Report of the Ad-hoc Group
on an International Height Reference System (IHRS)

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Motivation

To determine and investigate the global changes of the Earth, geodetic reference systems with long-term stability and homogeneous consistency worldwide are required. Thus, the sea level rise of a few millimeters per year can only be detected when a stable spatial reference over a long period with globally high accuracy is realized. For this, an integrated global geodetic reference frame with millimeter accuracy must be implemented. To reach this goal, the inconsistencies existing between analysis strategies, models, and products related to the Earth's geometry and gravity field must be solved. Consequently, this is at present a main objective of the International Association of Geodesy (IAG) and especially of the Global Geodetic Observing System (GGOS), see e.g., Plag and Pearlman (2009), Kutterer et al. (2012).

Physical heights are potential differences of the Earth's gravity field and a global vertical reference frame provides the reference for Earth's gravity field parameters. The geoid potential parameter \(W_0\) defines the zero-level of the global height reference system and determines the relationship between the physical heights and the body of the Earth. The parameter \(W_0\) must be consistent between systems to ensure that the relevant relations are reproducible.

The objective of this work is to define the necessary standards for a global physical height reference system based on existing developments and past project results, and to draft and implement relevant products from this information. Conventions and guidelines resulting from this work are directly related to the activities of several IAG sub-entities: GGOS Bureau for Products and Standards (Angermann et al. 2015), GGOS Theme 1 Unified Height System (Sideris 2013) and Theme 3 Understanding and Forecasting Sea-Level Rise and Variability (Schöne et al., 2013), the Inter-Commission Project 1.2 Vertical Reference Frames (Ihde 2007, Ihde et al. 2007), the working group Vertical Datum Standardization (Sánchez 2012); as well as the joint activities of IAG Commission 2 Gravity Field and the Consultative Committee for Mass and Related Quantities to agree about a Strategy for Metrology in Absolute Gravimetry (Marti et al., 2015). This work should provide a basis to homogenize the products of the geometry, the gravity field, and the time reference.
I. General Concepts

a) Earth Gravity Field and Physical Height

There is a basic relationship between Earth gravity and geopotential. The Earth gravity field can be represented by means of: The geopotential scalar field $W(X)$ or the outer Earth gravity vector field $g(X)$ at a spatial position $X$. Both fields are related by the theorem

$$
\mathbf{g} = \nabla W = -g \begin{pmatrix} \cos \Phi & \cos \Lambda \\ \cos \Phi & \sin \Lambda \\ \sin \Phi \end{pmatrix},
$$

(1)

with the natural coordinates astronomical latitude $\Phi$, and astronomical longitude $\Lambda$. For the gravity there is the relation:

$$
P_p = g(X) = \left| \nabla W \right| = \left( -\frac{\partial W}{\partial H} \right)_p.
$$

(2)

In a very general notation, equations (1) and (2) can be expressed as:

$$
P(X, W, g) = P(X, W, -\partial W/\partial H) \quad \text{or} \quad W(X) = W_p \text{ collocated with } g(X) = g_p = -\partial W_p/\partial H.
$$

(3a)

(3b)

The geopotential scalar field $W(X)$ and the outer Earth gravity vector field $g(X)$ are completely consistent with each other, and are functions of time in Euclidean space. Because of this, physical heights $H$ may be expressed as potential differences of the Earth gravity field.

Subsequently, the inverse relationship equation (3b) may be used to estimate the disturbing potential $T_p$, which is defined as:

$$
T_p = W_p - U_p,
$$

(4)

at any point $P(X)$ on the Earth’s surface, by solving the geodetic boundary value problem (GBVP) and integrating gravity over the whole Earth’s surface $\sigma$,

$$
T_p = \frac{R}{4\pi} \int_{\sigma} (\Delta g + G_1 \cdots) S(y) d\sigma
$$

(5)

or by applying a global gravity model (GGM) to obtain the real gravity potential $W_p$ at the point $P(X)$.

The aforementioned equivalent field configurations of the Earth gravity field require the consistent treatment of gravity, potential, and physical heights. For this reason, the interactions of the definition and realization of the International Height Reference System (IHRS) with the definition and realization of an International Gravity
Reference System (IGRS) as well as the International Terrestrial Reference System (ITRS) must be considered.

The gravity acceleration is the only measurable characteristic within the vector field, and in the vertical direction there are almost no observables or products. Potential values and potential differences cannot be measured directly and therefore they must be estimated within the gravity field modeling, where the physical height is a main component. The accuracy of the at present widely used gravity reference network (IGSN71: International Gravity Standardization Net 1971, Morelli et al. 1974) is one to two orders lower than the current accuracy of absolute gravity measurements and generated products (Jiang et al. 2012), yet IGSN71 is still officially recognized as a valid tool despite this shortcoming. Overall, the IAG has paid little attention to gravity (absolute gravity) and gravity variations. This deficiency creates an opportunity for the IAG to take a leadership role in gravity studies by developing a new gravity standard.

b) Physical Height Reference Systems

In general, a reference system defines constants, conventions, models, and parameters required for the mathematical representation of geometric and physical quantities. A reference frame realizes a reference system in two ways: physically, by a solid materialization of points; and mathematically, by the determination of coordinates referring to that reference system; i.e. the coordinates of the physical points are computed from the measurements, but following the definition of the reference system. The datum fixes univocally the relation between a reference frame and a reference system. In the case of a vertical or height reference system, the primary components are a reference surface (i.e. the zero-height level) and a vertical coordinate (i.e. a physical height or more general, level differences). Its realization is given by a vertical network, i.e. a set of points, whose heights are of the same type specified in the definition and refer to the vertical datum that establishes the level of the reference surface.

Physical Height Reference Systems (HRS) are related to the Earth's gravity field on or outside the solid Earth body. A global HRS is a geopotential reference system co-rotating with the Earth in its diurnal motion in space. In such a system, positions of points attached to the solid surface of the Earth are given by geopotential values and geocentric Cartesian coordinates $X$ in a defined Terrestrial Reference System (TRS). A height or vertical reference frame (HRF) is a set of physical points with precisely determined geopotential values $W_p$ or level differences $C_p$ with respect to a geopotential reference value $W_0$. Such a HRF is said to be a realization of the HRS. The disturbing potential (Eq. 4) undergoes only small variations in time, due to geophysical effects (mass transports and tectonic or tidal deformations).

The height components are differences $\Delta W_p$ between the potential $W_p$ of the Earth gravity field level surface passing through the considered point $P$ and the potential of the HRS zero-level $W_0$. The potential difference $-\Delta W_p$ is also designated as geopotential number $C_p$:

$$C_p = -\Delta W_p = W_0 - W_p.$$  \hspace{1cm} (6)
The zero-level \( W_0 \) to which the geopotential numbers \( C_p \) are related is called the vertical datum of the HRS.

### II. Standards, Conventions, Guidelines

#### a) Numerical standards

In 1979, the International Union of Geodesy and Geophysics (IUGG), International Association of Geodesy (IAG), and International Astronomical Union (IAU) have agreed upon the Geodetic Reference System 1980 (GRS80, Moritz 1980 and 2000) to define major parameters for a geodetic reference system related to a geocentric equipotential ellipsoid. At the IUGG General Assembly 1991 in Vienna, new values for the geocentric gravitational constant \( (GM) \), and the semi-major axis \( (a) \) of the level ellipsoid were recommended. The two other defining parameters (dynamical form factor \( J_2 \) and mean angular velocity of the Earth’s rotation \( \omega \)) of the equipotential ellipsoid were not changed. The value of the geocentric gravitational constant \( (GM) \) has not changed since 1991. These values (or others very close to them) have been used in the computation of global gravity models since 1991.

Table 1 of this paper contains defining parameters for different level ellipsoids. The gravitational constants \( GM \) of the GRS80 and IERS Conventions 2010 differ by about 0.9 m\(^3\)s\(^{-2}\); the semi-major axis \( a \) of both standards differs by 0.4 m. and the geopotential reference values \( (U_0 \text{ and } W_0) \) differ by 4.85 m\(^2\)s\(^{-2}\). Also noteworthy is that the IERS Conventions 2010 recommend different level ellipsoid parameters for different applications.

In the IERS Conventions 2010, Table 1.1 lists parameters that represent the current best estimations, and the best estimates for level ellipsoid parameters have not changed since 2003. It is not immediately evident how the 2010 estimates were determined. In addition, Table 1.2 of the same Conventions contains the parameters of the GRS80 ellipsoid and it is designated as convention for the conversion of Cartesian coordinates into ellipsoidal ones. This is new against the IERS Conventions 2003. These inconsistencies in the IAG and the IERS conventions shall be removed in view of the development of integrated geodetic products and applications.

Since the most accepted definition of the geoid is understood to be the equipotential surface that coincides (in the sense of the least-squares) with the worldwide mean ocean surface, the reference level for a global height system can be defined with the potential at the mean sea level, \( W_0 \). The value of \( W_0 \) depends from the Earth's gravity field, on the definition of mean sea level, and conventions about processing procedures including used models. This is independent from measurements. The definitions, conventions, and conditions shall documented for further comparisons and monitoring of mean sea level. It is to be expected that the mean sea level will change by mm/a. On the other hand, outgoing, that \( W_0 \) can be introduced as a defining parameter of the mean Earth ellipsoid, the semi-major axis \( (a) \) of the level ellipsoid would be a derived parameter and it would change if \( W_0 \) changes. To
provide a reference ellipsoid that remains unchanged with time, it would be necessary to decouple $W_0$ from the sea surface variations.

The IERS Conventions (2003 and 2010) include a $W_0$ value that was derived in 1998 (Burša et al. 1998, Groten 1999, Groten 2004). This value presents discrepancies of more than $-2 \text{ m}^2\text{s}^{-2}$ against recent computations (Sánchez et al. 2014). It must be decided whether a new value $W_0$ should be introduced as a more accurate estimate. As mentioned, for each new $W_0$ estimation, a new value for the semi-major axis ($a$) of the level ellipsoid would have to be derived. However, by a recalculation of the parameter $W_0$, the discrepancy existing between the value included in the IERS Conventions 2010 (see Table 1) and recent calculations will be eliminated and the estimation procedure can be documented to ensure the reproducibility of the new adopted $W_0$ value.

From this perspective, $W_0$ is the only measurable defining parameter of the level ellipsoid that depends on changes in the Earth system. $J_2$ also depends on changes in the Earth system, but these changes are not measurable yet. As a fundamental parameter, $W_0$ shall not be changed from time to time; i.e. a change of $W_0$ per year in $\text{m}^2\text{s}^{-2}\text{a}^{-1}$ cannot be applied practically. For a global height reference system, any value $W_0$ within a range of a few decimeters can be defined as conventional without affecting the task of defining and realizing a global height reference system. Like any reference system, $W_0$ should be based on adopted conventions that guarantee its uniqueness, reliability, and reproducibility; otherwise there would be as many $W_0$ reference values (i.e., global zero-height surfaces) as computations.

In any case, the complete set of ellipsoidal parameters must be computed for the best estimate of a new level ellipsoid as done for the GRS80. So far, this has not been the case.

<table>
<thead>
<tr>
<th>Ellipsoid</th>
<th>Semi-major axis $a$ in [m]</th>
<th>Dynamical form factor $J_2$</th>
<th>Geocentric gravitational constant $GM$ in $10^8$ [m$^3$s$^{-2}$]</th>
<th>Normal potential at ellipsoid $U_0$, geoidal potential $W_0$ in [m$^2$s$^{-2}$]</th>
<th>Normal gravity at equator $\gamma_e$ in [ms$^{-2}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRS 80</td>
<td>6.378 137</td>
<td>$1.082 \times 10^{-2a}$</td>
<td>3 986 005</td>
<td>$U_0 = 62 636 860.850$</td>
<td>9.780 326 7715</td>
</tr>
<tr>
<td>IERS 2010</td>
<td>6.378 136.6** ± 0.1</td>
<td>$1.082 \times 10^{-2\times 10}$</td>
<td>3 986 004.418 $\pm 0.008$</td>
<td>$W_0 = 62 636 856.0 \pm 0.5$</td>
<td></td>
</tr>
<tr>
<td>Conventions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Mean angular velocity of the Earth's rotation $\omega$ remains the same in any case ($7 292 115 \times 10^{-11}$ rad s$^{-1}$)

*Value given in tide-free system.
**Value given in zero-tide system.

In addition to the existing IERS numerical standards, other parameters shall be calculated and included in the IERS Conventions, for instance the normal gravity at equator ($\gamma_e$) and at pole ($\gamma_p$).

Independently of the decision to replace the GRS80 by a new conventional set of level ellipsoid parameters the current best-estimated value for $W_0$ shall be defined (and fixed) as the potential value of the geoid. To ensure the reproducibility and interpretability of these changes, the procedure applied for the determination of $W_0$ must be well documented including conventions and guidelines.
It is desirable that the recent best-estimates for the parameters of the level ellipsoid are applied for all products of measurements and modeling of the Earth’s gravity field and geometry, including the global height reference system. In this case, a new GRS shall be computed. If the GRS80 remains as the conventional level ellipsoid, all necessary parameters must be then derived in accordance with the GRS80 values. For combination products such as GNSS/leveling, the regulations for the reductions should be specified based on the different numerical parameters and underlying geometrical and gravity field relations.

Within the next four years, an IAG inter-commission working group should be established to investigate the necessity and usefulness of replacing GRS80 with a new GRS. If the computation of a new GRS is decided, this working group shall prepare and propose a full set of parameters to be presented and adopted at the IUGG 2019 General Assembly.

b) Permanent Tide

A HRS is comprised of geometric and gravity potential parameters, including their variations with time, and in particular those generated by Earth tides.

The foundations of the IAG Resolution Number 16, adopted in 1983 at the General Assembly in Hamburg (Tscherning 1984), have not changed. The zero-tide system is the most adequate tide system applicable to both gravity acceleration and gravity potential of the rotating and deforming Earth. The pendent for the geometry is the mean/zero crust concept, where the mean sea surface corresponds to a crust deformed by mean/zero tides.

Table 2. Tide systems used in the determination of physical and geometrical coordinates

<table>
<thead>
<tr>
<th>gravity</th>
<th>geoid</th>
<th>levelling height</th>
<th>altimetry</th>
<th>mean sea level</th>
<th>position</th>
</tr>
</thead>
<tbody>
<tr>
<td>g ↔ Δg</td>
<td>W ↔ N</td>
<td>ΔH</td>
<td>h</td>
<td>Msl</td>
<td>X ↔ h</td>
</tr>
</tbody>
</table>

Mean tidal system
Mean/zero crust
(Stokes is not valid if masses outside the Earth surface)

<table>
<thead>
<tr>
<th>Δg_m</th>
<th>N_m</th>
<th>ΔH_m</th>
<th>Relation to N_m for oceanographic studies h_msl</th>
</tr>
</thead>
</table>

Zero tidal system
Zero/mean crust
(Recommended by IAG Res. No. 16, 1983)

<table>
<thead>
<tr>
<th>Δg_z</th>
<th>Stokes N_z</th>
<th>ΔH_z</th>
<th>Cp</th>
</tr>
</thead>
</table>

Non-tidal system
Non-tidal crust
(far away from the real earth shape – there is no reason for the non-tidal concept)

<table>
<thead>
<tr>
<th>Δg_z</th>
<th>Stokes N_n</th>
<th>X_n</th>
<th>ITRF</th>
</tr>
</thead>
</table>
There is no justification for the application of a tide-free concept for both the geometry and the gravity field, since the tide-free crust and gravity are far away from the real Earth shape and are unobservable (Ekman 1989, 1996; Mäkinen and Ihde, 2009). For the mean-tide geopotential the condition of the Laplace equation is not fulfilled, and even if the tide-free concept is kept for the terrestrial reference system parameters, the IAG Resolution No. 16 adopted in Hamburg in 1983 shall be used for gravity and geopotential. (see also table 2)

For practical applications, parameters and products of a HRS shall be related to the mean-tide system or mean crust. This means that a consistent transformation between the three tidal systems must be considered before combining gravity field and geometrical products.

III. Definition of an International Height Reference System (IHRS)

The International Height Reference System (IHRS) is a geopotential reference system co-rotating with the Earth in its diurnal motion in space. Coordinates of points attached to the solid surface of the Earth are given by (1) geopotential values $W(X)$ (and their changes with time $dW(X)/dt$) defined within the Earth's gravity field and, (2) geocentric Cartesian coordinates $X$ (and their changes with time $dX/dt$) referring to the ITRS. For practical purposes, potential values $W(X)$ and geocentric positions $X$ can be transformed in vertical coordinates given with respect to a reference surface.

Five conventions define the IHRS:
1. The vertical reference level is the normal potential (or geopotential at the geoid or the geoid potential parameter) $W_0$ as an equipotential surface of the Earth gravity field. $U_0=W_0$ is a defining parameter of the conventional geocentric level ellipsoid.
   The relationship between $W_0$ and the Earth body must be defined and reproducible.
2. Parameters, observations, and data shall be related to the mean tidal system/mean crust.
3. The unit of length is the meter (SI). The unit of time is the second (SI). This scale is consistent with the TCG time coordinate for a geocentric local frame, in agreement with IAU and IUGG (1991) resolutions, and is obtained by appropriate relativistic modeling.
4. The vertical coordinates are the differences $-\Delta W_p$ between the potential $W_p$ of the Earth gravity field at the considered points $P$, and the geoidal potential of the level ellipsoid $W_0$. The potential difference $-\Delta W_p$ is also designated as geopotential number $c_p$:
   $$-\Delta W_p = c_p = W_0 - W_p.$$
5. The spatial reference of the position $P$ for the potential $W_p = W(X)$ is related as coordinates $X$ of the International Terrestrial Reference System.

IV. Conventions for the Realization of an International Height Reference System (IHRS)
The IHRS shall correlate the Earth gravity field (gravity, potential) with the geometry of the Earth and the timescale. The IHRS is to be realized by combining a global station network, a Global Gravity Model (GGM), and values for a set of parameters as an International Height Reference Frame (IHRF). The IHRF must be in accordance with the conventions underlying the definition of an International Height Reference System (IHRS), especially for conventions outlining how the elements can be derived. It is important to distinguish between the definition of IHRS, physical heights derived in the IHRF (important for applications and users), and the unification of existing physical height systems aligned to a defined and realized IHRS (Ihde and Sánchez 2005).

Proposal for the elements of an IHRF:

(1) The reference geopotential value $W_0$ is achieved through best estimates. The procedure of the $W_0$ determination must be documented in conventions and guidelines, to ensure the reproducibility and interpretability of changes.

(2) A central element of the IHRF is a Global Gravity Model (GGM). It is proposed to dedicate one satellite-only GGM for homogenous long wavelength approximation of the Earth gravity potential, as a matter of convention. One GGM combined with terrestrial data is recommended for applications in sparsely surveyed regions. For this, a maximum degree has to be defined as well, due to the fact that satellite-only models are usually regularized (constrained towards zero) in the high degrees, i.e. the higher degrees of the model do not contain the full signal. This is problematic if the models are combined with complementary (near-surface) data.

(3) The Earth gravity potential difference $\Delta WP$ in relation to a conventional $W_0$ shall be known through an existing highest-accuracy network of geodetic observation stations, where observations can be generated to derive the defining elements in a highest level of quality consistent with other reference systems/frames.

(4) The reference network conforming the IHRF shall follow the same hierarchy of the ITRF reference network; i.e. a global network with regional/national densifications. This network shall be collocates with:
- reference tide gauges (local vertical datum points);
- main nodal points of the levelling networks;
- border points connecting neighboring vertical datum zones;
- geometrical reference stations (ITRF and densifications);
- fundamental geodetic observatories (connection between $W_0$, TAI, and absolute gravity).

Conventions and guidelines for products are necessary for all the aforementioned elements.

Bibliography


