A Novel Connectionist System for Unconstrained Handwriting Recognition

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Abstract—**Recognising lines of unconstrained handwritten text is a challenging task. The difficulty of segmenting cursive or overlapping characters, combined with the need to exploit surrounding context, has led to low recognition rates for even the best current recognisers. Most recent progress in the field has been made either through improved preprocessing, or through advances in language modelling. Relatively little work has been done on the basic recognition algorithms. Indeed, most systems rely on the same hidden Markov models that have been used for decades in speech and handwriting recognition, despite their well-known shortcomings. This paper proposes an alternative approach based on a novel type of recurrent neural network, specifically designed for sequence labelling tasks where the data is hard to segment and contains long range, bidirectional interdependencies. In experiments on two large unconstrained handwriting databases, our approach achieves word recognition accuracies of 79.7% on online data and 74.1% on offline data, significantly outperforming a state-of-the-art HMM-based system. In addition, we demonstrate the network's robustness to lexicon size, measure the individual influence of its hidden layers, and analyse its use of context. Lastly we provide an in depth discussion of the differences between the network and HMMs, suggesting reasons for the network's superior performance.**

Index Terms—**Handwriting recognition, online handwriting, offline handwriting, connectionist temporal classification, bidirectional long short-term memory, recurrent neural networks, hidden Markov model**

I. Introduction

HANDWRITING recognition is traditionally divided into online and offline recognition. In online recognition a time series of coordinates, representing the movement of the pen-tip, is captured, while in the offline case only an image of the text is available. Because of the greater ease of extracting relevant features, online recognition generally yields better results [1]. Another important distinction is between recognising isolated characters or words, and recognising whole lines of text. Unsurprisingly, the latter is substantially harder, and the excellent results that have been obtained for digit and character recognition [2], [3] have never been matched for complete lines. Lastly, handwriting recognition can be split into cases where the writing style is constrained in some way—for example, only hand printed characters are allowed—and the more challenging scenario where it is unconstrained. Despite more than 30 years of handwriting recognition research [2], [3], [4], [5], developing a reliable, general-purpose system for unconstrained text line recognition remains an open problem.

A well known test bed for isolated handwritten character recognition is the UNIPEN database [6]. Systems that have been found to perform well on UNIPEN include: a writer-independent approach based on hidden Markov models [7]; a hybrid technique called cluster generative statistical dynamic time warping [8], which combines dynamic time warping with HMMs and embeds clustering and statistical sequence modelling in a single feature space; and a support vector machine with a novel Gaussian dynamic time warping kernel [9]. Typical error rates on UNIPEN range from 3% for digit recognition, to about 10% for lower case character recognition.

Similar techniques can be used to classify isolated words, and this has given good results for small vocabularies (for example a writer dependent word error rate of about 4.5% for 32 words [10]). However an obvious drawback of whole word classification is that it does not scale to large vocabularies.

For large vocabulary recognition tasks, such as those considered in this paper, the only feasible approach is to recognise individual characters and map them onto complete words using a dictionary. Naively, this could be done by presegmenting words into characters and classifying each segment. However, segmentation is difficult for cursive or unconstrained text, unless the words have already been recognised. This creates a circular dependency between segmentation and recognition that is sometimes referred to as Sayre's paradox [11].

One solution to Sayre's paradox is to simply ignore it, and carry out segmentation before recognition. For example [3] describes techniques for character segmentation, based on unsupervised learning and data-driven methods. Other strategies first segment the text into basic strokes, rather than characters. The stroke boundaries may be defined in various ways, such as the minima of the velocity, the minima of the y-coordinates, or the points of maximum curvature. For example, one online approach first segments the data at the minima of the y-coordinates, then applies self-organising maps [12]. Another, offline, approach uses the minima of the vertical histogram for an initial estimation of the character boundaries and then applies various heuristics to improve the segmentation [13].

A more promising approach to Sayre's paradox is to segment and recognise at the same time. Hidden Markov models (HMMs) are able to do this, which is one reason for their popularity in unconstrained handwriting recognition [14],

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[15], [16], [17], [18], [19]. The idea of applying HMMs to handwriting recognition was originally motivated by their success in speech recognition [20], where a similar conflict exists between recognition and segmentation. Over the years, numerous refinements of the basic HMM approach have been proposed, such as the writer independent system considered in [7], which combines point oriented and stroke oriented input features.

However, HMMs have several well-known drawbacks. One of these is that they assume the probability of each observation depends only on the current state, which makes contextual effects difficult to model. Another is that HMMs are generative, while discriminative models generally give better performance in labelling and classification tasks.

Recurrent neural networks (RNNs) do not suffer from these limitations, and would therefore seem a promising alternative to HMMs. However the application of RNNs alone to handwriting recognition have so far been limited to isolated character recognition (e.g. [21]). The main reason for this is that traditional neural network objective functions require a separate training signal for every point in the input sequence, which in turn requires presegmented data.

A more successful use of neural networks for handwriting recognition has been to combine them with HMMs in the so-called hybrid approach [22], [23]. A variety of network architectures have been tried for hybrid handwriting recognition, including multilayer perceptrons [24], [25], time delay neural networks [18], [26], [27], and RNNs [28], [29], [30]. However, although hybrid models alleviate the difficulty of introducing context to HMMs, they still suffer from many of the drawbacks of HMMs, and they do not realise the full potential of RNNs for sequence modelling.

This paper proposes an alternative approach, in which a single RNN is trained directly for sequence labelling. The network uses the connectionist temporal classification (CTC) output layer [31], [32], first applied to speech recognition. CTC uses the network to map directly from the complete input sequence to the sequence of output labels, obviating the need for presegmented data. We extend the original formulation of CTC by combining it with a dictionary and language model to obtain word recognition scores that can be compared directly with other systems. Although CTC can be used with any type of RNN, best results are given by networks able to incorporate as much context as possible. For this reason we chose the bidirectional Long Short-Term Memory (BLSTM; [33]) architecture, which provides access to long range context along both input directions.

In experiments on large online and offline handwriting databases, our approach significantly outperforms a state-of-the-art HMM-based system on unconstrained text line recognition. Furthermore, the network retains its advantage over a wide range of dictionary sizes.

The paper is organised as follows. Sections II and III describe the preprocessing and feature extraction methods for the online and offline data respectively. Section IV introduces the novel RNN-based recogniser. Section V describes the databases and presents the experimental analysis and results. Section VI discusses the differences between the new system and HMMs, and suggests reasons for the network's superior performance. Our conclusions are presented in Section VII.

Fig. 1. Illustration of the recording

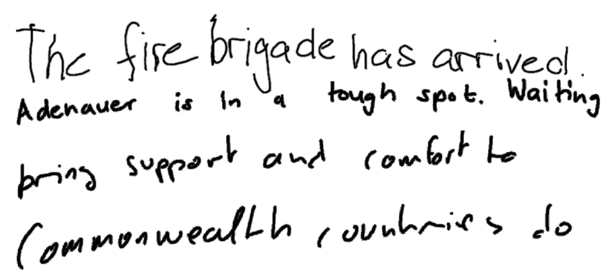

Fig. 2. Examples of handwritten text acquired from a whiteboard

II. Online Data Preparation

The online data used in our experiments were recorded from a whiteboard using the eBeam interface[1] [34]. As illustrated in Figure 1, the interface consists of a normal pen in a special casing, which sends infrared signals to a triangular receiver mounted in one of the corners of the whiteboard. The acquisition interface outputs a sequence of (x, y)-coordinates representing the location of the tip of the pen together with a time stamp for each location. The coordinates are only recorded during the periods when the pen-tip is in continuous contact with the whiteboard. We refer to these periods as *strokes*. After some standard steps to correct for missing and noisy points [35], the data was stored in xml-format, along with the frame rate, which varied from 30 to 70 frames per second.

A. Normalisation

The preprocessing begins with data normalisation. This is an important step in handwriting recognition because writing styles differ greatly with respect to the skew, slant, height and width of the characters.

Since the subjects stand rather than sit, and their arms do not rest on a table, handwriting rendered on a whiteboard

[1] eBeam System by Luidia, Inc. - www.e-Beam.com

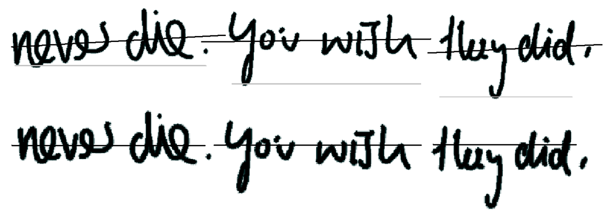

Fig. 3. Processing the text line. Top: text line split into individual parts with estimated skew in the middle of the text line; bottom: text line after skew normalisation. Note that baseline and corpus line detection (described below) give an improved final estimate of the skew.

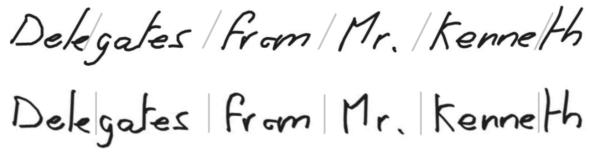

Fig. 4. Slant correction; grey lines indicate the estimated slant angle

is different from that produced with a pen on a writing tablet. In particular, it has been observed that the baseline on a whiteboard cannot usually be approximated by a simple straight line. Furthermore, the size and width of the characters become smaller the more the pen moves to the right. Examples of both effects can be seen in Figure 2. Consequently, online handwriting gathered from a whiteboard requires some special preprocessing steps.

Since the text lines on a whiteboard usually have no uniform skew, they are split into smaller parts and the rest of the preprocessing is done for each part separately. To accomplish the splitting, all gaps within a line are determined first. The text is split at a gap if it is wider than the median gap width, and if the size of both parts resulting from the split is larger than some predefined threshold. An example of the splitting process is shown in Figure 3 with the resulting parts indicated by lines below the text.

Next the parts are corrected with respect to their skew and slant. A linear regression is performed through all the points, and the orientation of the text line is corrected according to the regression parameters (see Figure 3). For slant normalisation, we compute the histogram over all angles enclosed by the lines connecting two successive points of the trajectory and the horizontal line [26]. The histogram ranges from -90° to 90° with a step size of 2°. We weight the histogram values with a Gaussian whose mean is at the vertical angle and whose variance is chosen empirically. This is beneficial because some words are not properly corrected if a single long straight line is drawn in the horizontal direction, which results in a large histogram value. We also smooth each histogram entry γ_i using its nearest neighbours, $\bar{\gamma}_i = (\gamma_{i-1} + 2\gamma_i + \gamma_{i+1})/4$, because in some cases the correct slant is at the border of two angle intervals and a single peak at another interval may be slightly higher. This single peak will become smaller after smoothing. Figure 4 shows a text line before and after slant correction.

Delayed strokes, such as the crossing of a ‘t’ or the dot of an ‘i’, are a well known problem in online handwriting recognition, because the order in which they are written varies between different writers. For this reason, delayed strokes (identified as strokes above already written parts, followed by a pen-movement to the right [26]) are removed. Note that some i-dots may be missed by this procedure. However, this usually occurs only if there was no pen-movement to the left, meaning that the writing order is not disrupted. A special *hat-feature* is used to indicate to the recogniser that a delayed stroke was removed.

To correct for variations in writing speed, the input sequences are transformed so that the points are equally spaced. The optimal value for this distance is found empirically.

The next step is the computation of the baseline and the corpus line, which are then used to normalise the size of the text. The baseline corresponds to the original line on which the text was written, i.e. it passes through the bottom of the characters. The corpus line goes through the top of the lower case letters. To obtain these lines two linear regressions through the minima and maxima are computed. After removing outliers the regression is repeated twice, resulting in the estimated baseline (minima) and corpus line (maxima). Figure 5 illustrates the estimated baseline and the corpus line of part of the example shown in Figure 3. The baseline is subtracted from all y-coordinates and the heights of the three resulting areas are normalised.

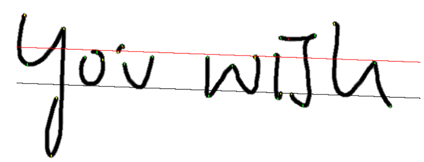

Fig. 5. Baseline and corpus line of an example part of a text line

The final preprocessing step is to normalise the width of the characters. This is done by scaling the text horizontally with a fraction of the number of strokes crossing the horizontal line between the baseline and the corpus line. This preprocessing step is needed because the x-coordinates of the points are taken as a feature.

B. Feature Extraction

The input to the recogniser consists of 25 features for each (x, y)-coordinate recorded by the acquisition system. These features can be divided into two classes. The first class consists of features extracted for each point by considering its neighbours in the time series. The second class is based on the spatial information given by the offline matrix representation.

For point (x, y), the features in the first class are as follows:

- A feature indicating whether the pen-tip is touching the board or not
- The hat-feature indicating whether a delayed stroke was removed at y
- The velocity computed before resampling
- The x-coordinate after high-pass filtering, i.e. after subtracting a moving average from the true horizontal position

- The y-coordinate after normalisation.
- The cosine and sine of the angle between the line segment starting at the point and the x-axis (writing direction)
- The cosine and sine of the angle between the lines to the previous and the next point (curvature)

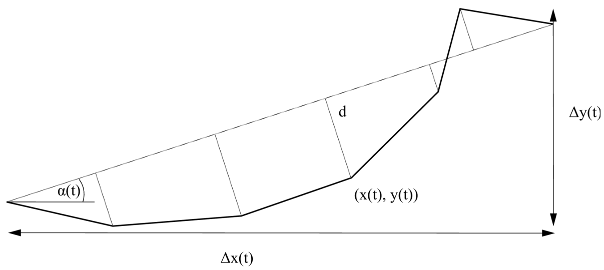

Fig. 6. Vicinity features of the point $(x(t), y(t))$. The three previous and three next points are considered in the example shown in this figure.

- *vicinity aspect:* this feature is equal to the aspect of the trajectory (see Figure 6):

$$\frac{\Delta y(t) - \Delta x(t)}{\Delta y(t) + \Delta x(t)}$$

- The cosine and sine of the angle α of the straight line from the first to the last vicinity point (see Figure 6)
- The length of the trajectory in the vicinity divided by $\max(\Delta x(t), \Delta y(t))$
- The average squared distance d^2 of each point in the vicinity to the straight line from the first to the last vicinity point

The features in the second class, illustrated in Figure 7, are computed using a two-dimensional matrix $B = b_{i,j}$ representing the offline version of the data. For each position $b_{i,j}$ the number of points on the trajectory of the strokes is stored, providing a low-resolution image of the handwritten data. The following features are used:

- The number of points above the corpus line (ascenders) and below the baseline (descenders) in the vicinity of the current point
- The number of black points in each region of the context map (the two-dimensional vicinity of the current point is transformed to a 3×3 map with width and height set to the height of the corpus)

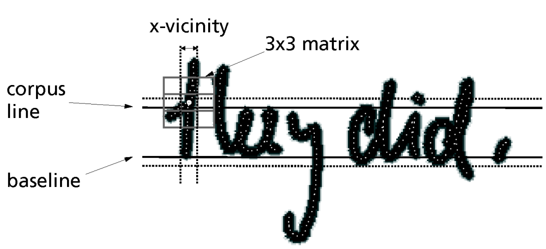

Fig. 7. Offline matrix features. The large white dot marks the considered point. The other online points are marked with smaller dots. The strokes have been widened for ease of visualisation.

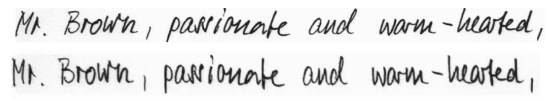

Fig. 8. Preprocessing of an image of handwritten text, showing the original image (top), and the normalised image (bottom).

III. Offline Data Preparation

The offline data used in our experiments consists of greyscale images scanned from handwritten forms, with a scanning resolution of 300 dpi and a greyscale bit depth of 8. The following procedure was carried out to extract the text lines from the images. First, the image was rotated to account for the overall skew of the document, and the handwritten part was extracted from the form. Then a histogram of the horizontal black/white transitions was calculated, and the text was split at the local minima to give a series of horizontal lines. Any stroke crossing the boundaries between two lines was assigned to the line containing their centre of gravity. With this method almost all text lines were extracted correctly.[2]

Once the line images were extracted, the next stage was to normalise the text with respect to writing skew and slant, and character size.

A. Normalisation

Unlike the online data, the normalisations for the offline data are applied to entire text lines at once. First of all the image is rotated to account for the line skew. Then the mean slant of the text is estimated, and a shearing transformation is applied to the image to bring the handwriting to an upright position. Next the baseline and the corpus line are normalised. Normalisation of the baseline means that the body of the text line (the part which is located between the upper and lower baselines), the ascender part (located above the upper baseline), and the descender part (below the lower baseline) are each scaled to a predefined height. Finally the image is scaled horizontally so that the mean character width is approximately equal to a predefined size. Figure 8 illustrates the offline preprocessing.

B. Feature Extraction

To extract the feature vectors from the normalised images, a sliding window approach is used. The width of the window is one pixel, and nine geometrical features are computed at each window position. Each text line image is therefore converted to a sequence of 9-dimensional vectors. The nine features are as follows:

- The mean grey value of the pixels
- The centre of gravity of the pixels
- The second order vertical moment of the centre of gravity
- The positions of the uppermost and lowermost black pixels
- The rate of change of these positions (with respect to the neighbouring windows)

[2] Only about 1 % of the text lines contain errors. These have been corrected manually in previous work [36].

- The number of black-white transitions between the uppermost and lowermost pixels
- The proportion of black pixels between the uppermost and lowermost pixels

For a more detailed description of the offline features, see [17].

IV. Neural Network Recogniser

A. Recurrent Neural Networks

Recurrent neural networks (RNNs) are a connectionist model containing a self-connected hidden layer. One benefit of the recurrent connection is that a 'memory' of previous inputs remains in the network's internal state, allowing it to make use of past context. Context plays an important role in handwriting recognition, as illustrated in Figure 9. Another important advantage of recurrency is that the rate of change of the internal state can be finely modulated by the recurrent weights, which builds in robustness to localised distortions of the input data.

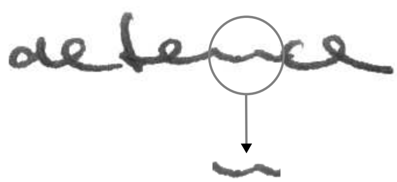

Fig. 9. Importance of context in handwriting recognition. The word 'defence' is clearly legible, but the letter 'n' in isolation is ambiguous.

B. Long Short-Term Memory (LSTM)

Unfortunately, the range of contextual information that standard RNNs can access is in practice quite limited. The problem is that the influence of a given input on the hidden layer, and therefore on the network output, either decays or blows up exponentially as it cycles around the network's recurrent connections. This shortcoming (referred to in the literature as the *vanishing gradient problem* [37], [38]) makes it hard for an RNN to bridge gaps of more than about 10 time steps between relevant input and target events [37]. The vanishing gradient problem is illustrated schematically in Figure 10.

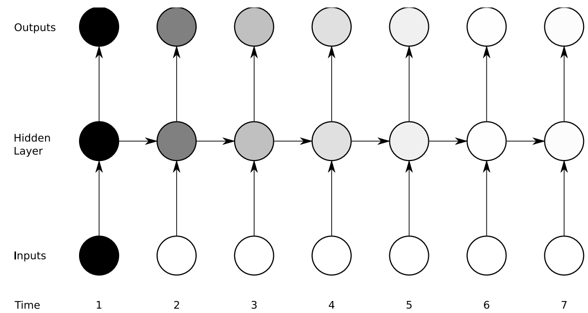

Fig. 10. Illustration of the vanishing gradient problem. The diagram represents a recurrent network unrolled in time. The units are shaded according to how sensitive they are to the input at time 1 (where black is high and white is low). As can be seen, the influence of the first input decays exponentially over time.

Long Short-Term Memory (LSTM) [39], [40] is an RNN architecture specifically designed to address the vanishing gradient problem. An LSTM hidden layer consists of recurrently connected subnets, called memory blocks. Each block contains a set of internal units, or cells, whose activation is controlled by three multiplicative gates: the input gate, forget gate and output gate. Figure 11 provides a detailed illustration of an LSTM memory block with a single cell.

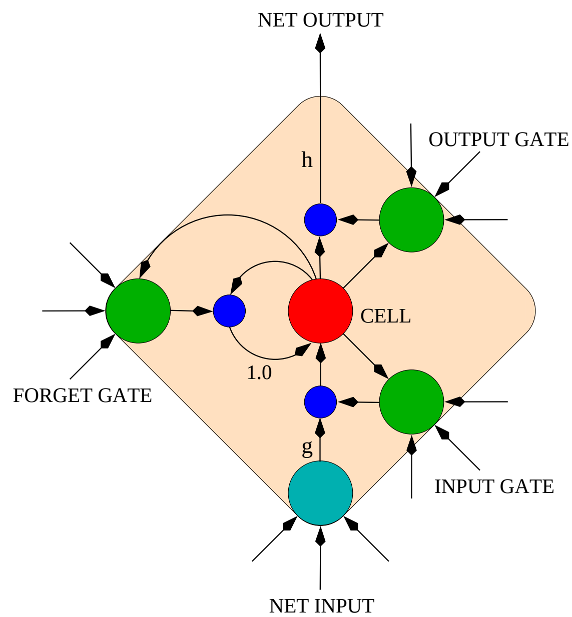

Fig. 11. LSTM memory block with one cell. The cell has a recurrent connection with fixed weight 1.0. The three gates collect input from the rest of the network, and control the cell via multiplicative units (small circles). The input and output gates scale the input and output of the cell, while the forget gate scales the recurrent connection of the cell. The cell squashing functions (g and h) are applied at the indicated places. The internal connections from the cell to the gates are known as *peephole weights*.

The effect of the gates is to allow the cells to store and access information over long periods of time. For example, as long as the input gate remains closed (i.e. has an activation close to 0), the activation of the cell will not be overwritten by the new inputs arriving in the network. Similarly, the cell activation is only available to the rest of the network when the output gate is open, and the cell's recurrent connection is switched on and off by the forget gate.

Figure 12 illustrates how an LSTM block maintains gradient information over time. Note that the dependency is 'carried' by the memory cell as long as the forget gate is open and the input gate is closed, and that the output dependency can be switched on and off by the output gate, without affecting the hidden cell.

C. Bidirectional Recurrent Neural Networks

For many tasks it is useful to have access to future as well as past context. In handwriting recognition, for example, the identification of a given letter is helped by knowing the letters both to the right and left of it.

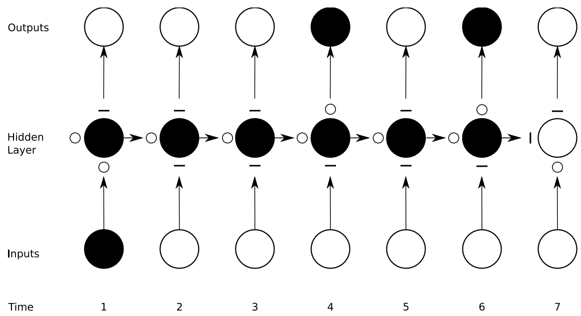

Fig. 12. Preservation of gradient information by LSTM. The diagram represents a network unrolled in time with a single hidden LSTM memory block. The input, forget, and output gate activations are respectively displayed below, to the left and above the memory block. As in Figure 10, the shading of the units corresponds to their sensitivity to the input at time 1. For simplicity, the gates are either entirely open ('O') or entirely closed ('—').

Bidirectional recurrent neural networks (BRNNs) [41], [42] are able to access context in both directions along the input sequence. BRNNs contain two separate hidden layers, one of which processes the input sequence forwards, while the other processes it backwards. Both hidden layers are connected to the same output layer, providing it with access to the past and future context of every point in the sequence. BRNNs have outperformed standard RNNs in several sequence learning tasks, notably protein structure prediction [43] and speech processing [41], [44].

Combining BRNNs and LSTM gives bidirectional LSTM (BLSTM). BLSTM has previously outperformed other network architectures, including standard LSTM, BRNNs and HMM-RNN hybrids, on phoneme recognition [33], [45].

D. Connectionist Temporal Classification (CTC)

Traditional RNN objective functions require a presegmented input sequence with a separate target for every segment. This has limited the applicability of RNNs in domains such as cursive handwriting recognition, where segmentation is difficult to determine. Moreover, because the outputs of such an RNN are a series of independent, local classifications, some form of post processing is required to transform them into the desired label sequence.

Connectionist temporal classification (CTC) [31] is an RNN output layer specifically designed for sequence labelling tasks. It does not require the data to be presegmented, and it directly outputs a probability distribution over label sequences. CTC has been shown to outperform both HMMs and RNN-HMM hybrids on a phoneme recognition task [31]. CTC can be used for any RNN architecture, though as we will discuss in a moment, it is better suited to some than others.

A CTC output layer contains as many units as there are labels in the task, plus an additional 'blank' or 'no label' unit. The output activations are normalised using the softmax function [46] so that they sum to 1 and are each in the range $(0, 1)$:

$$y_k^t = \frac{e^{a_k^t}}{\sum_{k'} e^{a_{k'}^t}}, \tag{1}$$

where a_k^t is the unsquashed activation of output unit k at time t, and y_k^t is the final output of the same unit.

The normalised outputs are used to estimate the conditional probabilities of observing label (or blank) k at time t in the input sequence $\mathbf{x}$, i.e. $y_k^t = p(k, t|\mathbf{x})$ (from now on we will use a bold font to denote sequences). Note that each output is conditioned on the entire input sequence. For this reason, CTC is best used in conjunction with an architecture capable of incorporating long range context in both input directions. One such architecture is bidirectional LSTM, as described in the previous section.

The conditional probability $p(\pi|\mathbf{x})$ of observing a particular *path* π through the lattice of label observations is found by multiplying together the label and blank probabilities at every time step:

$$p(\pi|\mathbf{x}) = \prod_{t=1}^{T} p(\pi_t, t|\mathbf{x}) = \prod_{t=1}^{T} y_{\pi_t}^t, \tag{2}$$

where π_t is the label observed at time t along path π.

Paths are mapped onto label sequences by an operator $\mathcal{B}$ that removes first the repeated labels, then the blanks. For example, both $\mathcal{B}(a, -, a, b, -)$ and $\mathcal{B}(-, a, a, -, -, a, b, b)$ yield the labelling (a,a,b). Since the paths are mutually exclusive, the conditional probability of some labelling $\mathbf{l}$ is the sum of the probabilities of all the paths mappend onto it by $\mathcal{B}$:

$$p(\mathbf{l}|\mathbf{x}) = \sum_{\pi \in \mathcal{B}^{-1}(\mathbf{l})} p(\pi|\mathbf{x}) \tag{3}$$

The above step is what allows the network to be trained with unsegmented data. The intuition is that, because we don't know where the labels within a particular transcription will occur, we sum over all the places where they *could* occur.

In general, a large number of paths will correspond to the same label sequence, so a naive calculation of (3) is unfeasible. However, it can be efficiently evaluated using a graph-based algorithm, similar to the forward-backward algorithm for HMMs [20].

E. CTC Forward Backward Algorithm

To allow for blanks in the output paths, we consider modified label sequences $\mathbf{l}'$, with blanks added to the beginning and the end of $\mathbf{l}$, and inserted between every pair of consecutive labels. The length of $\mathbf{l}'$ is therefore $2|\mathbf{l}| + 1$. In calculating the probabilities of prefixes of $\mathbf{l}'$ we allow all transitions between blank and non-blank labels, and also those between any pair of distinct non-blank labels.

For a labelling $\mathbf{l}$, define the *forward variable* α_s^t as the summed probability of all paths whose length t prefixes are mapped by $\mathcal{B}$ onto the length $s/2$ prefix of $\mathbf{l}$, i.e.

$$\alpha_s^t = \sum_{\substack{\pi: \\ \mathcal{B}(\pi_{1:t}) = \mathbf{l}_{1:s/2}}} \prod_{t'=1}^{t} y_{\pi_{t'}}^{t'}, \tag{4}$$

where, for some sequence $\mathbf{s}$, $\mathbf{s}_{a:b}$ is the subsequence $(\mathbf{s}_a, \mathbf{s}_{a+1}, ..., \mathbf{s}_{b-1}, \mathbf{s}_b)$, and $s/2$ is rounded down to an integer value. As we will see, α_s^t can be calculated recursively.

Allowing all paths to start with either a blank (b) or the first symbol in $\mathbf{l}$ (l_1), we get the following rules for initialisation:

$$\alpha_1^1 = y_b^1$$
$$\alpha_2^1 = y_{l_1}^1$$
$$\alpha_s^1 = 0, \ \forall s > 2$$

and recursion:

$$\alpha_s^t = y_{l'_s}^t \begin{cases} \sum_{i=s-1}^{s} \alpha_i^{t-1} & \text{if } l'_s = b \text{ or } l'_{s-2} = l'_s \\ \sum_{i=s-2}^{s} \alpha_i^{t-1} & \text{otherwise} \end{cases}$$

Note that $\alpha_s^t = 0 \ \forall s < |\mathbf{l}'| - 2(T-t) - 1$, because these variables correspond to states for which there are not enough time-steps left to complete the sequence.

The *backward variables* β_s^t are defined as the summed probability of all paths whose suffixes starting at t map onto the suffix of $\mathbf{l}$ starting at label $s/2$:

$$\beta_s^t = \sum_{\substack{\pi: \\ \mathcal{B}(\pi_{t:T}) = \mathbf{l}_{s/2:|\mathbf{l}|}}} \prod_{t'=t+1}^{T} y_{\pi_{t'}}^{t'} \tag{5}$$

The rules for initialisation and recursion of the backward variables are as follows

$$\beta_{|\mathbf{l}'|}^T = 1$$
$$\beta_{|\mathbf{l}'|-1}^T = 1$$
$$\beta_s^T = 0, \ \forall s < |\mathbf{l}'| - 1$$

$$\beta_s^t = \begin{cases} \sum_{i=s}^{s+1} \beta_i^{t+1} y_{l'_i}^{t+1} & \text{if } l'_s = b \text{ or } l'_{s+2} = l'_s \\ \sum_{i=s}^{s+2} \beta_i^{t+1} y_{l'_i}^{t+1} & \text{otherwise} \end{cases}$$

Note that $\beta_s^t = 0 \ \forall s > 2t$, because these variables correspond to unreachable states.

Finally, the label sequence probability is given by the sum of the products of the forward and backward variables at any time:

$$p(\mathbf{l}|\mathbf{x}) = \sum_{s=1}^{|\mathbf{l}'|} \alpha_s^t \beta_s^t. \tag{6}$$

F. CTC Objective Function

The CTC objective function is defined as the negative log probability of the network correctly labelling the entire training set. Let S be a training set, consisting of pairs of input and target sequences $(\mathbf{x}, \mathbf{z})$. Then the objective function O can be expressed as

$$O = -\sum_{(\mathbf{x},\mathbf{z}) \in S} \ln p(\mathbf{z}|\mathbf{x}). \tag{7}$$

The network can be trained with gradient descent by first differentiating O with respect to the outputs, then using backpropagation through time [47] to find the derivatives with respect to the network weights.

Noting that the same label (or blank) may be repeated several times for a single labelling $\mathbf{l}$, we define the set of positions where label k occurs as $lab(\mathbf{l}, k) = \{s : l'_s = k\}$, which may be empty. We then set $\mathbf{l} = \mathbf{z}$ and differentiate (6) with respect to the network outputs to obtain

$$\frac{\partial p(\mathbf{z}|\mathbf{x})}{\partial y_k^t} = \frac{1}{y_k^t} \sum_{s \in lab(\mathbf{z},k)} \alpha_s^t \beta_s^t \tag{8}$$

Substituting this into (7) gives

$$\frac{\partial O}{\partial y_k^t} = -\frac{1}{p(\mathbf{z}|\mathbf{x}) y_k^t} \sum_{s \in lab(\mathbf{z},k)} \alpha_s^t \beta_s^t. \tag{9}$$

To backpropagate the gradient through the output layer, we need the objective function derivatives with respect to the outputs a_k^t before the activation function is applied. For the softmax function

$$\frac{\partial y_{k'}^t}{\partial a_k^t} = y_{k'}^t \delta_{kk'} - y_{k'}^t y_k^t, \tag{10}$$

and therefore

$$\frac{\partial O}{\partial u_k^t} = y_k^t - \frac{1}{p(\mathbf{z}|\mathbf{x})} \sum_{s \in lab(\mathbf{z},k)} \alpha_s^t \beta_s^t \tag{11}$$

G. CTC Decoding

Once the network is trained, we would ideally transcribe some unknown input sequence $\mathbf{x}$ by choosing the labelling $\mathbf{l}^*$ with the highest conditional probability:

$$\mathbf{l}^* = \arg\max_{\mathbf{l}} p(\mathbf{l}|\mathbf{x}). \tag{12}$$

Using the terminology of HMMs, we refer to the task of finding this labelling as *decoding*. Unfortunately, we do not know of a tractable decoding algorithm that is guaranteed to give optimal results. However a simple and effective approximation is given by assuming that the most probable path corresponds to the most probable labelling:

$$\mathbf{l}^* \approx \mathcal{B}\left(\arg\max_{\pi} p(\pi|\mathbf{x})\right) \tag{13}$$

This is trivial to compute, since the most probable path is just the concatenation of the most active outputs at every time-step.

For some tasks we want to constrain the output labellings according to a grammar. For example, in continuous speech and handwriting recognition, the final transcriptions are usually required to form sequences of dictionary words. In addition it is common practice to use a language model to weight the probabilities of particular sequences of words.

We can express these constraints by altering the label sequence probabilities in (12) to be conditioned on some probabilistic grammar G, as well as the input sequence $\mathbf{x}$:

$$\mathbf{l}^* = \arg\max_{\mathbf{l}} p(\mathbf{l}|\mathbf{x}, G) \tag{14}$$

Note that absolute requirements, for example that $\mathbf{l}$ contains only dictionary words, can be incorporated by setting the probability of all sequences that fail to meet them to 0. Applying the basic rules of probability, we obtain

$$p(\mathbf{l}|\mathbf{x}, G) = \frac{p(\mathbf{l}|\mathbf{x}) p(\mathbf{l}|G) p(\mathbf{x})}{p(\mathbf{x}|G) p(\mathbf{l})} \tag{15}$$

where we have used the fact that $\mathbf{x}$ is conditionally independent of G given $\mathbf{l}$. If we assume that $\mathbf{x}$ is independent of G, (15) reduces to

$$p(\mathbf{l}|\mathbf{x}, G) = \frac{p(\mathbf{l}|\mathbf{x})p(\mathbf{l}|G)}{p(\mathbf{l})} \tag{16}$$

Note that this assumption is in general false, since both the input sequences and the grammar depend on the underlying generator of the data, for example the language being spoken. However it is a reasonable first approximation, and is particularly justifiable in cases where the grammar is created using data other than that from which $\mathbf{x}$ was drawn (as is common practice in speech and handwriting recognition, where separate textual corpora are used to generate language models).

If we further assume that, prior to any knowledge about the input or the grammar, all label sequences are equally probable, (14) reduces to

$$\mathbf{l}^* = \arg\max_{\mathbf{l}} p(\mathbf{l}|\mathbf{x})p(\mathbf{l}|G) \tag{17}$$

Note that, since the number of possible label sequences is finite (because both L and $|\mathbf{l}|$ are finite), assigning equal prior probabilities does not lead to an improper prior.

H. CTC Token Passing Algorithm

We now describe an algorithm, based on the *token passing algorithm* for HMMs [48], that allows us to find an approximate solution to (17) for a simple grammar.

Let G consist of a dictionary D containing W words, and a set of W^2 bigrams $p(w|\hat{w})$ that define the probability of making a transition from word $\hat{w}$ to word w. The probability of any label sequence that does not form a sequence of dictionary words is 0.

For each word w, define the modified word w' as w with blanks added at the beginning and end and between each pair of labels. Therefore $|w'| = 2|w| + 1$. Define a token $tok = (s, h)$ to be a pair consisting of a real valued score s and a history h of previously visited words. In fact, each token corresponds to a particular path through the network outputs, and the token score is the log probability of that path. The transition probabilities are used when a token is passed from the last character in one word to the first character in another. The output word sequence is then given by the history of the highest scoring end-of-word token at the final time step.

At every time step t of the length T output sequence, each character c of each modified word w' holds a single token $tok(w, c, t)$. This is the highest scoring token reaching that segment at that time. In addition we define the *input token* $tok(w, 0, t)$ to be the highest scoring token arriving at word w at time t, and the *output token* $tok(w, -1, t)$ to be the highest scoring token leaving word w at time t.

Pseudocode is provided in Algorithm 1.

1: **Initialisation:**
2: **for** all words $w \in D$ **do**
3: $\quad tok(w, 1, 1) = (\ln y_b^1, (w))$
4: $\quad tok(w, 2, 1) = (\ln y_{w_1}^1, (w))$
5: $\quad$ **if** $|w| = 1$ **then**
6: $\quad\quad tok(w, -1, 1) = tok(w, 2, 1)$
7: $\quad$ **else**
8: $\quad\quad tok(w, -1, 1) = (-\infty, ())$
9: $\quad tok(w, c, 1) = (-\infty, ())$ for all other l
10:
11: **Algorithm:**
12: **for** $t = 2$ to T **do**
13: $\quad$ sort output tokens $tok(w, -1, t-1)$ by rising score
14: $\quad$ **for** all words $w \in D$ **do**
15: $\quad\quad w^* = \arg\max_{\hat{w}} [tok(\hat{w}, -1, t-1).s + \ln p(w|\hat{w})]$
16: $\quad\quad tok(w, 0, t).s = tok(w^*, -1, t-1).s + \ln p(w|w^*)$
17: $\quad\quad tok(w, 0, t).h = tok(w^*, -1, t-1).h + w$
18: $\quad\quad$ **for** $c = 1$ to $|w'|$ **do**
19: $\quad\quad\quad P = \{tok(w, c, t-1), tok(w, c-1, t-1)\}$
20: $\quad\quad\quad$ **if** $w'_c \neq blank$ and $c > 2$ and $w'_{c-2} \neq w'_c$ **then**
21: $\quad\quad\quad\quad$ add $tok(w, c-2, t-1)$ to P
22: $\quad\quad\quad tok(w, c, t) =$ token in P with highest score
23: $\quad\quad\quad tok(w, c, t).s \mathrel{+}= \ln y_{w'_c}^t$
24: $\quad\quad R = \{tok(w, |w'|, t), tok(w, |w'|-1, t)\}$
25: $\quad\quad tok(w, -1, t) =$ token in R with highest score
26:
27: **Termination:**
28: $w^* = \arg\max_w tok(w, -1, T).s$
29: output $tok(w^*, -1, T).h$

Algorithm 1: CTC Token Passing Algorithm

The CTC token passing algorithm has a worst case complexity of $O(TW^2)$, since line 15 requires a potential search through all W words. However, because the output tokens $tok(w, -1, T)$ are sorted in order of score, the search can be terminated when a token is reached whose score is less than the current best score with the transition included. The typical complexity is therefore considerably lower, with a lower bound of $O(TWlogW)$ to account for the sort. If no bigrams are used, lines 15-17 can be replaced by a simple search for the highest scoring output token, and the complexity reduces to $O(TW)$.

V. Experiments and Results

The aim of our experiments was to evaluate the complete RNN handwriting recognition system, illustrated in Figure 13, on both online and offline handwriting. In particular we wanted to see how it compared to a state-of-the-art HMM-based system. The online and offline databases used were the IAM-OnDB and the IAM-DB respectively. Note that these do not correspond to the same handwriting samples: the IAM-OnDB was acquired from a whiteboard (see Section II), while the IAM-DB consists of scanned images of handwritten forms (see Section III).

To make the comparisons fair, the same online and offline preprocessing was used for both the HMM and RNN systems. In addition, the same dictionaries and language models were used for the two systems—see Section V-B for further details. As well as the main comparisons, extra experiments were carried out on the online database to determine the effect of varying the dictionary size, and of disabling the forward and backward hidden layers in the RNN.

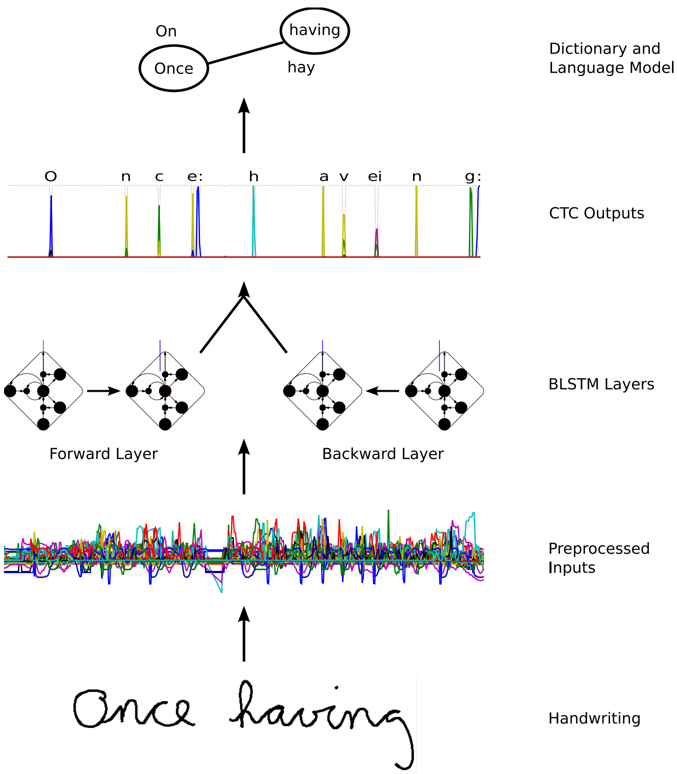

Fig. 13. The complete RNN handwriting recognition system. First the online or offline handwriting data is preprocessed with the techniques described in Sections II and III. The resulting sequence of feature vectors is scanned in opposite directions by the forwards and backwards BLSTM hidden layers. The BLSTM layers feed forward to the CTC output layer, which produces a probability distribution over character transcriptions. This distribution is passed to the dictionary and language model, using the token passing algorithm, to obtain the final word sequence.

For all the experiments, the task was to transcribe the text lines in the test set using the words in the dictionary. The basic performance measure was the *word accuracy*

$$acc = 100 * \left(1 - \frac{\text{insertions} + \text{substitutions} + \text{deletions}}{\text{total length of test set transcriptions}}\right)$$

where the number of word insertions, substitutions and deletions is summed over the whole test set. For the RNN system, we also recorded the *character accuracy*, defined as above except with characters instead of words.

A. Data Sets

For the online experiments we used the IAM-OnDB, a database acquired from a 'smart' whiteboard [49]. The database was divided into four disjoint sets, as defined by the IAM-OnDB-t2 benchmark task: a training set containing 5,364 lines; a first validation set containing 1,438 lines; a second validation set containing 1,518 lines which can be used, for example, to optimise a language model; and a test set containing 3,859 lines. After preprocessing, the input data consisted of 25 inputs per time step.

For the offline experiments we used the IAM-DB, a database acquired from scanned images of handwritten forms [36]. The IAM-DB consists of 13,040 fully transcribed handwritten lines, containing 86,272 instances of 11,050 distinct words. The division into datasets was defined by the benchmark 'large writer independent text line recognition task'[3]. The training set contains 6,161 lines from 283 writers, the validation set contains 920 lines from 56 writers, and the test set contains 2,781 lines from 161 writers. After preprocessing, the input data consisted of 9 inputs per time step.

Both databases were created using texts from the Lancaster-Oslo/Bergen (LOB) corpus [50] as prompts. Note however that the online and offline prompts were not the same.

Both the online and offline transcriptions contain 81 separate characters, including all lower case and capital letters as well as various other special characters, e.g., punctuation marks, digits, a character for garbage symbols, and a character for the space. Note that for the RNN systems, only 80 of these were used, since the garbage symbol is not required for CTC.

B. Language Model and Dictionaries

Dictionaries were used for all the experiments where word accuracy was recorded. All dictionaries were derived from three different text corpora, the LOB (excluding the data used as prompts), the Brown corpus [52], and the Wellington corpus [53]. The 'standard' dictionary we used for our main results consisted of the 20,000 most frequently occurring words in the three corpora. The figure 20,000 was chosen because it had been previously shown to give best results for HMMs [51]. Note that this dictionary was 'open', in the sense that it did not contain all the words in either the online or offline test set. To analyse the dependency of the recognition scores on the lexicon, we carried out extra experiments on the online data using open dictionaries with between 5,000 and 30,000 words (also chosen from the most common words in LOB). In addition, we tested both systems with two dictionaries that were 'closed' with respect to the online data (i.e. that contained every word in the online test set).

The bigram language model, used for some of the experiments, was based on the same corpora as the language model. It was then optimised for the online and offline experiments separately, using data from the validation sets. Note that, for the RNN system, the effect of the language model is to directly multiply the existing word sequence probabilities by the combined transition probabilities given by the bigrams. This contrasts with HMMs, where the language model weighting factor must be found empirically because HMMs are known to underestimate acoustic scores [54].

C. HMM Parameters

The HMM system used for the online and offline experiments was similar to that described in [35]. One linear HMM was used for each character. For the online data every character model contained eight states, while for the offline

[3]Both databases are available for public download, along with the corresponding task definitions
http://www.iam.unibe.ch/fki/databases/iam-handwriting-database
http://www.iam.unibe.ch/fki/databases/iam-on-line-handwriting-database

data, the number of states was chosen individually for each character [51]. The observation probabilities were modelled by mixtures of diagonal Gaussians. 32 Gaussians were used for online data and 12 for the offline data. In both cases the number of Gaussians was optimised by incrementally splitting the Gaussian component with the highest weight. The language model weighting factor and the word insertion penalty were determined empirically on the validation set.

D. RNN Parameters

The RNN had a bidirectional Long Short-Term Memory (BLSTM) architecture with a connectionist temporal classification (CTC) output layer (see Section IV for details). The forward and backward hidden layers each contained 100 LSTM memory blocks. Each memory block contains a memory cell, an input gate, an output gate, a forget gate and three peephole connections. Hyperbolic tangent was used for the block input and output activation functions, and the gate activation function was the logistic sigmoid. The CTC output layer had 81 nodes (one for each character occurring in the training set, and one extra for 'blank'). The size of the input layer was determined by the data: for the online data there were 25 inputs, for the offline data there were 9. Otherwise the network was identical for the two tasks. The input layer was fully connected to both hidden layers, and these were fully connected to themselves and to the output layer. This gave 117,681 weights for the online data and 105,082 weights for the online data.

The network weights were initialised with a Gaussian distribution of mean 0 and standard deviation 0.1. The network was trained using online gradient descent with a learning rate of 0.0001 and a momentum of 0.9. The error rate was recorded every 5 epochs on the validation set and training was stopped when performance had ceased to improve on the validation set for 50 epochs. Because of the random initialisation, all RNN experiments were repeated four times, and the results are stated as the mean $\pm$ the standard error.

E. Main Results

As can be seen from Tables I and II, the RNN substantially outperformed the HMM on both databases. To put these results in perspective, the Microsoft tablet PC handwriting recogniser [55] gave a word accuracy score of 71.32% on the online test set. This result is not directly comparable to our own, since the Microsoft system was trained on a different training set, and uses considerably more sophisticated language modelling than the systems we implemented. However, it suggests that our recogniser is competitive with the best commercial systems for unconstrained handwriting.

System	Word Accuracy	Char. Accuracy
HMM	65.0%	—
CTC	79.7 ± 0.3%	88.5 ± 0.05%

TABLE I
MAIN RESULT FOR ONLINE DATA

System	Word Accuracy	Char. Accuracy
HMM	64.5%	—
RNN	74.1 ± 0.8%	81.8 ± 0.6%

TABLE II
MAIN RESULT FOR OFFLINE DATA

F. Influence of Dictionary Size

We carried out two sets of experiments on the online database to gauge the effect of varying the dictionary size. In one we generated open dictionaries with between 5,000 and 30,000 words by taking the n most common words in the LOB, Brown and Wellington corpora. In the other, we took a closed dictionary containing the 5,597 words in the online test set, and measured the change in performance when this was padded to 20,000 words. Note that the language model was not used for the RNN results in the section, which would be substantially higher otherwise (for example, the accuracy for the 20,000 word open lexicon was 79.7% with the language model and 74.0% without).

The results for the first set of experiments are shown in Table III and plotted in Figure 14. In all cases the RNN system significantly outperforms the HMM system, despite the lack of language model. Both RNN and HMM performance increased with size (and test set coverage) up to 25,000 words. Note however that the RNN is less affected by the dictionary size, and that for the 30,000 word dictionary, performance continues to increase for the RNN but drops for the HMM. The tendency of HMMs to lose accuracy for very large handwriting lexicons has been previously observed [51].

Dictionary Size	Coverage(%)	RNN(%)	HMM(%)
5,000	86.1	66.2 ± 0.4	47.7
10,000	90.7	70.3 ± 0.4	54.8
15,000	92.9	72.5 ± 0.3	60.6
20,000	94.4	74.0 ± 0.3	63.7
25,000	95.1	74.6 ± 0.3	65.0
30,000	95.7	75.0 ± 0.3	61.5

TABLE III
ONLINE WORD ACCURACY WITH OPEN DICTIONARIES

The results of the second set of experiments are shown in Table IV. Unsurprisingly, the closed dictionary containing only the test set words gave the best results for both systems. The scores with the 20,000 word closed dictionary were somewhat lower in both cases, due to the increased perplexity, but still better than any recorded with open dictionaries.

Dictionary Size	RNN(%)	HMM(%)
5,597	85.3 ± 0.3	70.1
20,000	81.5 ± 0.4	68.8

TABLE IV
ONLINE WORD ACCURACY WITH CLOSED DICTIONARIES

In summary the RNN retained its advantage over the HMM regardless of the dictionary used. Moreover, the differences

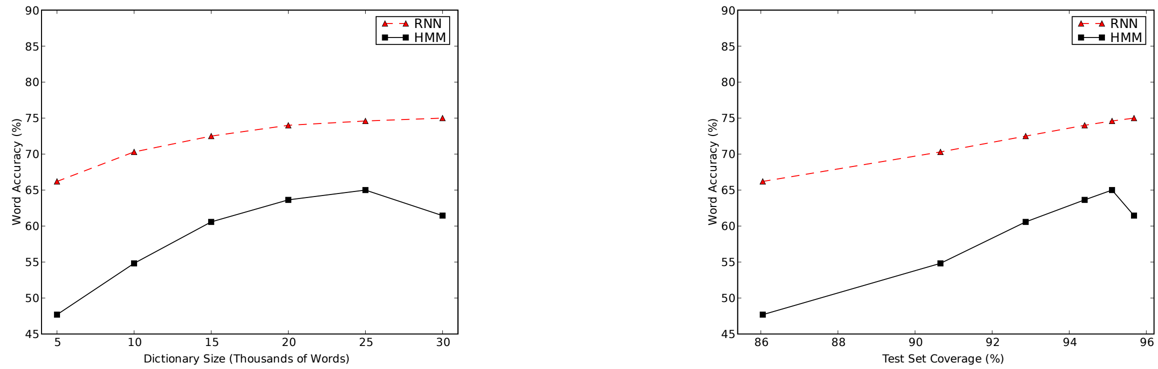

Fig. 14. HMM and RNN word accuracy plotted against dictionary size (left) and test set coverage (right)

tended to be larger when the dictionary had higher perplexity. We believe this is because the RNN is better at recognising characters, and is therefore less dependent on a dictionary or language model to constrain its outputs.

G. *Influence of RNN Hidden Layers*

To test the relative importance of the forward and backward hidden layers, we evaluated the RNN system on the online database with each of the hidden layers disabled in turn. We give the character error rate only, since this is the clearest indicator of network performance. The results are displayed in Table V. It is interesting to note that significantly higher accuracy was achieved with a reverse layer only than with a forward layer only (all differences are significant using a standard z-test with $\alpha < 0.001$). This suggests that the right-to-left dependencies are more important to the network than the left-to-right ones.

Architecture	Character Accuracy	Training Iterations
Forward Only	81.3 ± 0.3%	182.5 ± 24.3
Reverse Only	85.8 ± 0.3%	228.8 ± 51.7
Bidirectional	88.5 ± 0.05%	41.25 ± 2.6

TABLE V
ONLINE CHARACTER ACCURACY WITH DIFFERENT RNN HIDDEN LAYERS

H. *Learning Curve*

Figure 15 shows the decrease in training and validation error over time for a typical RNN training run on the online database.

I. *Use of Context by the RNN*

We have previously asserted that the BLSTM architecture is able to access long-range, bidirectional context. Evidence of this can be found by analysing the partial derivatives of the network outputs at a particular time t in a sequence with respect to the inputs at all times t'. We refer to the matrix of these derivatives as the *sequential Jacobian*. The larger the values of the sequential Jacobian for some t, t', the more sensitive the network output at time t is to the input at time t'.

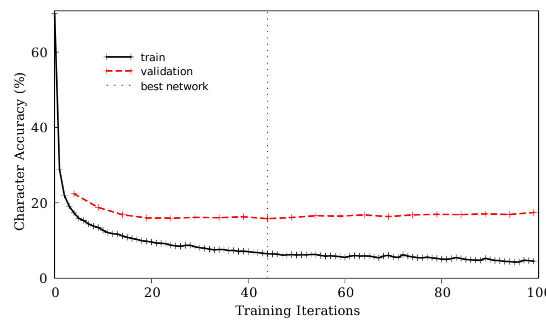

Fig. 15. RNN character error rate during training. The performance ceased to improve on the validation set after 45 passes through the training set (marked with 'best network')

Figure 16 plots the sequential Jacobian for a single output during the transcription of a line from the online database. As can be seen, the network output is sensitive to information from about the first 120 time steps of the sequence, which corresponds roughly to the length of the first word. Moreover, this area of sensitivity extends in both directions from the point where the prediction is made.

VI. Discussion

Our experiments reveal a substantial gap in performance between the HMM and RNN systems, with a relative error reduction of over 40% in some cases. In what follows, we discuss the differences between the two systems, and suggest reasons for the RNN's superiority.

Firstly, standard HMMs are generative, while an RNN trained with a discriminative objective function (such as CTC) is discriminative. That is, HMMs attempts to model the conditional probability of the input data given the internal state sequence, then use this to find the most probable label sequence, while the RNN directly models the probability of the label sequence given the inputs. The advantages of the generative approach include the possibility of adding extra models to an already trained system, and being able to generate synthetic data. However, for discriminative tasks,

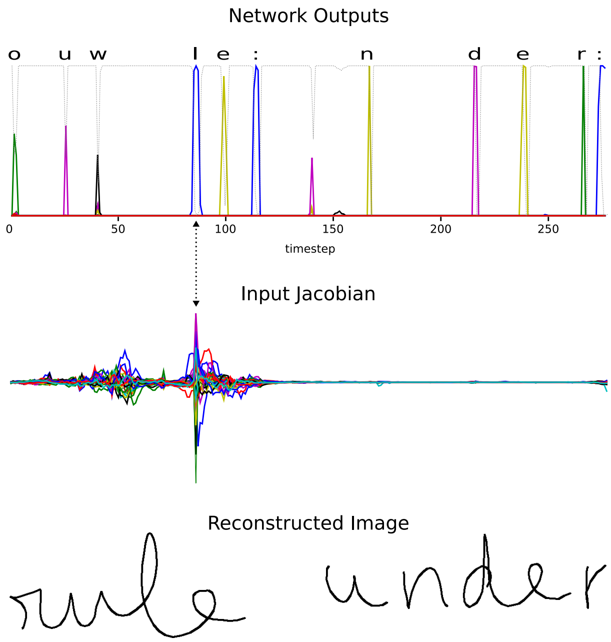

Fig. 16. Sequential Jacobian for a sequence from the IAM-OnDB. The Jacobian is evaluated for the output corresponding to the label 'l' at the time step when 'l' is emitted (indicated by the vertical dotted line).

determining the input distribution is unnecessary. Additionally, for tasks such as handwriting recognition where the prior data distribution is hard to determine, generative approaches can only provide unnormalised likelihoods for the label sequences. Discriminative approaches, on the other hand, yield normalised label probabilities, which can be used to assess prediction confidence, or to combine the outputs of several classifiers. In most cases, discriminative methods achieve better performance than generative methods on classification tasks.

A second difference is that RNNs provide more flexible models of the input features than the mixtures of diagonal Gaussians used in standard HMMs. Gaussian's with diagonal covariance matrices are limited to modelling distributions over independent variables. This assumes that the input features are decorrelated, which can be difficult to ensure for real world tasks such as handwriting recognition. RNNs, on the other hand, do not assume that the features come from a particular distribution, or that they are independent, and can model non-linear relationships among features. However, it should be noted that RNNs typically perform better using input features with simpler relationships.

A third difference is that the internal states of a standard HMM are discrete and univariate. This means that for an HMM with n states, only $O(logn)$ bits of information about the past observation sequence are carried by the internal state. RNNs, on the other hand, have a continuous, multivariate internal state (the hidden layer) whose information capacity grows linearly with its size.

A fourth difference is that HMMs are constrained to segment the input into a sequence of states or units. This is often problematic for continuous input sequences, since the precise boundary between units can be ambiguous. A further problem with segmentation is that, at least with standard Markovian transition probabilities, the probability of remaining in a particular state decreases exponentially with time. Exponential decay is in general a poor model of state duration, and various measures have been suggested to alleviate this [56]. However, an RNN trained with CTC does not need to segment the input sequence, and therefore avoids both of these problems.

A final, and perhaps most crucial, difference is that unlike RNNs, HMMs assume that the probability of each observation depends only on the current state. One consequence of this is that data consisting of continuous trajectories (such as the sequence of pen coordinates for online handwriting, and the sequence of window positions in offline handwriting) are difficult to model with standard HMMs, since each observation is heavily dependent on those around it. Similarly, data with long-range contextual dependencies is troublesome, because individual sequence elements are influenced by the elements surrounding them. The latter problem can be mitigated by adding extra models to account for each sequence element in all different contexts (e.g., using triphones instead of phonemes for speech recognition). However, increasing the number of models exponentially increases the number of parameters that must be inferred which, in turn, increases the amount of data required to reliably train the system. For RNNs on the other hand, modelling continuous trajectories is natural, since their own hidden state is itself a continuous trajectory. Furthermore, the range of contextual information accessible to an RNN is limited only by the choice of architecture, and in the case of BLSTM can in principle extend to the entire input sequence.

In summary, the observed difference in performance between RNNs and HMM in unconstrained handwriting recognition can be explained by the fact that, as researchers approach handwriting recognition tasks of increasing complexity, the assumptions HMMs are based on lose validity. For example, unconstrained handwriting or spontaneous speech are more dynamic and show more marked context effects than hand-printed scripts or read speech.

VII. Conclusions

We have introduced a novel approach for recognising unconstrained handwritten text, using a recurrent neural network. The key features of the network are the bidirectional Long Short-Term Memory architecture, which provides access to long range, bidirectional contextual information, and the connectionist temporal classification output layer, which allows the network to be trained on unsegmented sequence data. In experiments on online and offline handwriting data, the new approach outperformed a state-of-the-art HMM-based system, and also proved more robust to changes in dictionary size.

Acknowledgments

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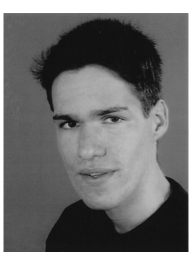

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