be used directly from the SINEX files.

A combination of normal equations is done in the process at two different stages (1) the combination of the normal equation time series of one technique to one normal equation per technique and (2) the combination of the technique normal equations. The ITRF2005 was the first ITRF based on time series of weekly or session-wise input data (*Altamimi et al.* (2007), *Angermann et al.* (2007)). The advantage of input time series compared to multi-year input solutions is, that the analysis of the parameter time series (station positions, datum parameters and EOP) can be done consistently for all the techniques within the combination process.

The parameters, which are relevant in the ITRS realization DTRF2008 are specified in Fig. (6). The figure gives an overview, about the contribution of the techniques to the determination of the several parameters. While station coordinates and the coordinates of the terrestrial pole are provided by all the techniques, UT1-UTC can only be derived from VLBI in an absolute sense. GNSS and SLR contribute to the UT1-UTC estimates by the first derivative in time. The actually provided parameter is LOD, which is related to the UT1-UTC by LOD=$-d/dt$(UT1-UTC). Concerning the datum parameters, the origin is derived from SLR observations only, while for the realization of the scale SLR and VLBI observations are considered. Since also GPS and DORIS observations provide information on origin and scale (which should not be used for ITRF datum realization as they are affected by systematics) the normal equations are extended by scale as well as geocentre parameters, which correspond to the degree 0 and degree 1 gravity field coefficients, respectively.

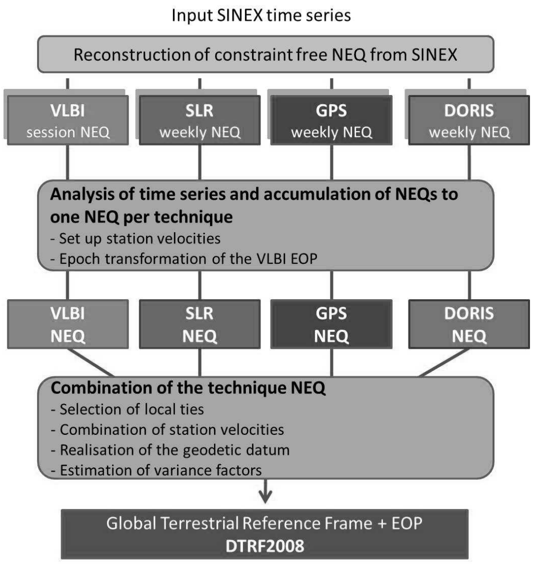

Fig. 5 Flowchart of ITRS realization at DGFI.

Examples for the combination at parameter level are e.g. the computation of the ITRF2008 (*Altamimi et al.*, 2011) and that of EOP series IERS C04 (ftp://hpiers.obspm.fr/iers/eop/eopc04/C04.guide.pdf, 2011-0-02). As the combination at observation level

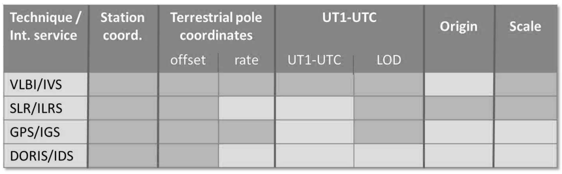

Fig. 6 Parameters of the ITRS realization DTRF2008. Dark colour means that the technique contributes to the parameter.

is the most intricate approach, ITRS related products are not yet computed by combination at observation level. Therefore, the IERS installed the Working Group on the Combination at the Observation Level (COL), which deals with the development of methods, strategies and software necessary for the combination of the different techniques at the observation level (http://www.iers.org, 2011-05-02).

6 Summary and conclusions

The combination of different observation types can be performed at the three levels of least squares adjustment within a Gauß-Markov model: the observation, the normal equation and the parameter level. For applications in ITRF computation the approaches show differences with respect to the data processing and/or to the mathematical concepts and do not lead to identical results. The combination at observation level is the most rigorous combination method as the whole processing line starting from the preprocessing steps is run using the full amount of available observation data.

The combination at normal equation level is a good approximation of the combination at observation level, if the observation equations are written by using the same standards for models and parametrization and if no constraints are added to the observation or normal equations before the combination. Thus, both, the combination on observation and on normal equation level can be recommended for the computation of geodetic products.

The combination at solution level differs clearly from the two other approaches. Technique-wise solutions are performed and the resulting parameters are combined afterwards by a second least squares adjustment process. The multiple addition of pseudo-observations and the subsequent reconstruction of rank deficiencies by set ups of transformation parameters is problematic as the latter step induce a stronger dependency of the final solution on operator impacts (impact of selected stations on the results) and numerical aspects (correlations between the parameters of similarity transformation).

While combination at normal equation level is a standard, combination at the observation level is not performed for ITRS realization today since an analysis software, which allows the processing of all the different data types in a contemporary way, is not available yet. In order to overcome this problem, different groups working on the development of multi-technique softwares. The IERS initiated the Working Group on the Combination at the Observation Level which deals with the development of combination strategies for the IERS products of the future by the use of the expertise of the different groups.

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