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Energy efficiency maximization for 5G multi-antenna receivers

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Abstract

In a digital communication system, the analog signal that the receiver receives with its radio frequency front end is converted into digital format by using the analog-to-digital converter (A/D converter, ADC). Quantisation takes place during the conversion from continuous amplitude signal to discrete amplitude signal, leading inevitably to losses in information which are dependent on the number of bits that is used to represent each sample. Although employing a higher bit resolution reduces the quantisation error, a higher power dissipation of the ADC is incurred at the same time. This trade-off is essential to the *energy efficiency* of the receiver, which is commonly measured by the number of information bits conveyed per consumed Joule of energy. We investigate, in this work, the adaptation of ADC resolutions of a multi-antenna receiver based on instantaneous channel knowledge, with the goal of maximising receiver energy efficiency. The formulated optimisation is a combinatorial problem, and we propose several algorithms which yield near-optimal solutions. Results from numerical simulations are presented and analysed, which provide guidelines to operation and deployment of the system. Copyright © 2014 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The next generation of wireless communication technologies, that is, 5G, is best known for its prediction and promise of supporting 1000 times data traffic as today beyond the year 2020. The energy consumption of wireless systems and networks, from an operation point of view, cannot and should not increase with the same pace. Therefore, improving the energy efficiency of wireless systems and networks has also become a key target of 5G [1]. Driven by economical interest and environmental responsibility, *green communication* has already drawn significant research and industrial attention over the past years, for example, [2][3]. For portable devices and wireless sensors, prolonging the lifetime of batteries is a major concern. For base stations in cellular networks, minimising the energy consumption while satisfying the quality-of-service requirements of the end users is desirable in reducing the operation and maintenance costs. In both scenarios, it is critical to consider and appropriately model the circuit power consumed by the hardware, which comes to the same order of magnitude as the radiated power for short-range communications [4]. For multi-antenna systems such as the multiple-input multiple-output (MIMO) system, while a larger channel capacity and higher spectrum

efficiency can be achieved, the increased number of radio frequency (RF) chains lead also to higher power dissipation, which could become dominant and unaffordable if a very large antenna array is deployed. This is to say, for the key enabling technologies of 5G such as femtocells [5] and massive MIMO [6], circuit power plays an important role in the improvement of system energy efficiency.

The focus of this work is on the impact of analog-to-digital converter (ADC) on the energy efficiency of a multi-antenna receiver. In modern receiver design, more and more receive functions are implemented by digital hardware due to its high speed and low cost. Therefore, the importance of ADCs, which enable the subsequent digital signal processing by sampling and converting the received analog signal into digital format, is evident and has long been realised [7]. However, the ADC is expected to be a limiting factor of the system as it consumes a significant amount of power when operating at high sampling rate and resolution. It was reported in [8] that the power dissipation of an eight-bit ADC with sampling rate 20 GS/s reaches as much as 10 Watt, which is obviously impractical for most mobile devices. In the recent paper [9], the authors pointed out that the ADC is one of the major challenges for millimetre wave communication when hardware constraints are concerned. From a communication point

of view, this motivates the employment of low ADC resolution, because high sampling frequency is required by many applications such as cognitive radio. In recent years, there have been various works investigating the performance limit of communications over quantised channels, for example, [10][11], where the focus was on the derivation of capacity loss when low ADC resolution is used, as well as design of the quantiser. As the ADC naturally links the system performance in terms of capacity or achievable rate to the power consumption, that is, based on the trade-off between quantisation error and power dissipation, we propose to treat the ADC resolution as an adaptable parameter and attempt to maximise the energy efficiency of the system, which is quantified as the number of information bits conveyed per Joule of energy consumption [12]. Significant gain over systems employing fixed, although low, ADC resolution has been observed with the simulation results provided in the paper.

At the receiver side, diversity can be achieved via the deployment of multiple antennas. With a single transmit antenna, independent signal paths and maximal ratio combining (MRC), the gain in receive signal-to-noise ratio (SNR) equals the number of receive antennas [13]. With multiple transmit antennas, spatial multiplexing techniques can be realised to provide linear growth of the channel capacity with the minimum of transmit and receive antenna numbers [14]. However, the hardware complexity and power consumption of the receiver scale as well with the number of RF chains, which include, besides the ADC, also the low noise amplifier, the mixer, the automatic gain control and some filters. As a result, energy efficiency analysis for massive MIMO systems has put its emphasis on circuit power [15][16]. In both papers, however, the dependency of ADC power on the bit resolution employed is not taken into account.

A common and long-developed technique to alleviate this burden of affording too many RF chains, known as *antenna selection* [17], is to choose signals received by a subset of antennas based on the channel conditions for further processing, thus reducing the necessary RF chains and the associated power consumption. When jointly optimising the ADC resolutions for all receive antennas, antenna selection can be performed in an implicit way by allowing the 0 bit resolution, which suggests that the signal from the corresponding antenna is not processed. Or, we can explicitly perform antenna selection and use positive bit resolutions for chosen active antennas. This motivates the proposal of suboptimal algorithms, which interactively chooses antennas and their bit resolutions. As processing power also accounts for part of the circuit power, we are in particular interested in low-complexity algorithms, which do not require large amount of online computations. Besides the design of ADC resolution adaptation algorithms, we also aim at gaining insight into the energy efficient operation of the system, for instance, which parameters influence the optimal bit resolution that should be employed, and how are the dependencies characterised. Due to the lack of analytical solutions, we mainly illustrate

these results via numerical simulations. Part of this work has been published in conference contributions [12] and [18], where the idea of ADC resolution adaptation was proposed and studied for SIMO systems. We summarise and refine the concept as well as algorithms here, and more importantly, extend the investigation to the MIMO case where the task of antenna selection becomes nontrivial.

The rest of the paper is organised as follows. In Section 2, we introduce the system model and, in particular, give expressions of a lower bound on channel capacity and the total power dissipation as functions of the ADC resolution vector. The problem of maximising energy efficiency is then formally given. The application of the particle swarm optimisation technique to tackle the combinatorial problem at hand is described in Section 3, where its effectiveness is shown in comparison to global optimal solutions obtained with exhaustive search. We propose several antenna selection-based ADC adaptation schemes in Section 4, followed by simulation results and analysis exhibited in Section 5. Section 6 concludes the paper with a summary of ideas, methods and results, which have been presented, and shortly discusses related open issues as well as possible future works.

Notations: we use boldfaced letters to represent vectors and matrices throughout the paper. The operator $(\cdot)^H$ stands for the Hermitian of a matrix, the symbol \mathbf{I}_M denotes the identity matrix of dimension $M \times M$, and $|\mathbf{A}|$ and $\text{diag}(\mathbf{A})$ denote the determinant of \mathbf{A} and the diagonal matrix with the same diagonal elements as \mathbf{A} , respectively.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the point-to-point communication between a transmitter with $N \geq 1$ antennas and a receiver with $M \geq 1$ antennas over a flat fading channel. Throughout the paper, it is assumed that the receiver has perfect channel state information, which is an idealised condition for evaluating the performance limit of the system. In practice, a training sequence for which both the transmitter and the receiver have a priori knowledge is sent at the beginning of the communication, based on which the receiver can perform channel estimation [19]. It should be noted that the obtained channel knowledge is imperfect due to the additive noise of the channel, as well as quantisation of the received training symbols. This problem is interesting and important in itself but is beyond the scope of this paper.

The communication scenario under investigation is depicted in Figure 1. The vector channel output $\mathbf{y} \in \mathbb{C}^M$ before quantisation is given as

$$\mathbf{y} = \sqrt{\alpha} \mathbf{H} \mathbf{x} + \boldsymbol{\eta},$$

where α is the average power gain of the wireless channel, $\mathbf{H} \in \mathbb{C}^{M \times N}$ is the matrix containing i.i.d. complex Gaussian distributed channel coefficients with zero mean and unit variance, $\mathbf{x} \in \mathbb{C}^N$ is the vector of transmitted symbols and $\boldsymbol{\eta} \in \mathbb{C}^M$ is the i.i.d. zero-mean complex circular

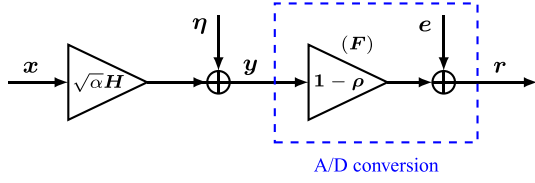


Figure 1. Communication over a quantised channel.

Gaussian noise vector. We assume that α and \mathbf{H} are perfectly known to the receiver, and their values stay constant over the period of time that the data transmission takes place. The covariance matrix of \mathbf{y} is computed as

$$\mathbf{R}_{yy} = \mathbb{E}[\mathbf{y}\mathbf{y}^H] = \mathbf{R}_{\eta\eta} + \alpha\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^H,$$

where \mathbf{R}_{xx} and $\mathbf{R}_{\eta\eta}$ are the covariance matrices of the transmitted signal and the additive noise, respectively. Assuming uniform power allocation at the transmitter and uncorrelated transmit symbols, we have that \mathbf{R}_{xx} is equal to a scaled identity matrix given by

$$\mathbf{R}_{xx} = \frac{P_{\text{tx}}}{N} \cdot \mathbf{1}_N,$$

where P_{tx} stands for the total transmit power. Letting σ^2 denote the noise power at each receive antenna, we further have $\mathbf{R}_{\eta\eta} = \sigma^2 \cdot \mathbf{1}_M$. Consequently, the average receive SNR at each antenna is given by

$$\gamma = \frac{\alpha P_{\text{tx}}}{\sigma^2},$$

which allows us to write \mathbf{R}_{yy} as

$$\mathbf{R}_{yy} = \sigma^2 \left(\mathbf{1}_M + \frac{\gamma}{N} \cdot \mathbf{H}\mathbf{H}^H \right).$$

The ADC at each receive antenna samples the received signal and uses a finite number of bits to represent each sample. We assume here that the M A/D converters all act as scalar quantisers, and let $\mathbf{b} \in \{0, 1, \dots, b_{\text{max}}\}^M$ be the vector of resolutions employed by each ADC, where b_{max} is the maximal number of bits that an ADC could use for a single sample. Given the instantaneous channel state information, the vector of bit resolutions \mathbf{b} can be adapted accordingly in order to optimise certain performance metrics, for example, the mutual information between channel input and the quantised output, the mean squared error after receive filtering and so on. The associated cost to account for is the power or energy consumption of the receiver circuitry. We introduce in the sequel the computation of a capacity lower bound of the MIMO quantised channel and the power consumption of the receiver, both as functions of the vector \mathbf{b} . Note that the capacity lower bound we derive is more general than [21] in that non-uniform ADC resolutions are allowed and has a simpler form than

the expression given in [12]. This means, the following derivation is a new contribution of the paper.

2.1. Capacity lower bound of the quantised channel

The quantisation operation is in general nonlinear, and the resulting quantisation error is correlated with the input data. The Bussgang theorem [20][21] suggests a decomposition of the output of the nonlinear quantiser into a desired signal part and an uncorrelated distortion, which provides us a convenient analytical approach to formulating the quantisation process. To this end, the quantised output vector \mathbf{r} can be written as

$$\mathbf{r} = \mathbf{F}\mathbf{y} + \mathbf{e},$$

where the noise vector \mathbf{e} is uncorrelated with \mathbf{y} , and the linear operator \mathbf{F} is taken as the MMSE estimator of \mathbf{r} from \mathbf{y} :

$$\mathbf{F} = \mathbb{E}[\mathbf{r}\mathbf{y}^H] \mathbb{E}[\mathbf{y}\mathbf{y}^H]^{-1} = \mathbf{R}_{ry} \mathbf{R}_{yy}^{-1}.$$

Consequently, we have

$$\begin{aligned} \mathbf{r} &= \mathbf{F}(\sqrt{\alpha}\mathbf{H}\mathbf{x} + \boldsymbol{\eta}) + \mathbf{e} = \sqrt{\alpha}\mathbf{F}\mathbf{H}\mathbf{x} + \mathbf{F}\boldsymbol{\eta} + \mathbf{e} \\ &= \mathbf{H}'\mathbf{x} + \boldsymbol{\eta}', \end{aligned}$$

where the effective channel \mathbf{H}' , the effective noise $\boldsymbol{\eta}'$ and its covariance matrix are given respectively by

$$\mathbf{H}' = \sqrt{\alpha}\mathbf{F}\mathbf{H},$$

$$\boldsymbol{\eta}' = \mathbf{F}\boldsymbol{\eta} + \mathbf{e},$$

$$\mathbf{R}_{\eta'\eta'} = \mathbf{F}\mathbf{R}_{\eta\eta}\mathbf{F}^H + \mathbf{R}_{ee}.$$

Note that the effective noise $\boldsymbol{\eta}'$ is not necessarily Gaussian. Therefore, if we define a new MIMO Gaussian channel with the input-output relation $\mathbf{r}_G = \mathbf{H}'\mathbf{x} + \boldsymbol{\eta}_G$ and assume that $\mathbb{E}[\boldsymbol{\eta}_G\boldsymbol{\eta}_G^H] = \mathbf{R}_{\eta'\eta'}$, the capacity of the new channel provides a lower bound on that of the quantised channel, for Gaussian distributed noise minimises the mutual information [22]. Based on this observation and assuming that the channel input \mathbf{x} is Gaussian distributed, we have

$$I(\mathbf{x}; \mathbf{r}) \geq \log_2 \left| \mathbf{1}_M + \mathbf{R}_{\eta'\eta'}^{-1} \mathbf{H}' \mathbf{R}_{xx} \mathbf{H}'^H \right|. \quad (1)$$

Given Gaussian distributed channel input \mathbf{x} , a deterministic channel matrix \mathbf{H} , and uncorrelated Gaussian noise $\boldsymbol{\eta}$, the channel output \mathbf{y} is also Gaussian. For such a quantisation source and a distortion-minimising scalar non-uniform quantiser, the distortion factor ρ , which is the inverse of the signal-to-quantisation-noise ratio, attains the values given in Table I [23] with respect to different b . For $b > 5$, the asymptotic approximation $\rho = \frac{\pi\sqrt{3}}{2} \cdot 2^{-2b}$ can be used [24].

Table I. Distortion factor ρ for different analog-to-digital converter resolutions b .

b	1	2	3	4
ρ	0.3634	0.1175	0.03454	0.009497
b	5	6	7	8
ρ	0.002499	0.0006642	0.0001660	0.00004151

Let ρ_i denote the distortion factor of the ADC associated with antenna i , and $\boldsymbol{\rho} \in \mathbb{R}^{M \times M}$ denote the diagonal matrix containing all M distortion factors. Analytical expressions for \mathbf{H}' and $\mathbf{R}_{\eta'\eta'}$ have been derived in [21][12] as

$$\mathbf{H}' = \sqrt{\alpha} \mathbf{R}_{ry} \mathbf{R}_{yy}^{-1} \mathbf{H} = \sqrt{\alpha} (\mathbf{1}_M - \boldsymbol{\rho}) \mathbf{H},$$

$$\mathbf{R}_{\eta'\eta'} = (\mathbf{1}_M - \boldsymbol{\rho}) \left(\sigma^2 (\mathbf{1}_M - \boldsymbol{\rho}) + \boldsymbol{\rho} \text{diag}(\mathbf{R}_{yy}) \right),$$

which enable the calculation of the capacity lower bound (1). With the denotation of \mathbf{h}_i , $i = 1, \dots, M$ as the M columns of \mathbf{H}^H , we have

$$\text{diag}(\mathbf{R}_{yy}) = \sigma^2 \cdot \text{diag} \left\{ 1 + \frac{\gamma}{N} \cdot \mathbf{h}_i^H \mathbf{h}_i \right\}_{i=1}^M$$

$$\triangleq \sigma^2 \cdot \text{diag} \{ 1 + \beta_i \}_{i=1}^M,$$

where $\beta_i = \frac{\gamma}{N} \cdot \|\mathbf{h}_i\|_2^2$, $i = 1, \dots, M$. The inverse of $\mathbf{R}_{\eta'\eta'}$ is then given by

$$\mathbf{R}_{\eta'\eta'}^{-1} = \frac{1}{\sigma^2} \text{diag} \left\{ \frac{1}{1 + \rho_i \beta_i} \right\}_{i=1}^M (\mathbf{1}_M - \boldsymbol{\rho})^{-1},$$

which leads to

$$I(\mathbf{x}; \mathbf{r}) \geq \log_2 \left| \mathbf{1}_M + \mathbf{R}_{\eta'\eta'}^{-1} \mathbf{H}' \mathbf{R}_{xx} \mathbf{H}'^H \right|$$

$$= \log_2 \left| \mathbf{1}_N + \frac{P_{\text{tx}}}{N} \cdot \mathbf{H}'^H \mathbf{R}_{\eta'\eta'}^{-1} \mathbf{H}' \right|$$

$$= \log_2 \left| \mathbf{1}_N + \frac{\gamma}{N} \cdot \mathbf{H}^H \text{diag} \left\{ \frac{1 - \rho_i}{1 + \rho_i \beta_i} \right\}_{i=1}^M \mathbf{H} \right|.$$

Defining the real-valued diagonal matrix

$$\mathbf{D} = \text{diag} \left\{ \frac{1 - \rho_i}{1 + \rho_i \beta_i} \right\}_{i=1}^M,$$

which depends both on the employed bit resolutions and the channel coefficients, we write the capacity lower bound C_L (in bit/s) in a very similar form with the well-known MIMO capacity formula as

$$C_L = B \log_2 \left| \mathbf{1}_N + \frac{\gamma}{N} \cdot \mathbf{H}^H \mathbf{D} \mathbf{H} \right|, \quad (2)$$

where B is the signal bandwidth. As it has been shown in [21] that this lower bound on channel capacity is quite tight

especially in the low SNR regime, we employ it in our performance measure of energy efficiency for the achievable rate of the system.

2.2. Power consumption model

Power dissipation of the ADC associated with the i th antenna can be calculated as [25]

$$P_{\text{ADC},i} = \begin{cases} c_0 \cdot \sigma^2 \cdot 2^{b_i}, & b_i > 0, \\ 0, & b_i = 0, \end{cases} \quad (3)$$

where c_0 is a constant determined by the specific design of the ADC. When positive bit resolution is employed, power dissipation of the ADC grows exponentially with b_i . Otherwise, the corresponding RF chain is considered to be turned off, and no power consumption is incurred. We model the total power consumption of the receiver by

$$P = c_0 \cdot \sigma^2 \sum_{i:b_i>0} 2^{b_i} + c_1, \quad (4)$$

where c_1 is a constant accounting for the power consumption of baseband components of the receiver. This means, we roughly model the power consumption of the RF chain as equal to the power consumption of the ADC and assume that the processing power at the baseband is independent of data rate. In practice, the specific design of the receiver, especially the complexity of decoding, determines whether the power consumption of A/D converters is the dominant part in the total power consumption [26]. For now, we settle for the generic and general model (4) to have a clearer structure of our problem. For the simulations, we have taken $B = 1$ MHz, $b_{\text{max}} = 8$, $c_0 = 1 \times 10^{-4} / \sigma^2$ Watt and $c_1 = 0.02$ Watt with reference to [27].

2.3. Maximisation of energy efficiency

Motivated by the demand to increase the lifetime of mobile terminals and other communication devices powered by batteries, as well as by the desire to conduct green communications, energy efficiency has become another important performance metric for communication systems over the past years. For different applications, one might want to minimise the energy needed to transmit/receive a certain amount of data or to maximise the operation time given fixed available energy while a constant data rate is provided. In this work, we focus on the unconstrained maximisation of the bit per Joule metric at the receiver with the ADC resolution vector \mathbf{b} as the optimisation variable, which is defined as

$$\max_{\mathbf{b} \in \{0, 1, \dots, b_{\text{max}}\}^M} \frac{C_L(\mathbf{b})}{P(\mathbf{b})} \triangleq \eta \quad (5)$$

where the expressions for $C_L(\mathbf{b})$ and $P(\mathbf{b})$ are given by (2) and (4), respectively. This problem is of particular interest

as it potentially provides insight to the solutions of the constrained optimisations we mentioned earlier. We denote the optimisation objective, that is, energy efficiency of the receiver, with η and its optimal value with η^* . The optimal bit resolution vector is denoted with \mathbf{b}^* accordingly.

The optimisation problem (5) with respect to integer-valued bit resolutions is a combinatorial problem with a search space growing exponentially with M and $b_{\max} + 1$, leading to prohibited computational complexity if the global optimum is to be found. For a practical ADC, the maximal bit resolution b_{\max} is typically a rather small number. Therefore, when the number of receive antennas M is also small, an exhaustive search for the optimal \mathbf{b} is possible. Yet for a large receive antenna array, we would need more effective search algorithms. Common techniques for tackling integer programming, such as the branch and bound method, often requires the computation of upper bounds on the optimal objective, which is usually achieved via solving a relaxation of the original problem [28]. In our case, this does not seem an option due to the non-linearity and discontinuity of the objective function, that is, even if we allow b_i to take real values on $[0, b_{\max}]$, it is still very hard to solve (5) to optimality. We propose to apply the *particle swarm optimisation* (PSO) technique [29], which serves as a good candidate here because it is rather independent of the problem structure and can be implemented easily and is able to produce near-optimal solutions for systems of small-to-medium scale. For large-scale systems, however, more effective suboptimal algorithms need to be designed to concur the problem of computational complexity.

3. ADAPTING ANALOG-TO-DIGITAL CONVERTER RESOLUTIONS VIA PARTICLE SWARM OPTIMISATION

The PSO method, although originally proposed for solving unconstrained optimisations with real-valued variables [29], can be applied to tackle integer programming without any complicated modification [30]. It is a stochastic optimisation technique based on the social behaviour of a population of individuals. Each individual, termed as a *particle*, moves in the feasible region of the optimisation problem probing for good solutions and shares information with the whole set of particles, called the *swarm*. The movement of the particles in each *generation* of the algorithm is random but also depends on the memory of the individual particles as well as of the swarm. Let the swarm contain S particles. For initialisation of the algorithm, S feasible solutions of the optimisation are generated at random, which are denoted with $\mathbf{b}_1^0, \mathbf{b}_2^0, \dots, \mathbf{b}_S^0$. In the $(k+1)$ th generation, the particle s evolves according to the formula

$$\begin{aligned} \mathbf{v}_s^{k+1} &= w\mathbf{v}_s^k + s_1 r_1 (\mathbf{p}_s^k - \mathbf{b}_s^k) + s_2 r_2 (\mathbf{p}_g^k - \mathbf{b}_s^k), \\ \mathbf{b}_s^{k+1} &= \mathbf{b}_s^k + \mathbf{v}_s^{k+1}, \end{aligned}$$

that is, the position of particle s is incremented by a velocity vector \mathbf{v}_s^{k+1} , which is computed based on the velocity vector \mathbf{v}_s^k in the previous generation, the displacement between the particle and the best self-found solution $\mathbf{p}_s^k - \mathbf{b}_s^k$, and the displacement between the particle and the global best solution found by the swarm $\mathbf{p}_g^k - \mathbf{b}_s^k$. The parameter w is called the *inertia weight* and is usually chosen as a decreasing function in the generation index, facilitating global search in early generations of the algorithm and local search in later generations. The weights s_1 and s_2 are called the *cognition and social learning rates*, and they are usually kept constant throughout the generations. On the other hand, the uniformly distributed random numbers r_1 and r_2 on $[0, 1]$, which are renewed in every generation, add randomness into the trajectory of each particle. In our numerical studies, the parameter settings $s_1 = s_2 = 1$, $w = 1 - 0.02k$ are used.

In the application of the PSO method to solving (5), the particle positions correspond to the ADC resolution vectors \mathbf{b} , hence each fractional valued particle position obtained from the update needs to be mapped to a feasible solution by rounding and fitting the values into the set $\{0, \dots, b_{\max}\}$. After the mapping and fitting, the objective function is evaluated at each new particle position, and the local best solutions as well as the global best solution need to be updated. The algorithm terminates when a maximal number of generations G is reached. Depending on the dimension of the optimisation, the values of S and G can be increased to improve the performance of the PSO. One could also repeat the algorithm for several runs and pick the best solution among all obtained global best solutions. From simulations, we have found that combining both schemes yields better performance than relying solely on either one of them. For the simulation results shown in the sequel, we take $G = 20$ and run the algorithm 20 times for each given array size and channel realisation, and choose S to be dependent on the scale of the system.

To verify the effectiveness of the PSO algorithm, we set up a test scenario with $N = 2$, $\gamma \in \{0, 30\}$ dB and restrict M to small numbers such that an exhaustive search for \mathbf{b}^* is possible. The successful rate, that is, the ratio that the PSO method finds the global optimum, and the averaged value of η_{PSO}/η^* are used as performance measures, which are computed after 1000 simulation trials and listed in Table II. One immediately sees that the PSO method is able to find solutions very close to the global optimum, even though, in some cases, the unsuccessful rate is not trivial. Expanding the swarm further helps improve the performance of the algorithm, yet it should be noted that the number of cost function evaluations scales linearly with the swarm size.

The PSO method requires $S \times G$ evaluations of the cost function per run. According to (2), this involves the calculation of the determinant of an $N \times N$ matrix and some matrix multiplications with complexity $O(N^2M)$. Therefore, the complexity order of the PSO algorithm is approximately $O(N^3M + N^2M^2)$, if S is chosen to be linear in M . From Table II, we see that the performance of the algorithm deteriorates even though S increases linearly

Table II. Effectiveness of the particle swarm optimisation method.

	γ (dB)	M	η_{PSO}/η^* (%)	Succ. rate (%)
$S = M$	0	3	100	100
		4	100	99.80
		5	100	98.01
	30	6	99.97	86.50
		3	99.96	99.70
		4	99.87	84.90
$S = 4M$	0	5	99.82	79.30
		6	99.78	73.20
		3	100	100
	30	4	100	100
		5	100	100
		6	99.99	99.99

with M , which implies that the PSO might not be suitable for systems of a large M . To this end, we discuss several alternative suboptimal algorithms in the next section.

4. ANTENNA SELECTION-BASED RECEIVE STRATEGIES

Both from intuitions and results of the PSO method, we find that in order to have better system energy efficiency, a number of receive antennas could be switched off. These are usually antennas with weaker channel conditions, the usage of which does not lead to a capacity increment that pays off the boost in power consumption of the corresponding RF chains. In other words, we can consider the optimisation problem (5) as the selection of a number of active antennas, and the choice of positive bit resolutions for each selected antenna. Antenna selection is a well-investigated method for reducing the hardware cost and computational complexity of MIMO systems, while the channel capacity can be preserved to a large portion [31][32]. Existing receive antenna selection schemes mostly employ the channel capacity of the selected subsystem as performance measure and assume that the receiver has infinite precision to access the received signal [33][34]. We shall investigate in this section, the influence of quantisation by A/D conversion on the antenna selection process. Moreover, we propose different schemes for finding the bit resolutions of the selected active antennas.

4.1. Uniform bit resolution for selected antennas

Receive antenna selection is in general a combinatorial problem which is difficult to solve to optimality. When there is only one transmit antenna, we can simply, and also optimally, select a subset of antennas with the largest channel gains. When $N > 1$, this norm-based selection scheme

is suboptimal because of the correlation between the channel vectors. A number of heuristic algorithms have been proposed in the literature, for example, [33][34], which exhibit performance very close to optimal. For our optimisation problem, we do not know from the beginning, how many antennas should be selected. An incremental greedy selection scheme can be a good choice in this case, due to its flexibility and low complexity. To this end, we design the antenna selection scheme for quantised received signal based on the algorithm proposed in [33].

On the other hand, as the channel coefficients are i.i.d., one can think of a receive strategy where uniform bit resolution is applied to all selected antennas. Let the chosen resolution be b and the corresponding distortion factor be ρ . The essential idea here is that, in each iteration, the antenna that leads to the largest performance gain is selected. As the ADC power consumption of all selected antennas is identical, we simply choose the antenna that results in the largest capacity increase. The selection process is terminated when η decreases on the addition of one more antenna. Mathematically, let us suppose that antenna k is selected for the $(l+1)$ th iteration, $l > 0$. At this point, l antennas have already been selected, and the corresponding channel vectors form the matrix $\mathbf{H}_l \in \mathbb{C}^{l \times N}$. The diagonal matrix \mathbf{D}_l can also be computed based on the norms of the selected channel vectors. After antenna k is chosen, the two matrices are updated according to

$$\mathbf{H}_{l+1}^H = [\mathbf{H}_l^H \mathbf{h}_k],$$

$$\mathbf{D}_{l+1} = \begin{bmatrix} \mathbf{D}_l & \mathbf{0} \\ \mathbf{0}^T & \frac{1-\rho}{1+\rho\beta_k} \end{bmatrix},$$

which further lead to

$$\mathbf{H}_{l+1}^H \mathbf{D}_{l+1} \mathbf{H}_{l+1} = \mathbf{H}_l^H \mathbf{D}_l \mathbf{H}_l + \frac{1-\rho}{1+\rho\beta_k} \mathbf{h}_k \mathbf{h}_k^H.$$

Define $\mathbf{B}_l = (\mathbf{1}_N + \frac{\gamma}{N} \mathbf{H}_l^H \mathbf{D}_l \mathbf{H}_l)^{-1}$. The channel capacity achieved after iteration l is then given by

$$C_l = \log_2 \left| \mathbf{1}_N + \frac{\gamma}{N} \mathbf{H}_l^H \mathbf{D}_l \mathbf{H}_l \right| = \log_2 \left| \mathbf{B}_l^{-1} \right|.$$

After the selection of antenna k , the increased channel capacity is computed as

$$\begin{aligned} C_{l+1} &= \log_2 \left| \mathbf{1}_N + \frac{\gamma}{N} \mathbf{H}_{l+1}^H \mathbf{D}_{l+1} \mathbf{H}_{l+1} \right| \\ &= \log_2 \left| \mathbf{B}_l^{-1} + \frac{\gamma}{N} \cdot \frac{1-\rho}{1+\rho\beta_k} \mathbf{h}_k \mathbf{h}_k^H \right| \\ &= \log_2 \left| \mathbf{B}_l^{-1} \right| + \log_2 \left| \mathbf{1}_N + \frac{1-\rho}{1+\rho\beta_k} \mathbf{B}_l \mathbf{h}_k \mathbf{h}_k^H \right| \\ &= C_l + \log_2 \left(1 + \frac{1-\rho}{1+\rho\beta_k} \mathbf{h}_k^H \mathbf{B}_l \mathbf{h}_k \right) \\ &\triangleq C_l + \log_2 \left(1 + \frac{1-\rho}{1+\rho\beta_k} Q_{l,k} \right). \end{aligned}$$

Therefore, the selected antenna k satisfies

$$k = \operatorname{argmax}_{j \in \mathcal{A}_l} \frac{1 - \rho}{1 + \rho\beta_j} \cdot Q_{lj}, \quad (6)$$

where \mathcal{A}_l is the set of antennas that have not been selected after the l th iteration, which should be updated by

$$\mathcal{A}_{l+1} = \mathcal{A}_l - \{k\}.$$

Obviously, the selection governed by (6) depends not only on the channel conditions but also on the chosen ADC resolution. The update of \mathbf{B}_l , by using the matrix inversion lemma, does not require a matrix inversion operation:

$$\begin{aligned} \mathbf{B}_{l+1} &= \left(\mathbf{1}_N + \frac{\gamma}{N} \mathbf{H}_{l+1}^H \mathbf{D}_{l+1} \mathbf{H}_{l+1} \right)^{-1} \\ &= \left(\mathbf{B}_l^{-1} + \frac{\gamma}{N} \cdot \frac{1 - \rho}{1 + \rho\beta_k} \mathbf{h}_k \mathbf{h}_k^H \right)^{-1} \\ &= \mathbf{B}_l - \left(\frac{N}{\gamma} \cdot \frac{1 + \rho\beta_k}{1 - \rho} + Q_{l,k} \right)^{-1} \mathbf{B}_l \mathbf{h}_k \mathbf{h}_k^H \mathbf{B}_l. \end{aligned}$$

The algorithm can be initialised with $\mathbf{B}_0 = \mathbf{1}_N$ and terminated if $\eta_{l+1} < \eta_l$ or $l = M$. The steps of the algorithm are summarised in Algorithm 1. We denote the number of selected antennas with L . The complexity order of the algorithm is given by $O(MNL)$ [33].

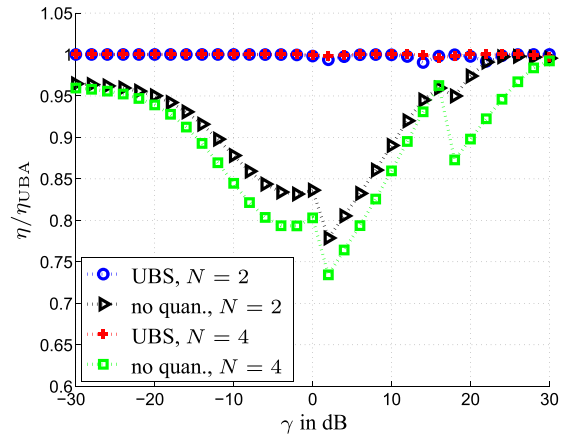
Algorithm 1 Antenna selection procedure with given uniform bit resolution

Given \mathbf{H} , b , ρ
 Initialisation: $\mathcal{A} \leftarrow \{1, \dots, M\}$, $\mathbf{B} \leftarrow \mathbf{1}_N$
 $\beta_i \leftarrow \frac{\gamma}{N} \cdot \|\mathbf{h}_i\|_2^2$, $Q_i \leftarrow \|\mathbf{h}_i\|_2^2$, $i = 1, \dots, M$
 $C \leftarrow 0$, $P \leftarrow c_1$, $\eta \leftarrow 0$
while $\mathcal{A} \neq \emptyset$ **do**
 $k \leftarrow \operatorname{argmax}_{j \in \mathcal{A}} \left\{ \frac{Q_j}{1 + \rho\beta_j} \right\}$
 $C' \leftarrow C + \log_2 \left(1 + \frac{1 - \rho}{1 + \rho\beta_k} Q_k \right)$
 $P' \leftarrow P + c_0 \sigma^2 \cdot 2^b$
 $\eta' \leftarrow C'/P'$
 if $\eta' < \eta$ **then**
 break
 end if
 $\eta \leftarrow \eta'$
 $\mathcal{A} \leftarrow \mathcal{A} - \{k\}$
 $\mathbf{B} \leftarrow \mathbf{B} - \left(\frac{N}{\gamma} \cdot \frac{1 + \rho\beta_k}{1 - \rho} + Q_k \right)^{-1} \mathbf{B} \mathbf{h}_k \mathbf{h}_k^H \mathbf{B}$
 for $j \in \mathcal{A}$ **do**
 $Q_j \leftarrow \mathbf{h}_j^H \mathbf{B} \mathbf{h}_j$
 end for
end while
Return $\{1, \dots, M\} - \mathcal{A}$

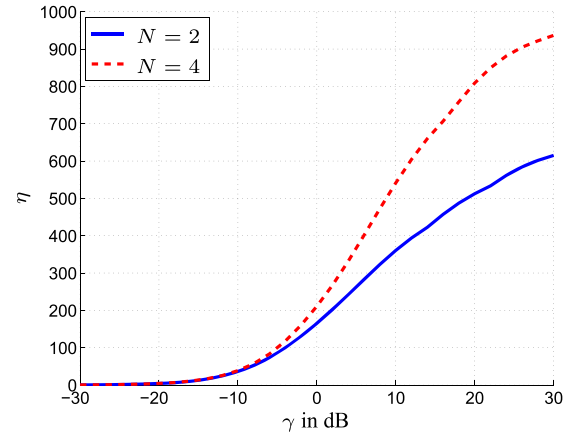
The antenna selection method described earlier requires a given bit resolution for all chosen antennas. One could

employ an outer loop and enumerate all available bit resolutions $1, \dots, b_{\max}$ to determine the best b . We refer to this scheme with the abbreviation *UBA*, that is, uniform bit resolution adaptive scheme. Accordingly, we also propose a uniform bit resolution static scheme, termed *UBS*, in which the employed bit resolution only depends on the average receive SNR γ . More specifically, we first apply the UBA to a large number of channel realisations and obtain the average optimal bit resolution. This value is then rounded and given to the UBS, where the subsequent antenna selection process is the same as in UBA. This means, we can set up a lookup table to record the average optimal bit resolutions for different γ and directly apply the corresponding rounded value to the UBS. The performances of UBA and UBS are presented and compared in the following.

We first show the energy efficiency performance of the proposed schemes in Figure 2, where the results are the averaged outcome of 1×10^4 independent simulation trials. Besides UBS, we also implement the original antenna selection scheme from [33], which does not take quantisation into account, and depict the results for reference with the



(a) Comparison with respect to UBA

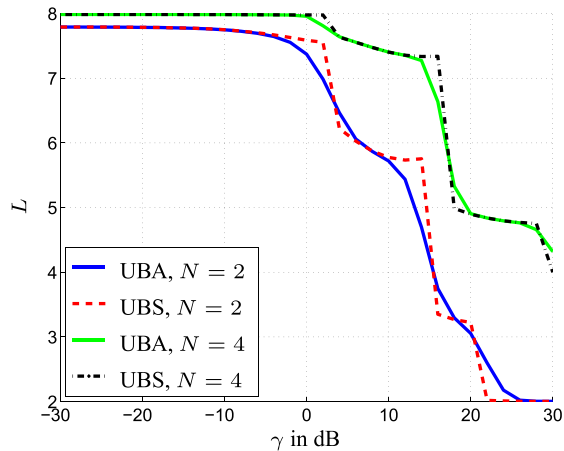


(b) Energy efficiency achieved by UBS

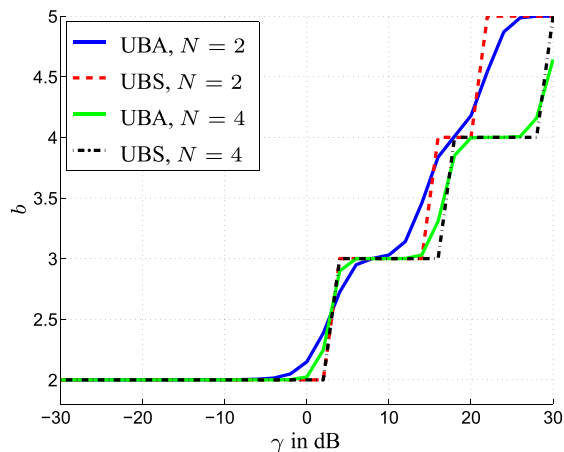
Figure 2. Performance of UBA and UBS in terms of energy efficiency ($M = 8$, unit of η is Mbit/Joule).

label *no quan.* in Figure 2(a). This means, the algorithm is given the same optimised bit resolution as UBS but presumes $\rho = 0$ during the antenna selection process. From the curves, we find that this would lead to a significant performance loss in the medium SNR range. On the other hand, the performance gap between UBS and UBA is negligible. In Figure 2(b), we see that increasing the number of transmit antennas is only helpful to energy efficiency of the receiver in the high SNR regime, which is in accordance with the conclusion made with respect to channel capacity.

For the same simulation setup, we illustrate the average number of active antennas and the average bit resolution for the active antennas in Figure 3, as dependent on the receive SNR γ . The curves for UBA and UBS have similar shapes, except that those for the UBA are smoother. In the low SNR regime, most receive antennas are used with low bit resolution, while in the high SNR regime, a few receive antennas are used with high bit resolution. In fact, in the latter case, the number of selected antennas converges to the number of transmit antennas N .



(a) Average number of active antennas as dependent on γ



(b) Average bit resolution as dependent on γ

Figure 3. Optimisation results of UBA and UBS ($M = 8$).

4.2. Non-uniform bit resolution for selected antennas: greedy algorithm

When non-uniform bit resolutions are allowed for the selected antennas, we can select one antenna together with its bit resolution in each iteration of the algorithm based on the improvement of energy efficiency after the selection. To this end, we use the same procedure as the UBA except that in any iteration l , instead of solving (6), we solve the joint optimisation

$$(k, b) = \operatorname{argmax}_{i \in \{1, \dots, b_{\max}\}} \frac{C_l + \log_2 \left(1 + \frac{(1 - \rho_i) Q_{l,j}}{1 + \rho_i \beta_j} \right)}{P_l + c_0 \sigma^2 \cdot 2^{b_i}},$$

where P_l indicates the power consumption of the receiver after l iterations and include the differently chosen bit resolutions in the diagonal matrix \mathbf{D} and the auxiliary matrix \mathbf{B} . We term this scheme as *NBG*, which stands for non-uniform bit resolution greedy algorithm, because now not only the antennas are chosen in a greedy fashion but also the bit resolution. This scheme is of the same complexity order as the UBA, yet we shall see in the next section that its performance is not as good.

4.3. Non-uniform bit resolution for selected antennas: decremental algorithm

An undesirable situation in the execution of NBG is that a high bit resolution can be chosen for one of the first selected antennas, which turns out to be too high and causes the corresponding antenna to occupy too much resources. We therefore propose another algorithm for the non-uniform bit resolution case, which allows reduction of the bit resolutions of selected antennas and is therefore less greedy. We call this scheme non-uniform bit resolution decremental algorithm, abbreviated as *NBD*.

In the first iteration, the NBD behaves the same as NBG, that is, it chooses the antenna and the bit resolution that give the maximal energy efficiency. In each of the following iterations, a new antenna is chosen according to (6) where the distortion factor of the last selected antenna is used. Then the resolutions of all selected antennas are examined according to the order they have been selected. From the first chosen antenna, we try to find out whether reducing its bit resolution by one could lead to a higher energy efficiency. Note that in this attempt, all subsequent antennas sharing the same bit resolution also reduce their

Table III. Proposed and reference algorithms.

Abbr.	Meaning
PSO	Particle swarm optimisation
UBS	Uniform bit resolution static algorithm
NBG	Non-uniform bit resolution greedy algorithm
NBD	Non-uniform bit resolution decremental algorithm
NAS	Without antenna selection

resolution by one. In this way, one antenna cannot use higher bit resolutions than the antennas that are selected before it, which is consistent with the idea of the greedy antenna selection process. Whether or not the bit resolution of the first antenna is reduced, the algorithm proceeds to try reducing the bit resolution of the second antenna, so on and so forth until the antenna chosen in the current iteration. If,

after this decremental process, the energy efficiency of the system is not increased, then we resort to the bit resolution vector of the last iteration and terminate the algorithm; otherwise, the current best bit resolution vector is recorded, and the next iteration begins to seek for another antenna.

During each iteration of the NBD, previously chosen bit resolutions can be changed. Therefore, the algorithmic

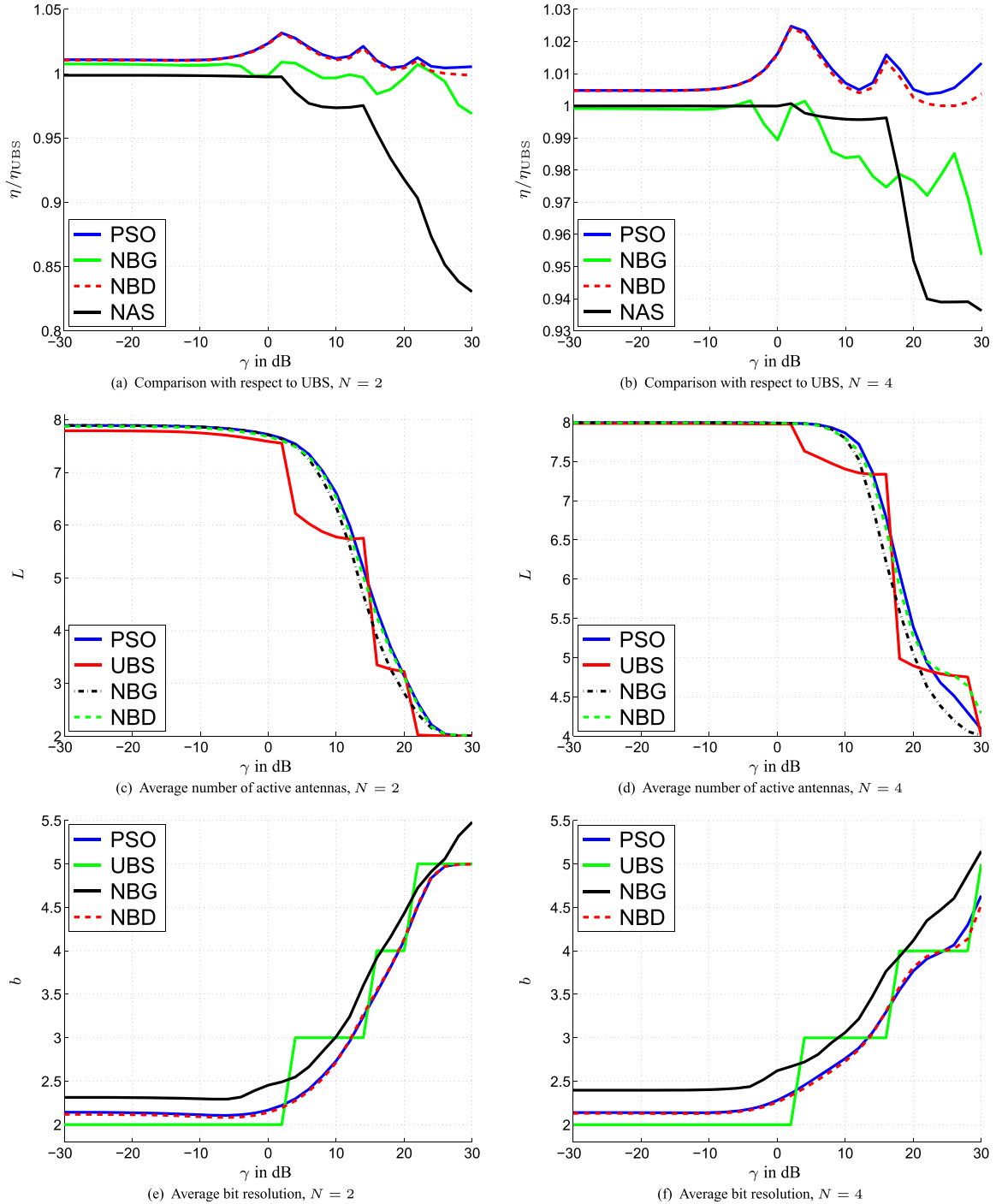


Figure 4. Performance comparison of proposed schemes with varying γ ($M = 8$).

structure of UBA and UBS does not apply anymore, and we need to evaluate the capacity lower bound repeatedly. This suggests that the NBD has a reasonably higher computational complexity than the UBS, which, from the simulation time, we have experienced, is still much lower than the PSO method.

5. SIMULATION RESULTS

We present results obtained with Monte Carlo simulations in this section, with the purpose to compare proposed algorithms as well as to summarise operational guide lines. The abbreviations used for the algorithms are listed in Table III. Each result, as shown in the figures, is the averaged outcome of 1×10^4 independent simulation trials. For the PSO method, the swarm size is chosen as $S = 4M$.

We first study the scenario with fixed receive antenna number $M = 8$, and varying average receive SNR γ in Figure 4. We use the results of UBS as comparison benchmark, and depict the ratio of achieved energy efficiency of other proposed algorithms in Figure 4(a) and (b). The method termed *NAS* refers to the scheme of using all receive antennas with one optimised uniform resolution. It turns out except *NAS*, the performance of all other proposed schemes are rather close. In the low SNR regime,

since almost all receive antennas should be used with a low bit resolution, the *NAS* scheme also performs very well. In the high SNR regime, however, the number of active antennas approaches N , and the *NAS* scheme exhibits a significant performance gap. The PSO and NBD schemes have a very small gain over the UBS, which is slightly larger at turning SNR values where the optimal bit resolution changes. The *NBG* scheme performs fine in low SNR regime but suffers at high SNR values. Taken the computational complexity also into account, we conclude that the UBS method is the most promising solution. The NBD scheme could potentially be useful for other channel models or communication scenarios, for example, for spatially correlated channel or in a multiuser scenario. The PSO method can provide some comparison benchmark to small-to-medium scale systems but is, in general, rather expensive to apply.

With fixed representative receive SNR values of $-30, 0, 30$ dB and a varying receive antenna number, we also observe some interesting results in Figure 5. As shown in Figure 5(a), the PSO method fails to outperform other proposed algorithms for large M , while the NBD scheme constantly performs a bit better than the UBS. From the remaining three figures, it can be seen that employing more receive antennas is more helpful in the low SNR regime

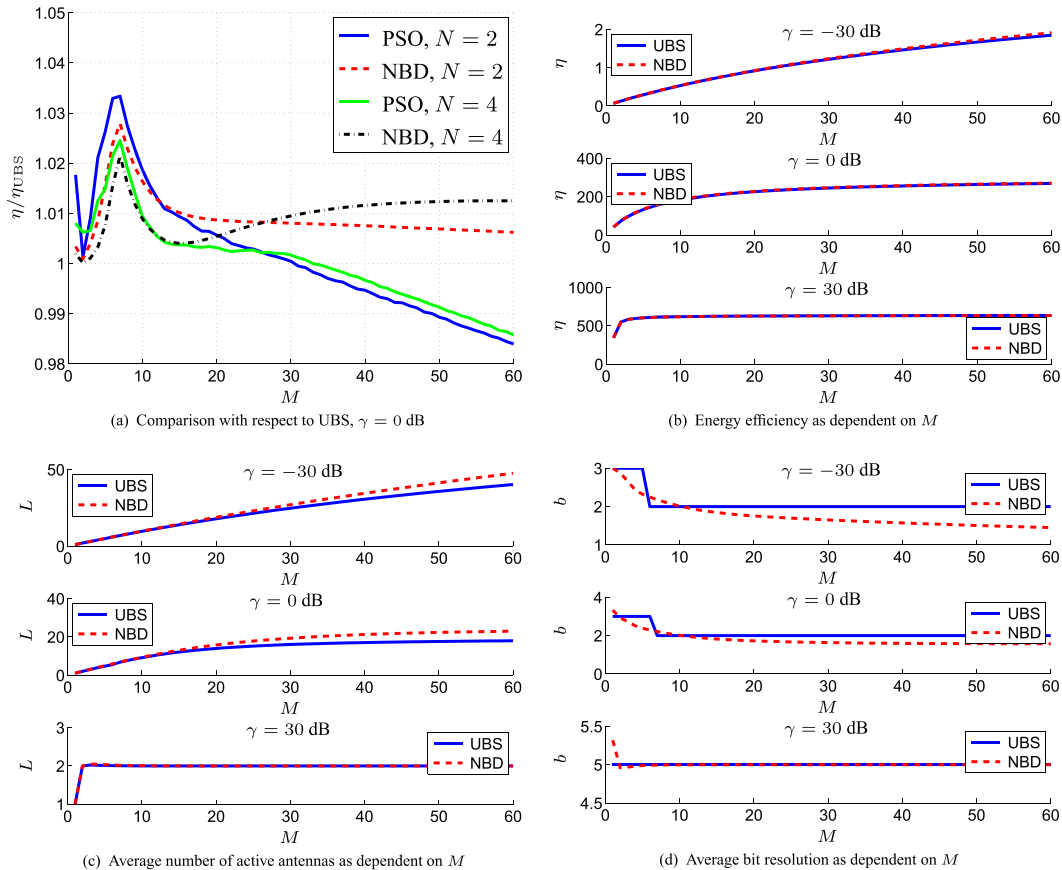


Figure 5. Optimised system performance as dependent on M ($N = 2$, unit of η is Mbit/Joule).

due to the increased array gain, and a large portion of them should be active. The optimised bit resolution decreases with M , meaning that if the receiver is employed with a large antenna array, lower bit resolution could be used for each RF chain, which is a potentially interesting feature for massive MIMO with low-cost RF front ends.

6. CONCLUSION AND OUTLOOK

We have investigated in this work the impact of A/D conversion at a multi-antenna receiver. Based on the trade-off between quantisation error and power consumption of the ADC, an energy efficiency maximisation problem of the vector of ADC resolutions is formulated. We propose several optimisation methods for finding near-optimal solutions: the PSO method, which is a stochastic optimisation technique, and a number of antenna selection based methods. From the results of numerical simulations, we find that the combination of antenna selection and the offline optimisation of a uniform ADC resolution for the active antennas achieves good balance between performance and computational complexity. A decremental resolution algorithm also shows potential to suit in other communication scenarios such as a multi-user situation. Moreover, based on the optimised solutions, we learn that:

- The optimal number of active antennas and the optimal bit resolutions depend heavily on the average receive SNR;
- With fixed receive antenna number, more antennas should be selected for reception with a low bit resolution in the low SNR regime, while a few antennas should be active with a high bit resolution in the high SNR regime;
- Employing a large receive antenna array is especially helpful to system performance in the low SNR regime, where lower bit resolution can be used for each RF chain.

There are many possibilities and directions to extend this work for future research, and in this short outlook, we try to discuss a few of them. First of all, we have assumed that the receiver has perfect knowledge about the instantaneous channel state information, which is not exactly realistic especially when we consider the quantisation that has to be taken at the receiver. The impact of quantisation of received pilots which in turn influences the quality of the channel estimation should be carefully investigated, and the ADC resolution applied for the pilot symbols should be chosen appropriately. In our recent work [35], we have touched upon this issue, but the consideration therein is by no means exclusive. Secondly, an information theoretic approach has been pursued where we employ a capacity lower bound to measure the achievable rate of the system. Practical signaling and signal processing at the receiver for coarsely quantised data should also be investigated, which in turn could provide us with a more detailed and refined power consumption model. Thirdly, at

the transmitter side, a digital-to-analog converter performs the inverse function of the ADC at the receiver. Therefore, a similar optimisation problem can be formulated to improve the energy efficiency of the transmitter. Even more interestingly, a joint optimisation of system parameters at the transmitter and the receiver could potentially lead to further performance gain, as well as provide insight into the energy efficient operation modes of point-to-point communications.

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