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Cost-benefit analysis for optimization of risk protection under budget constraints

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Abstract

Cost-benefit analysis (CBA) is commonly applied as a tool for deciding on risk protection. With CBA, one can identify risk mitigation strategies that lead to an optimal trade-off between the costs of the mitigation measures and the achieved risk reduction. In practical applications of CBA, the strategies are typically evaluated through efficiency indicators such as the benefit-cost ratio (BCR) and the marginal cost (MC) criterion. In many of these applications, the BCR is not consistently defined, which, as we demonstrate in this paper, can lead to the identification of sub-optimal solutions. This is of particular relevance when the overall budget for risk reduction measures is limited and an optimal allocation of resources among different subsystems is necessary. We show that this problem can be formulated as a hierarchical decision problem, where the general rules and decisions on the available budget are made at a central level (e.g. central government agency, top management), whereas the decisions on the specific measures are made at the subsystem level (e.g. local communities, company division). It is shown that the MC criterion provides optimal solutions in such hierarchical optimization. Since most practical applications only include a discrete set of possible risk protection measures, the MC criterion is extended to this situation. The findings are illustrated through a hypothetical numerical example. This study was prepared as part of our work on the optimal management of natural hazard risks, but its conclusions also apply to other fields of risk management.

Keywords

Risk-based decision-making; risk protection optimization; cost-benefit analysis; marginal cost criterion; benefit-cost ratio; natural hazards.

1 Introduction

Cost-benefit analysis (CBA) can be used for the identification of risk mitigation strategies that provide an optimal trade-off between the cost of the implemented measures and the achieved risk reduction. It is widely applied in various fields of engineering, health management and policy making. Exemplarily, CBA is used for economic evaluation of natural hazard mitigation projects in the USA (Rose et al. 2007), Great Britain (Defra, 2009), Switzerland (Bründl, 2009; Bründl et al., 2009), Austria (lebensministerium.at, 2009) and in developing countries (United Nations, 2011; Hochrainer-Stigler et al., 2011). Other examples of engineering applications of CBA include risk-based optimization of climate change adaptation of offshore structures (Garré and Friis-Hansen, 2013) or the management of man-made risks such as the risk of fire related to transport of hazardous material (Paltrinieri et al., 2012). CBA is also applied to assess the effect of policies and regulations, e.g. on terrorist prevention measures in aviation (Willis and LaTourrette, 2008; Stewart and Mueller, 2012), on retrofitting of buildings to reduce the impact of earthquakes (Li et al., 2009), on air pollution (Nemet et al., 2010; Fann et al., 2011) or on testing and use of pharmaceutics (Meckley et al., 2010).

CBA is limited by its focus on the economic efficiency of risk protection measures. Many aspects, such as the value of human life or environmental and social impacts of measures, cannot be easily quantified in monetary terms for inclusion in CBA (Ramsberg, 2000). Alternatives have therefore been proposed, such as multi-criteria analysis (MCA) that allows considering different attributes without monetizing them (Mysiak et al., 2005; ECA, 2009). However, in spite of its limitations, CBA supports the fair distribution of resources for risk protection in society (Paté-Cornell, 2002; Cox, 2012; Michel-Kerjan et al., 2012). This will remain essential for society in the future, as the frequency of natural and man-made hazards, as well as their potential consequences, are likely to increase (Johnson et al., 2007; Bonstrom et al., 2011) and the resources that can be invested into risk protection remain limited.

This paper presents an overview on CBA for the economic optimization of risk protection, with special focus on engineering applications. The study is motivated by our work on development of a methodology for planning and optimization of flood risk measures in the Bavarian Alps (Špačková et al., 2014). A majority of the methods and examples presented in this paper thus relate to risk posed by floods and related natural hazards. The findings of this paper are nevertheless general and are valid for many fields of risk management.

The paper considers both continuous and discrete optimization. The former is applied when cost and risk can be expressed as continuous functions of the optimization parameters. This situation is often discussed in the literature and in textbooks, but in engineering practice it is typically limited to subsidiary decision problems. In most real-life applications, only a countable number of risk mitigation strategies can be analysed. For example, when planning flood protection of a town, a first strategy can correspond to building an 800 m long dike and a flood storage reservoir with capacity of 2×10^5 m^3 and a second strategy can correspond to building no dike but a larger reservoir with capacity of 3×10^5 m^3. Many additional strategies with other measures and/or other parameter values are feasible, and the analysis of each strategy implies computationally demanding model evaluations. The space of possible solutions is high-dimensional. The engineers will therefore preselect a set of discrete strategies based on experience. Each of these strategies is associated with specific values of (residual) risk and expected costs.

In the field of natural hazard protection, CBA is often implemented through the calculation of efficiency measures, such as the benefit-cost ratio (BCR) and the marginal cost (MC) criterion, which are reviewed in Section 3. These measures allow prioritizing the risk protection strategies when the overall available budget is limited. BCR and MC are not commonly formulated in a rigorous manner, neither in practical guidelines nor in the scientific literature, which gives rise to inconstancies in their application. Particularly the BCR is defined differently from one country to another and from one area of application to another. For example, the benefits and costs are in some instances compared against the current level of protection, and in other instances against a hypothetical do-nothing option. This impedes the use of BCR as a measure for comparing the efficiency of mitigation strategies in different areas of risk protection.

The differences and inconsistencies in the application of CBA stem not least from the fact that risk assessment experts have different backgrounds: Experts in structural reliability use different methods than those dealing with natural hazard protection and those are again different to those used by health risk analysts. In a CBA carried out by engineers or life scientists, the economic optimization is typically perceived as a minor part of the analysis. The analysts focus on the detailed quantitative analysis of the hazard processes, while only limited efforts are devoted to the estimation of the damage, the quantification of the risk and the interpretation of the results for the actual decision (Messner and Meyer, 2005; Merz et al., 2010). As a result, the way CBA is used in engineering assessments is often in contradiction to the economic theory.

In this paper, we first discuss the optimization of risk protection in a single subsystem (Section 2.1) and thereafter extend the problem to the selection of a set of strategies in a system (Section 2.2). The subsystems can be, for example, individual municipalities or river catchments that are administered by the same state agency, or different plants owned by a company. The problem is treated as a hierarchical optimization, where the individual subsystems compete for the resources distributed from a common budget. The aim of the optimization is finding optimal risk levels in the individual subsystems from the point of view of the owner/administrator. As demonstrated in Section 3, the commonly applied efficiency measures BCR and MC can be utilized as coordination parameters in the hierarchical system optimization. However, only the MC leads to a theoretically optimal selection of risk mitigation strategies at the system level. The effect of different choices of efficiency measures on the resulting overall risk and cost at the system level is illustrated through a numerical example in Section 4. Finally, the limitations of the proposed approach and the challenges for future research are discussed in Section 5.

2 Risk optimization with cost-benefit analysis

The cost-benefit analysis (CBA) concept is commonly used for the purpose of optimizing risk protection measures. The optimal strategy is then defined as the one that maximizes the difference between expected benefits and expected costs. The risk, which is defined as expected damage, can be considered as an additional expected cost, and the objective function becomes:

$$\max_{\mathbf{a}} [B(\mathbf{a}) - C(\mathbf{a}) - R(\mathbf{a})] \tag{1}$$

where $\mathbf{a}$ is a set of optimization parameters, $B(\mathbf{a})$ and $C(\mathbf{a})$ are the expected net present values of benefits and costs, and $R(\mathbf{a})$ is the net present value of risk. The net present values aggregate the benefits, costs and risks over the planning horizon. The discounting procedure and selection of discount rate is not explicitly described in this paper, for a more detailed discussion of this topic we refer to (Brent, 1996; Just et al., 2004; Rackwitz, 2004; Groom et al., 2005; Nishijima et al., 2007; Gollier and Weitzman, 2010).

The risk $R(\mathbf{a})$ is defined as the expected value of the damages caused by the analysed hazards, and costs $C(\mathbf{a})$ are the expected value of the cost for construction (establishment), operation and maintenance of the mitigation measures. These two types of expenses are commonly paid from different sources. In case of natural hazards, the costs of mitigation measures are commonly included in government investment expenditures, while the risk is typically covered by special governmental funds, insurance companies and the private sector. In such cases it is useful to clearly distinguish the risk and costs throughout the analysis.

The benefit B of the activity is often assumed to be independent of the risk protection measures described by optimization parameters $\mathbf{a}$ and is outside of the scope of the analysis. $B(\mathbf{a})$ is thus considered as constant with $\mathbf{a}$ and is neglected. The optimization problem is then formulated as

$$\min_{\mathbf{a}} [C(\mathbf{a}) + R(\mathbf{a})] \tag{2}$$

CBA assumes that both the risks and costs are expressed in monetary units. Alternatively, the optimization can be formulated within the framework of the expected utility principle (Neumann and Morgenstern, 1944; Keeney and Raiffa, 1993). This facilitates accounting for the risk attitude of the decision-maker, in particular risk aversion (Paté-Cornell, 2002; Jonkman et al., 2003; Ditlevsen, 2003). It also enables the inclusion of alternative criteria, e.g. environmental and societal objectives, into a single objective function (Li et al., 2009). The approaches and findings presented in this paper are also applicable if risk and costs are expressed as expected utility and not in monetary units.

In many instances, the decisions on risk protection are made in a hierarchical manner, whereby the decisions on the available budget are taken at a higher/central level (central government agency, top management), whereas the decisions on the specific measures are taken at a lower/local level. In this case, the optimal allocation of resources for risk protection can be considered as a hierarchical optimization problem, where the objectives are formulated at the system level, but the actual decisions are made at the level of sub-systems, possibly at different times. Because of this distributed decision process, it is not possible to optimize all risk protection measures described by parameters $\mathbf{a}$ jointly. Instead, the aim is to find a set of criteria, defined at the system level and applied in the sub-systems, which ensure optimal risk protection in the entire system.

In the following Section 2.1, the optimization problem is first described at the level of subsystems, since this is the formulation that is typically used in the field of risk protection optimization. Thereafter, Section 2.2 presents the formulation of the optimization problem at the system level.

2.1 Optimization at the level of a subsystem

A subsystem is here understood as a part of a system, where the risk mitigation measures can be planned separately from other parts of the system. The risk protection measures applied in a subsystem do not influence the level of risk in other subsystems. The only connection between the subsystems is the financial budget that they share, which introduces dependence among them. In the case of flood risk, subsystems are e.g. individual municipalities or river catchments that are administered by the same state agency; in the case of risks to an oil and gas operator, individual offshore oil fields can represent such subsystems. Since both costs C and risks R are expected values, the framework is also valid if there is stochastic dependence among the hazards in different subsystems.

2.1.1 Continuous formulation

The optimization of risk protection at the subsystem level is formulated first for the case of a continuous optimization parameter representing the protection level l, which is the case that is mostly described in the literature and textbooks. Examples of continuous optimization problems include the selection of an optimal height of a coastal dyke (Danzig, 1956), the decision on the optimal raise of an offshore platform deck as an adaptation measure to climate change (Garré and Friis-Hansen, 2013) or the selection of an optimal bridge maintenance frequency (Fischer et al., 2013). These applications all have only one optimization parameter defining the protection level.

In accordance with Eq. (*2*) the optimal level of risk protection can be identified by minimizing the sum of the net present values of risk $R(l)$ and of expected costs $C(l)$:

$$\min_{l}[R(l) + C(l)] \tag{3}$$

The optimization problem can be constrained by a maximal available budget C_{max}. Then the optimization of Eq. (*3*) is subject to $C(l) \leq C_{max}$.

Two alternative graphical representations of the continuous optimization are shown in Figure 1. In Figure 1(a), risk and cost are plotted against the protection level; in Figure 1(b), risk is plotted against cost. The black dots in both figures represent the unconstrained optimum, Eq. (*3*), and the constrained optimum, which is subject to the budget constraint C_{max}.

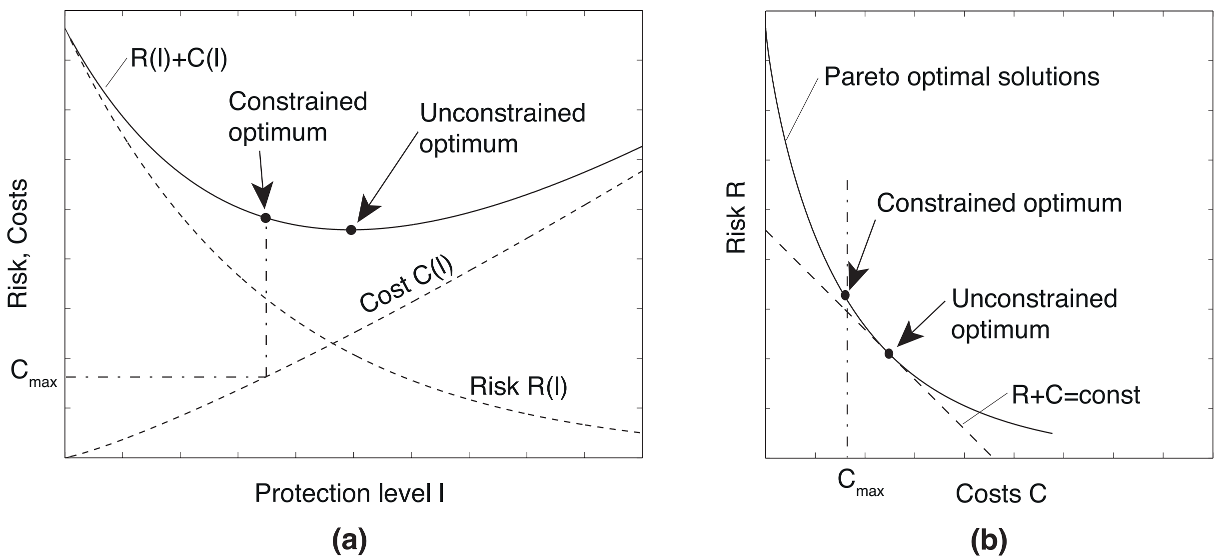

Figure 1. Alternative illustrations of the continuous risk optimization problem.

In Figure 1(a), the dependence of risk and cost on the protection level l is explicitly illustrated and the solid line represents their sum that is to be minimized. This representation requires an unambiguous definition of the protection level and it is thus applicable if only one optimization parameter is considered (e.g. dyke height, air-gap in an offshore platform) or if the protection level can be related to a single parameter describing the safety of the optimized system (e.g. safety factor, design return period, probability of failure). In many cases, however, it is not straightforward to define a protection level as a scalar variable, because different elements of the subsystem can have different protection levels. Additionally, even if the protection level is applicable, it often does not unambiguously determine the risk and cost of the protection measures. For example, the protection against a design flood defined by its return period (the so-called N-year flood) can be achieved with different types of measures. These will exhibit different costs and typically lead to different residual risks, because their failure mechanisms in case of a flood with higher return period will differ. If flood protection is ensured by a dam, a failure of the dam associated with erosion will lead to significant consequences immediately following the failure. If, on the other hand, the protection is ensured by a sufficiently wide river bed, the consequences of exceeding the design flood will be increasing only slowly with the flood level.

If the introduction of the protection level is not possible and helpful, the alternative representation of Figure 1(b) is more appropriate. This illustration depicts directly the residual risk as a function of cost. The set of Pareto optimal solutions represented by the solid line delineates the domain of feasible optimization alternatives. Note that the solid line in Figure 1(b) represents the same set of solutions as the solid line in Figure 1(a), because in the illustration of Figure 1(a) it is implicitly assumed that the costs (and risk) associated with each protection level are minimal.

The set of Pareto optimal solutions is here defined as all mitigation options for which there are no other options that have simultaneously lower costs and lower risks. Note that this is the definition of Pareto optimality found in engineering applications (Misra, 2008; Reed et al., 2013), which relaxes the rigorous conditions commonly used in the socio-economic theory. Following the socio-economic definition, a Pareto optimal solution corresponds to a state where nobody is ultimately made worse off. In engineering applications, one typically does not test the change on the level of individuals and it is not assured that all individuals are compensated for their potential loses. Strictly, one should thus speak of the Kaldor-Hicks efficiency (Corkindale, 2007), but this term is not well known in the engineering field. Hence we continue to use the term Pareto optimality.

In the unconstrained case, the optimal solution corresponds to the point in the domain of the feasible solutions that minimizes the sum of risk and costs. Graphically this can be illustrated by plotting a line through this optimum with $R + C = const$ (the dashed line in Figure 1b). There is no feasible solution to the left of this line. If the optimization is constrained by the available budget, the feasible space in Figure 1(b) is restricted by the vertical line at C_{max} . The solutions that are to the right of this line are not feasible because their costs exceed the available budget.

2.1.2 Discrete formulation

In the scientific literature, it is commonly assumed that explicit functions for cost $C(l)$ and risk $R(l)$ as a function of the protection level l can be established. However, as discussed in Section 2.1.1, it is often not possible or reasonable to define such a protection level. Furthermore, determining the Pareto optimal set depicted in Figure 1(b) is too demanding in most practical applications. This is because a large variety of different protection measures, and combinations thereof, can be implemented (e.g. dykes combined with retention areas, mobile flood barriers, warning systems) and each measure has one or more parameters to be optimized (height of a dyke, volume of the retention, type of the mobile barriers etc.). In such cases it is only realistic to evaluate risk and cost for a countable number of protection strategies, each one consisting of a set of measures. Therefore, the continuous optimization is replaced by a discrete optimization, which is illustrated in Figure 2.

Let $S_1, S_2 \dots S_m$ denote the possible strategies. We are searching for a strategy, which minimizes the value of future expenses (sum of risk and costs). The optimization problem is then formulated as

$$\min_{j}\left[R(S_j) + C(S_j)\right], \qquad j = 1,2, \dots m \tag{4}$$

where $R(S_j)$ and $C(S_j)$ are the net present values of the risk and expected costs of the jth strategy. If the optimization is constrained by a maximal available budget C_{max}, the optimization of Eq. (4) is subject to $C(S_j) < C_{max}$.

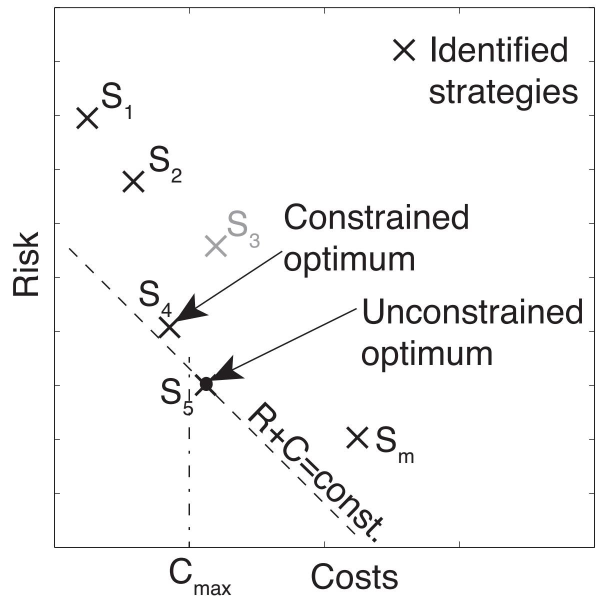

Figure 2. Illustration of the discrete risk optimization problem.

The strategies displayed in Figure 2 with the black crosses are Pareto optimal solutions. The gray strategy S_3 is not Pareto optimal, because it has both higher risk and higher cost than strategy S_4. The unconstrained optimum is strategy S_5, which minimizes the sum of risk and costs. The dashed line shown in Figure 2, for which $R + C = const = R(S_5) + C(S_5)$, illustrates this fact. It can be observed that all other strategies are to the right of this line, indicating that their sum of risk and costs is higher than the one of strategy S_5. If the optimization is constrained by the budget C_{max}, strategy S_5 lies in the infeasible space and strategy S_4 becomes the constrained optimum.

2.2 Hierarchical optimization for the system

Optimization of risk protection in a system is considered, where the risk mitigation measures are planned at the level of the subsystems, which, however, share a common budget. The situation is illustrated in Figure 3 for the discrete formulation.

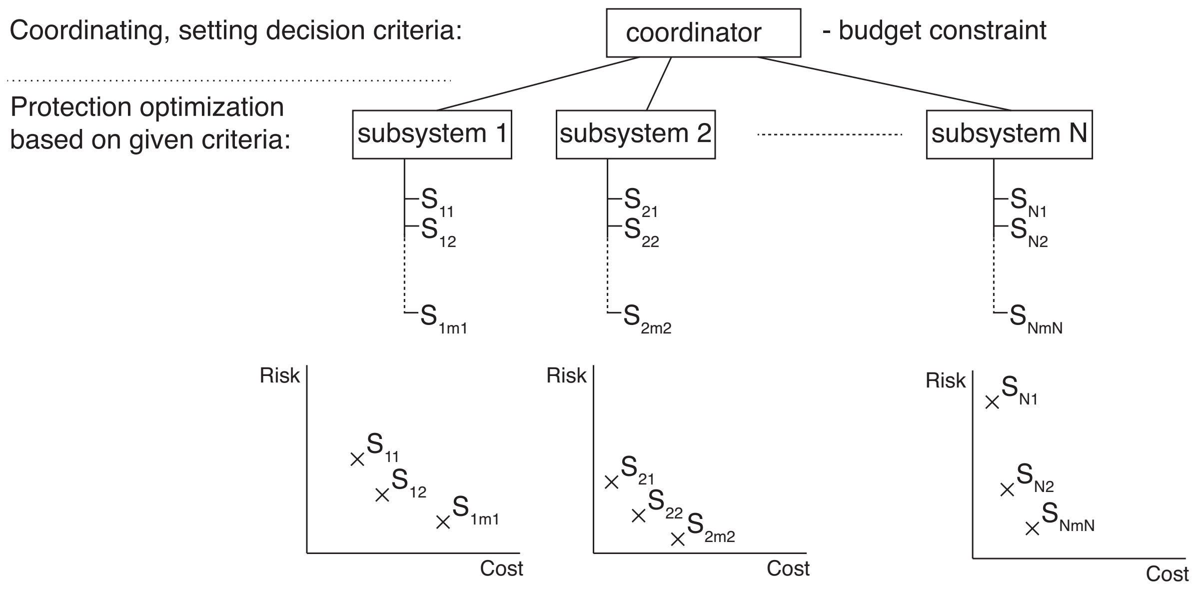

Figure 3. Illustration of the risk optimization problem for a portfolio of subsystems.

2.2.1 Continuous formulation

We first consider the situation where the risk and expected cost in the ith subsystem can be obtained as a function of the protection level l_i . In this case, the optimization problem in a system with N subsystems can be formulated as:

$$\min_{l_1,l_2,\ldots,l_N}\left[\sum\nolimits_{i=1}^{N}\big(C_i(l_i)+R_i(l_i)\big)\right] \tag{5}$$

$R_i(l_i)$ and $C_i(l_i)$ are the net present values of risk and costs in the ith subsystem.

If the budget is unlimited, i.e. if the optimization problem of Eq. (*5*) is unconstrained, then it holds:

$$\min_{l_1,l_2,\ldots,l_N}\left[\sum\nolimits_{i=1}^{N}\big(C_i(l_i)+R_i(l_i)\big)\right]=\sum_{i=1}^{N}\min_{l_i}[C_i(l_i)+R_i(l_i)] \tag{6}$$

Therefore, with unlimited budget, the optimal solution at the system level can be found by finding the optimum in each of the subsystems individually.

If the budget is limited to C_{max}, we must solve

$$\begin{aligned}&\min_{l_1,l_2,\ldots,l_N}\left[\sum\nolimits_{i=1}^{N}\big(C_i(l_i)+R_i(l_i)\big)\right]\\&\quad\text{s.t. } \textstyle\sum_{i=1}^{N}C_i(l_i)\le C_{max}\end{aligned} \tag{7}$$

In this case, Eq. (*6*) does not hold and the risk mitigation measures cannot be optimized independently in the individual subsystems. Additionally, because the protection strategies in the subsystems are often not planned at the same time or by the same engineers, it is generally impossible to optimize the risk protection measures in the whole system at once.

A solution to this problem is provided by hierarchical optimization (Stoilov and Stoilova, 2008). In hierarchical optimization, the coordinator sets criteria, so called *coordination parameters*, at the system level for the optimization in the subsystems. The optimization can then be performed at the individual subsystems for given values of the coordination parameters. At the system level, only these coordination parameters must be optimized. If these are chosen correctly, this procedure may lead to the same solution as the direct optimization of Eq. (7). The optimization can furthermore be carried out iteratively: The coordinator can adjust the coordination parameters depending on the results of optimizations in individual subsystems and with changing constraints such as availability of resources.

In current practice, the efficiency of risk mitigation in subsystems is commonly quantified through the benefit-cost ratio (BCR) or the marginal cost (MC) criterion. As we will show in Section 3.2, the MC criterion can be applied as a coordination parameter and leads to an optimal solution at the system level, ensuring an optimal allocation of resources among the subsystems. However, the more commonly used average BCR is not a correct coordination parameter and can therefore lead to suboptimal solutions.

2.2.2 Discrete formulation

As discussed in Sec. 2.1.2, in practice one typically selects a risk mitigation strategy from a countable number of options. In each of N subsystems one can identify a number of risk protection strategies denoted as S_{ij_i}, where $i \in \{1, \dots, N\}$ is the index of the subsystem and $j_i \in \{1, \dots, m_i\}$ is the index of the strategy in subsystem i and m_i is the number of strategies in subsystem i. The optimization can now be formulated as

$$\min_{j_1, j_2, \dots j_N} \sum_{i=1}^{N} \left[R_i\left(S_{ij_i}\right) + C_i\left(S_{ij_i}\right)\right] \tag{8}$$

where $R_i\left(S_{ij_i}\right)$ and $C_i\left(S_{ij_i}\right)$ are the net present values of risk and cost of the j_ith strategy in the ith subsystem. In accordance with Eq. (*6*), for an unconstrained problem it holds

$$\min_{j_1, j_2, \dots j_N} \sum_{i=1}^{N} \left[R_i\left(S_{ij_i}\right) + C_i\left(S_{ij_i}\right)\right] = \sum_{i=1}^{N} \min_{j_i}\left[R_i\left(S_{ij_i}\right) + C_i\left(S_{ij_i}\right)\right] \tag{9}$$

If the budget is constrained, it is

$$\begin{gathered}\min_{j_1, j_2, \dots j_N} \sum_{i=1}^{N} \left[R_i\left(S_{ij_i}\right) + C_i\left(S_{ij_i}\right)\right] \\ \text{s.t.} \sum_{i=1}^{N} C_i\left(S_{ij_i}\right) \leq C_{max}\end{gathered} \tag{10}$$

In this case, the equality (9) cannot be invoked and the hierarchical optimization is applied, as described in the next section.

3 Measures of efficiency/coordination criteria

When selecting risk protection strategies, the efficiency of the investment should be assessed. For this purpose, measures such as benefit-cost Ratio (BCR) or marginal costs (MC) are commonly used in the practice. Their advantages and limitations are discussed in the following.

3.1 Benefit-cost ratio (BCR)

BCR[1] is a commonly used criterion in the field of economic project appraisal; it was originally developed for evaluating projects where uncertainties are not explicitly considered. BCR is defined as ratio of benefits over costs. The criterion examines if the benefits of a project are high enough to justify the costs; the project is only acceptable if $BCR \geq 1$. Two types of BCR can be distinguished: the average BCR and the incremental BCR (sometimes called marginal BCR) (Lee and Jones, 2004; Corkindale, 2007).

The average BCR is calculated as the total benefits over total cost associated with each project. The average BCR criterion can be applied to select from projects that are not mutually exclusive, i.e. where several projects can be implemented in parallel. If projects are independent, the optimal combination of projects can be found by ranking the projects according to their average BCRs and selecting those with the highest BCRs until either the budget is exhausted or all projects with $BCR \geq 1$ are implemented (Vinod, 1988).

The incremental BCR should be used for selection from mutually exclusive projects, i.e. in situations where one selects only one project from available options (Irvin, 1978; Hendrickson and Matthews, 2011). To calculate the incremental BCR, the projects are first ordered from the cheapest to the most expensive. Project 1 is set as the initial reference project; the incremental BCR of project 2 is calculated as the ratio of increment of benefits over increment of costs compared to project 1. Project 2 is preferable if its incremental BCR is larger than one (or some minimum required value $BCR_{req} > 1$). If project 2 is preferable, it becomes the new reference project, otherwise project 1 is kept as reference. Then the incremental BCR of project 3 with respect to the reference is calculated, and if it is larger than one (or BCR_{req}), project 3 becomes the reference. This process is repeated until all projects are checked. The final reference project is the optimal one.

In the field of risk protection optimization, the BCR is used in many countries, see e.g. (Defra, 2009; lebensministerium.at, 2009; Bründl, 2009; United Nations, 2011). The application of the average BCR and the incremental BCR to the optimization of risk protection is illustrated in Figure 4. In the context of risk protection, the benefit is the risk reduction $-\Delta R$ and the cost is the increase in expected cost ΔC. The average BCRs for both strategies S_1 and S_2, denoted as $BCR_{1,Aver}$ and $BCR_{2,Aver}$, are calculated with respect to strategy S_0, which is here used as the reference. In contrast, when calculating the incremental BCR, the reference strategy is changing. For strategy S_1, the $BCR_{1,Incr}$ is calculated with respect to S_0 and it is thus equal to $BCR_{1,Aver}$. Because $BCR_{1,Incr}$ is here larger than 1 (the angle β is larger than 45°), S_1 is superior to S_0 and is thus selected as the new reference for calculating $BCR_{2,Incr}$ of strategy S_2.

[1] The BCR is sometimes referred to as the present value to capital (PV/C) ratio, if it is assumed that all the costs (capital investment) are spent at the beginning of the project.

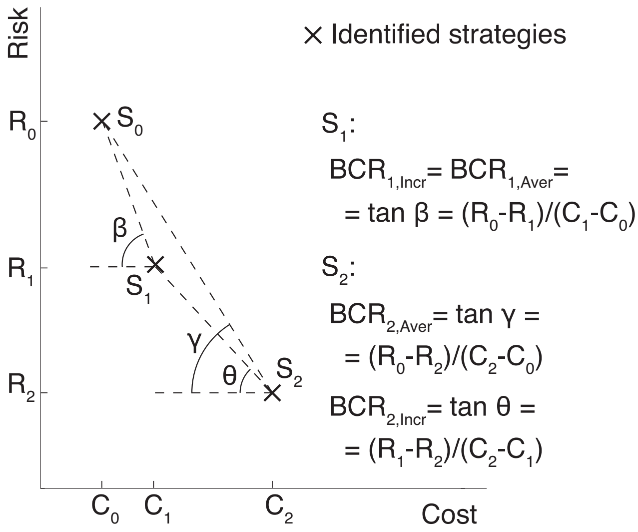

Figure 4. Illustration of incremental and average BCR.

When selecting the optimal risk protection strategy in a subsystem among a set of potential strategies, one deals with mutually exclusive options. Therefore, the incremental BCR should be applied, as recommended in (Riddell and Green, 1999; Defra, 2010). However, in practice it is mostly the average BCR that is used (e.g. Lucarelli et al., 2011; Scottish Executive, 2011; United Nations, 2011). In the rest of this chapter we thus limit ourselves to the average BCR. We will return to the incremental BCR in Section 3.2, as it is equivalent to the MC criterion for the discrete case.

The optimal strategy is commonly defined as the one maximizing the average BCR:

$$\max BCR = \max\left[\frac{-\Delta R}{\Delta C}\right]$$
$$\text{s.t. } \frac{-\Delta R}{\Delta C} \geq BCR_{req} \tag{11}$$

The minimal required value BCR_{req} should be larger or equal to one. BCR_{req} could be perceived as a coordination parameter that is used to efficiently distribute resources at the system level. Only subsystems in which the minimum BCR_{req} can be reached will receive funding for increased risk protection. However, despite its common use, the BCR utilized in this way does not generally lead to optimal solutions at the system level, as we demonstrate later in the numerical example.

The definition of the reference state, which is required in computing the average BCR, differs among countries and areas of application. It is often defined as maintaining the current level of protection or as a so-called “Do-nothing option”, aka “Null option”, which corresponds to no active intervention in the area and no maintenance of existing measures (Defra, 2009). The effect of the reference state is illustrated in Figure 5. The two figures (a) and (b) show an identical situation evaluated in two ways.

In Figure 5(a) the reference state corresponds to the current state, the BCR is thus defined as:

$$BCR_I = \frac{R_C - R}{C - C_C} \tag{12}$$

where R_C and C_C are the risk and costs corresponding to the current state of protection.

In Figure 5(b) the reference state corresponds to the Null (Do-nothing) option, which is associated with the maximal level of risk R_O and zero cost $C_0 = 0$ (United Nations, 2011; Keenan and Oldfield, 2012; Woodward et al., 2013). The BCR then equals:

$$BCR_{II} = \frac{R_0 - R}{C - C_0} = \frac{R_0 - R}{C} \tag{13}$$

Additionally, in practice one often encounters a mixed approach, where the reference point for risk is the current state, while the costs of maintaining the current state of protection are disregarded (e.g. Lucarelli et al., 2011; Krummenacher et al., 2011; Zahno et al., 2012). The BCR then equals:

$$BCR_{III} = \frac{R_C - R}{C} \tag{14}$$

For BCR_{III}, the coordination system shown in Figure 5(a) is shifted to the left.

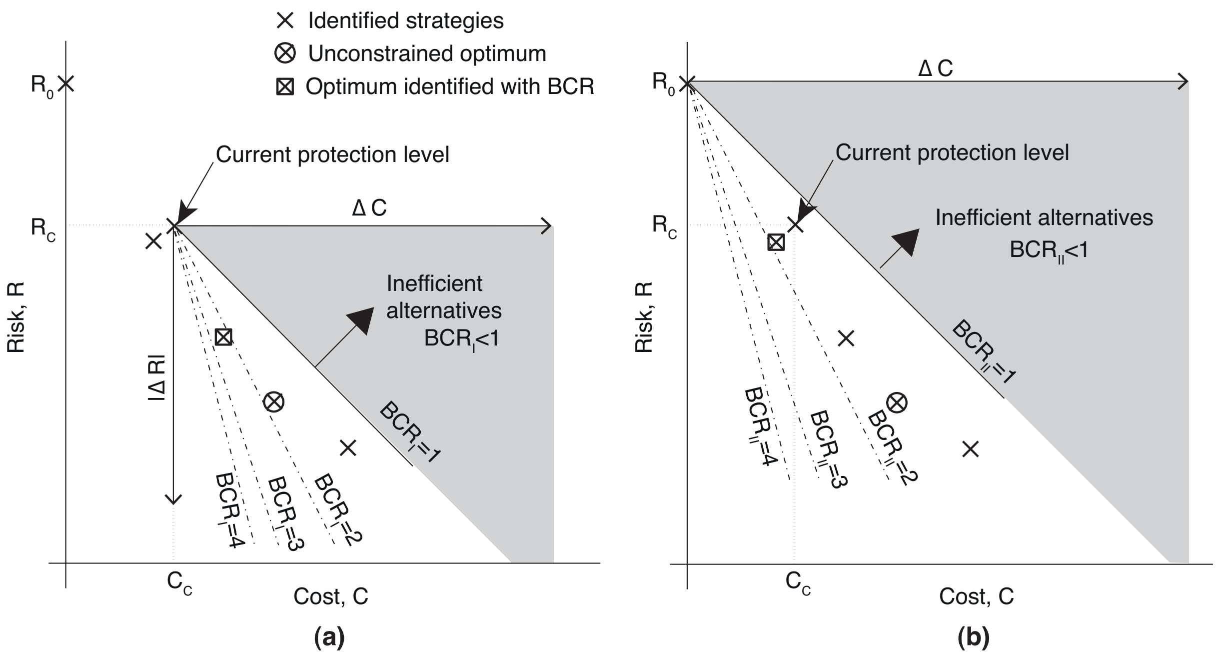

Figure 5. The effect of reference state on the average benefit-cost ratio: (a) the current protection level considered as reference state, (b) the do-nothing option considered as a reference state. The set of identified Pareto optimal strategies is the same in (a) and (b).

It can be observed from Figure 5(a) and (b) that maximizing the BCR according to Eq. (*11*) may not lead to the optimal solution: Assuming that the Pareto optimal border has the shape displayed in Figure 5, the solutions with the maximum BCR are the solutions with the smallest cost increase ΔC - they are marked as crosses in squares in the Figure 5 (a) and (b). The unconstrained optimum, depicted as crosses in circles, would not be identified with neither BCR_I nor BCR_{II}. Additionally, the definition of BCR_I with respect to the current state can lead to negative values for some of the possible solutions, as can be observed in Figure 5(a). These include solutions that are superior to current state, i.e. have lower risk and costs than the current state. BCR_{II}, defined with respect to the do-nothing option (in Figure 5b) can be associated with significant uncertainty, because estimating the risk associated with the null option R_O is often difficult. If BCR_{III} would be applied in this example, i.e. the coordinate system in Figure 5(a) would be shifted to the left, none of the identified solutions would have a BCR higher than one and none of them would thus be acceptable.

Note: The BCR was defined as the ratio between risk reduction and the increase in expected cost. Following (Baecher et al., 1980), instead the expected value of the ratio between damage reduction ΔD and cost ΔC should be used. If the cost is deterministic, the two definitions are identical, since we have $BCR = \mathrm{E}[(-\Delta D)/\Delta C] = \mathrm{E}[(-\Delta D)]/\Delta C = -\Delta R/\Delta C$, where $\mathrm{E}[\cdot]$ denotes the expectation operator and the risk reduction is $\Delta R = \mathrm{E}[(-\Delta D)]$. In case the costs are uncertain, the two definitions differ; in practice, however, the uncertainty in the cost will be significantly lower than in the damages and we thus have $BCR = \mathrm{E}[(-\Delta D)/\Delta C] \approx -\Delta R/\Delta C$.

3.2 Marginal costs (MC)

An alternative approach to risk protection optimization is the marginal cost criterion. It has been applied in the field of natural hazard protection in Switzerland (Bohnenblust and Troxler, 1987; Bohnenblust and Slovic, 1998; Bründl, 2009). In other fields of risk mitigation the utilization of MC criterion appears not to be common (Li et al., 2009).

The marginal costs δC are the costs for reducing the risk by an additional unit δR. This definition is only meaningful in the continuous case, e.g. when both C and R are differentiable functions of the protection level l. The marginal cost criterion for the continuous case is illustrated in Figure 6. If the cost of risk reduction is higher than the value of the risk reduction, i.e. $\delta C > -\delta R$, the strategy is inefficient. If the risk is a differentiable function of C and the budget is unlimited, the optimal solution is one for which it holds:

$$\frac{\delta R}{\delta C} = -1 \tag{15}$$

If the budget is limited and the unconstrained optimum is not feasible, the optimal strategy will have $\delta C \leq -\delta R$. For the optimization at the system level, it is convenient to introduce a parameter $\alpha \geq 1$, which represents the required minimum efficiency of the investment. Assuming differentiability, the optimal protection level for given α is one for which it holds:

$$\frac{\delta R}{\delta C} = -\alpha \tag{16}$$

Eq. (16) reduces to Eq. (15) with $\alpha = 1$. The parameter α determines how many units of risk must be reduced with an investment of one unit of costs, i.e. the required marginal risk reduction. In other words, $1/\alpha$ is the maximal acceptable marginal cost of reducing risk by one unit. Higher values of α will lead to smaller investments in risk protection.

The parameter α is a coordination criterion, which can be set at the system level to optimize the distribution of resources among subsystems. As we will show in Section 3.2.1, it leads to optimal solutions at the system level.

The criteria of Eq. (15) and Eq. (16) uniquely define an optimal solution when R is a differentiable, convex function of C (Figure 6a). When the function is not convex, multiple local optima may exist, and the global optimum has to be identified among these local maxima. The same applies if the function is not differentiable and in the discrete case. In the unconstrained case, i.e. with $\alpha = 1$, the global optimum is the local optimum that minimizes $R - C$. This solution can be found graphically. When plotting the line $-\delta R = \delta C$ through the global maxima, no other feasible solution can be to the left of this line, as illustrated in Figure 6b. By extending this graphical solution to the case with $\alpha > 1$ (Figure 6a), we obtain a more general marginal cost criterion that is applicable in all cases.

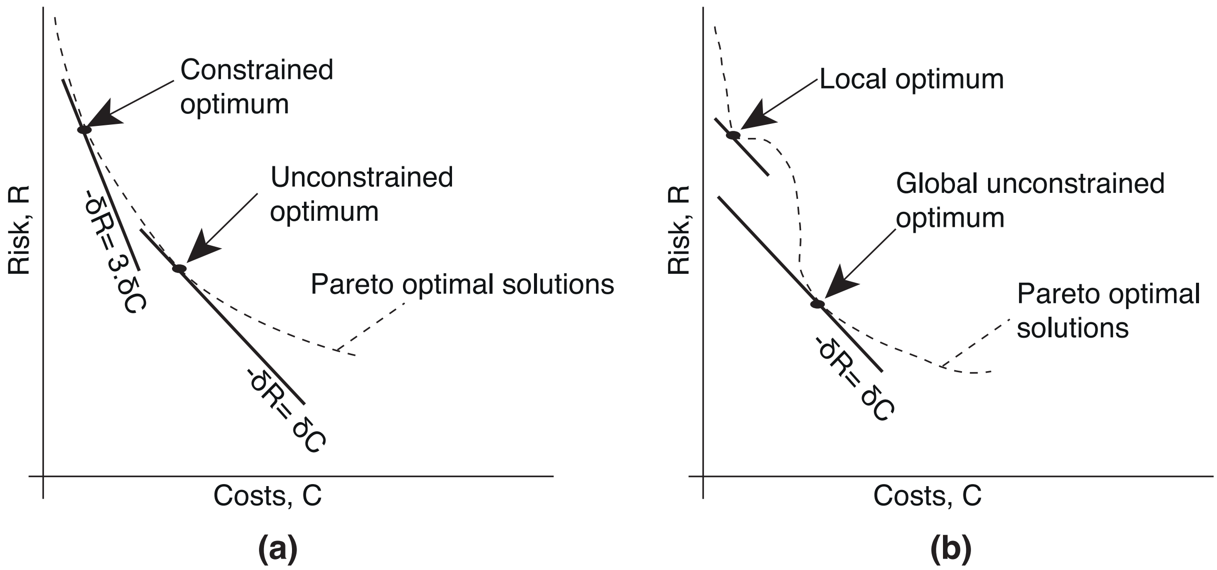

Figure 6. Illustration of the marginal cost criterion for the continuous case; (a) with R being a convex, differentiable function of C for $\alpha = 1$ and $\alpha = 3$; (b) with R being a general function of C for $\alpha = 1$.

In the discrete case (and for non-differentiable continuous functions), the marginal cost is not defined. However, we extend the marginal cost criterion to these situations, based on the graphical solution discussed above. We identify the optimal strategy for a required efficiency α graphically, by shifting the line with gradient $-\alpha$ from the origin to the right (see Figure 7). The optimal solution is the one that is first reached by this line. For the example of Figure 7, the optimal solution for $\alpha = 2$ corresponds to strategy S_2. Computationally this can be implemented by finding the strategy whose distance d^α in the direction perpendicular to this line (see Figure 7) is minimal for given α. The distance d_j^α associated with the pair $C(S_j)$ and $R(S_j)$ is $d_j^\alpha = C(S_j)\frac{\alpha}{\sqrt{\alpha^2+1}} + R(S_j)\frac{1}{\sqrt{\alpha^2+1}}$. The optimal strategy is thus the one found by the following minimization:

$$\min_j d_j^\alpha = \frac{1}{\sqrt{\alpha^2+1}} \min_j \left[\alpha C(S_j) + R(S_j)\right] \tag{17}$$

Since the constant $1/\sqrt{\alpha^2+1}$ is irrelevant, the optimal solution is the one minimizing $\alpha C(S_j) + R(S_j)$.

The approach is general, i.e. it can also be applied in the continuous case, when it is necessary to select among different local optima or when the relation between risk and cost is described by a non-differentiable function. For convex differentiable functions, it will identify the solution in accordance with Eq. (16).

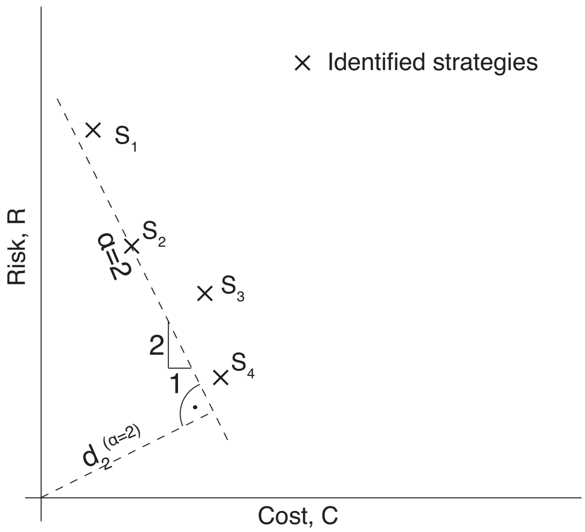

Figure 7. Illustration of the generalized marginal cost criterion for the discrete case. S_2 is the optimal strategy for a marginal cost criterion $\alpha = 2$.

While the marginal cost is not defined for discrete strategies, it is possible to provide an interval of the efficiency parameter α for which a strategy is optimal, as illustrated in Figure 8. From this figure it can also be observed that the MC criterion only identifies those strategies, which lie on the convex envelope of the set of all Pareto optimal strategies. The strategy S_3 in Figure 8 is not optimal for any value of α, even though it is a Pareto optimal solution.

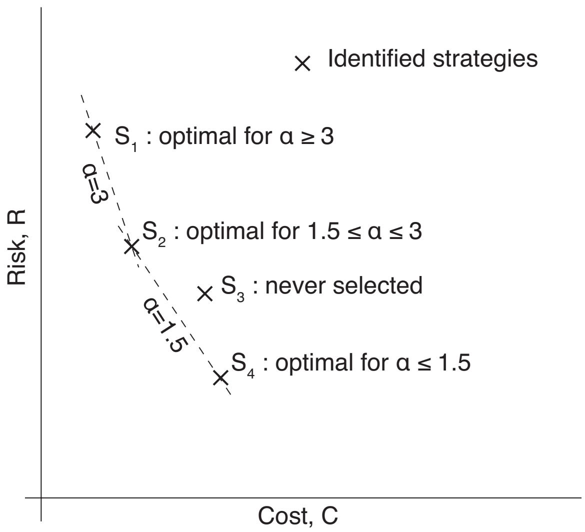

Figure 8. Intervals of the efficiency parameter α for which specific solutions are optimal.

For the discrete case, the generalized marginal cost criterion following Eq. (17) leads to the same solution as the incremental BCR algorithm described in Section 3.1. That algorithm also identifies only solutions that lie on the convex envelope of the set of all Pareto optimal strategies, and for required values of BCR_{req} equal to α, the optimal strategy identified with the two methods is the same. This can be observed when applying the incremental BCR algorithm to the example of Figure 8. For $BCR_{req} > 3$, S_1 will be optimal because none of the other solutions has $BCR_{incr} > 3$ with respect to S_1. Analogously, for $3 > BCR_{req} > 1.5$, S_2 will be optimal because S_3 and S_4 do not have $BCR_{incr} > 1.5$ with respect to S_2 and for $BCR_{req} < 1.5$, S_4 will be optimal. For $BCR_{req} = 1.5$ and $BCR_{req} = 3$, the two strategies S_2 and S_4, resp. S_1 and S_2, are equivalent. The solutions are thus equal to those identified with the proposed generalized marginal cost criterion.

3.2.1 Derivation of the MC criterion for system optimization

We show that the application of the MC criterion at the subsystem level leads to the optimal solution at the system level for the continuous differentiable case. The system optimization problem for a continuous case was stated in Eq. (7). By changing the minimization to a maximization problem and by reformulating the constraint to an equality constraint by the use of a so-called slack variable b (Jordaan, 2005; Nocedal and Wright, 2006), the optimization is formulated as follows:

$$\max_{l_1, l_2 \dots l_N} \left[-\sum_{i=1}^{N} \left(C_i(l_i) + R_i(l_i) \right) \right]$$
$$\text{s.t.} \quad \sum_{i=1}^{N} C_i(l_i) + b^2 = C_{max} \tag{18}$$

where $R_i(l_i)$ and $C_i(l_i)$ are the net present values of risk and cost in the ith region.

We formulate the Lagrangian function:

$$L(l_1, l_2, \dots, l_N, \lambda, b) = -\sum_{i=1}^{N} \left(C_i(l_i) + R_i(l_i) \right) - \lambda \left[\sum_{i=1}^{N} C_i(l_i) + b^2 - C_{max} \right] \tag{19}$$

By differentiating the Lagrangian function with respect to each variable, the following conditions for optimality are obtained:

$$\frac{\partial L}{\partial l_i} = -\frac{\partial R_i(l_i)}{\partial l_i} - \frac{\partial C_i(l_i)}{\partial l_i} - \lambda \frac{\partial C_i(l_i)}{\partial l_i} = 0, \quad i = 1, \dots, N \tag{20}$$

$$\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{N} C_i(l_i) - b^2 + C_{max} = 0 \tag{21}$$

$$\frac{\partial L}{\partial b} = -2\lambda b = 0 \tag{22}$$

Eq. (*20*) can be rewritten to

$$\frac{\partial R_i(l_i)}{\partial C_i(l_i)} = -(1+\lambda), \quad i = 1, \dots, N \tag{23}$$

We note that $(1+\lambda)$ is equal to the required efficiency α of Eq. (*16*).

Eq. (21) corresponds to the initial constraint. Eq. (22) holds if

$$\lambda = 0 \cup b = 0 \tag{24}$$

We can thus distinguish two cases: (a) When $\lambda = 0$, sufficient budget is available to implement the optimal risk protection in all subsystems. The condition of Eq. (23) is equal to Eq. (15), i.e. the optimal protection in all subsystems has marginal cost equal to one. (b) When $b = 0$, the full budget is used. In this case it is $\lambda \geq 0$ and the required efficiency in all subsystems is $\alpha = (1+\lambda) \geq 1$. This shows that the optimal solution at the system level is found as one where the required efficiency α in Eq. (16) is the same in all subsystems.

Unfortunately, this derivation cannot be extended to the proposed generalized marginal cost criterion, which is applicable to discrete sets of risk protection strategies and to non-differentiable continuous functions. In fact, as we show in the numerical example of Section 4, in the discrete case the marginal cost criterion (and thus the incremental BCR) is not able to identify the optimal solution at the system level for all budget levels. However, from the generalized marginal cost criterion, Eq. (17), it follows that the total cost $C = \sum_i C(S_i)$ and total risk $R = \sum_i R(S_i)$ of the identified solution minimize $\alpha C + R$. Therefore, there is no other solution with simultaneously lower total cost C and lower total risk R. Any solution identified with the marginal cost criterion is thus a Pareto optimal solution at the system level. In other words, the generalized MC criterion is not able to identify all Pareto optimal solutions on the system level, but the solutions that are identified with this criterion are Pareto optimal at the system level.

4 Numerical investigation

This numerical study is motivated by our work on defining procedures for optimizing flood protection measures in multiple regions that are managed by one government agency (Špačková et al., 2014). We consider a set of 6 regions (subsystems), in which optimal risk mitigation strategies should be identified. The identification of possible strategies and the assessment of the risks and costs associated with these strategies take place at the regional level, but the budget is administered by the agency. The utilized input data are hypothetical, but they are based on real case studies and they thus reflect an achievable ratio between risk reduction and costs.

In each region, a set of candidate strategies have been identified, including a Null option n (no measures are taken), the option of maintaining the current level of protection c and two to three alternative protection strategies x, y, z. For all strategies, the net present value of risk and cost are evaluated. These values are summarized in Table 1. Exemplarily, the risks and costs of strategies identified for regions 1 and 3 are shown in Figure 9. The risk protection in all regions is financed from a common budget C_{max}. We aim to select one strategy in each region to minimize the sum of the net present value of risk and costs over all regions, so that the total costs do not exceed C_{max}, following Eq. (10).

Table 1. Net present value of risk and cost of alternative strategies in the six analyzed regions [x10^6 Euro].

Strategy		**Region:**	**i=1**	**i=2**	**i=3**	**i=4**	**i=5**	**i=6**
Null opt. (n)	**j=1**	Risk, $R_i(S_{ij})$	34.50	0.90	92.40	8.75	3.50	17.20
		Cost, $C_i(S_{ij})$	0	0	0	0	0	0
Current state (c)	**j=2**	Risk, $R_i(S_{ij})$	11.50	0.50	44.00	2.50	1.40	8.60
		Cost, $C_i(S_{ij})$	10.00	0.01	10.00	3.10	0.90	8.00
Option x	**j=3**	Risk, $R_i(S_{ij})$	0.35	0.76	50.00	0.03	0.01	4.30
		Cost, $C_i(S_{ij})$	17.00	0.002	8.00	6.00	1.12	10.00
Option y	**j=4**	Risk, $R_i(S_{ij})$	0.05	0.03	0.69	0.025	0.01	0.10
		Cost, $C_i(S_{ij})$	18.80	0.009	19.00	7.00	1.2	15.00
Option z	**j=5**	Risk, $R_i(S_{ij})$	-	0.01	0.45	-	-	0.005
		Cost, $C_i(S_{ij})$	-	0.02	23.00	-	-	30.00

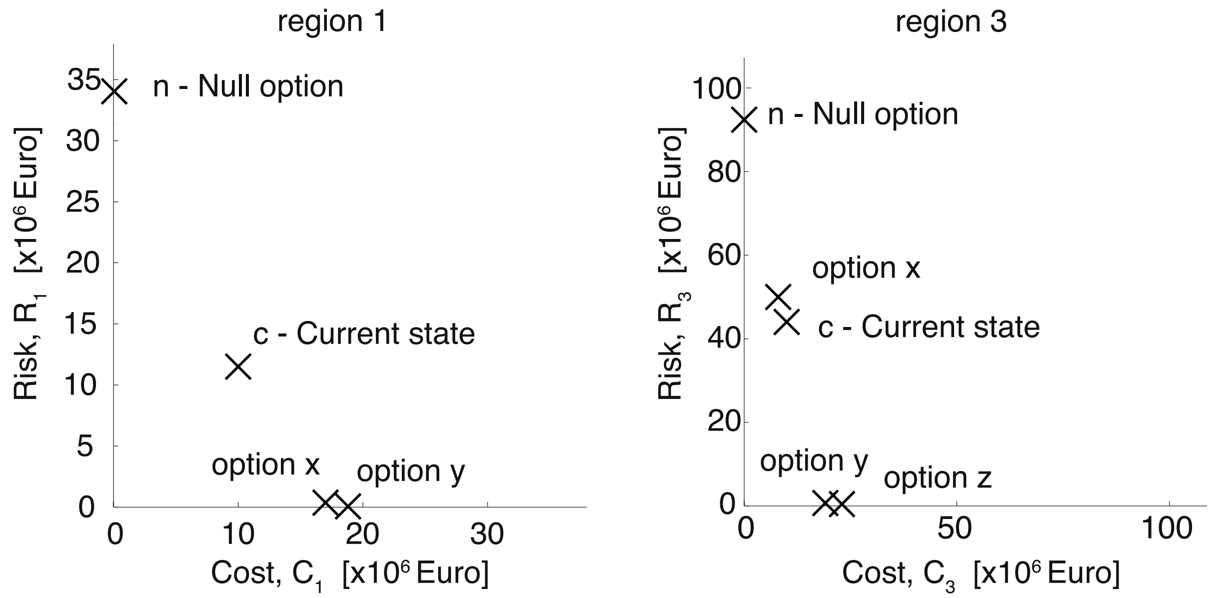

Figure 9. Net present value of risk and cost for identified strategies in regions 1 and 3.

The optimization of the risk protection strategies is carried out with five different evaluation methods:

1) BCR_I, which is defined with respect to the current state according to Eq. (*12*). The Null option is not considered a feasible strategy in this case.
2) BCR_{II}, which is defined with respect to the Null option following Eq. (*13*).
3) BCR_{III}, which is defined with respect to the current state, but the cost for maintaining the current level of protection is neglected as shown in Eq. (*14*).

In methods 1-3, the optimum is found by maximizing the BCR following Eq. (*11*).

4) MC, the marginal cost criterion following Eq. (*17*).
5) *Complete search*, which is obtained by evaluating all possible combinations of strategies in all regions.

It is noted that the complete search is not applicable in practice, where the strategies in different regions are not evaluated jointly. It is included here to provide the reference solution, which allows assessing the other methods.

Section 4.1 presents results of an unconstrained optimization with these five evaluation methods. Sections 4.2 and 4.3 compare the results of an optimization constrained with maximum budget C_{max}.

4.1 Results of unconstrained optimization

The results of the unconstrained optimization are summarized in Table 2. It shows for each method the total cost $C = \sum_i C_i(S_i)$ and the total residual risk $R = \sum_i R_i(S_i)$ of the identified optimal solution and the sum of total risk and costs $R + C$. In the unconstrained optimization, BCR_{req} and α are equal to 1.

Table 2. Portfolios selected with different methods for unconstrained optimization. [x10^6 Euro].

Method	Criterion	Strategies	C	R	$R + C$
1. BCR_I – Current state	$BCR_{req} = 1$	x,z,y,c,x,x	50.32	7.86	58.18
2. BCR_{II} – Null option	$BCR_{req} = 1$	c,y,x,c,x,x	32.31	68.34	100.65
3. BCR_{III} – Current st.excl.cost	$BCR_{req} = 1$	x,y,y,x,x,y	58.21	1.21	59.42
4. MC	$\alpha = 1$	x,z,y,c,x,x	50.32	7.86	58.18
5. Complete search		x,z,y,c,x,x	50.32	7.86	58.18

As expected, the MC criterion identifies the optimal solution (as found by a complete search). The BCR_I also identifies this optimal solution, whereas BCR_{II} and BCR_{III} do not. Clearly, the definition of the reference option has a significant influence on the solutions identified with the BCR criterion. With the Null option as a reference (BCR_{II}), inexpensive strategies are identified, which lead to a strongly sub-optimal solution with high residual risks. In contrast, BCR_{III} leads to a more expensive and conservative solution, which however is close to the optimal solution in terms of $C + R$.

4.2 Results of optimization with a limited budget $C_{max} = 35\times10^6$ Euro.

Table 3 summarizes the results of the optimization constrained by a limited budget of $C_{max} = 35\times10^6$ Euro. The coordination parameters BCR or α are varied to find those whose total cost C most closely comply with the budget constraint.

None of the methods 1–3 using the BCR identifies the optimal solution (complete search), only the marginal cost criterion does (method 4). This solution is obtained with the MC criterion set to $\alpha = 1.9$.

All three methods based on the BCR identify suboptimal solutions that have similar costs, but imply a risk that is more than double that of the optimal solution. The corresponding minimum required BCR values differ substantially among the different BCR definitions, from 1.2 to 4.9.

Table 3. Portfolios selected with different methods. Optimization constrained with a budget $C_{max} = 35\times10^6$ Euro. All values in [x10⁶ Euro].

Method	Criterion	Strategies	C	R	$R + C$
1. BCR_I – Current state	$BCR_{req} = 4.9$	c,z,c,c,c,c	32.02	68.01	100.03
2. BCR_{II} – Null option	$BCR_{req} = 1.2$	c,y,x,c,x,x	32.31	68.34	100.65
3. BCR_{III} – Current st.excl.cost	$BCR_{req} = 2.3$	c,y,c,c,c,c	32.01	68.03	100.04
4. MC	$\alpha = 1.9$	c,z,y,c,x,n	33.32	31.91	65.23
5. Complete search		c,z,y,c,x,n	33.32	31.91	65.23

The solutions identified with the BCR perform poorly mainly because of the suboptimal strategies identified in region 3 (see Figure 9). By applying the BCR as a coordination parameter and requiring the same minimum BCR in all regions, the optimal strategy y in region 3 is rejected since its BCR is less than the minimum or, in case of BCR_{II}, is less than that of strategy x.

4.3 Results of optimization for different levels of budget C_{max}

Extending the results of section 4.2, the constrained optimization is carried out for varying budget constraints $C_{max} = \{1,2,\ldots,51\}\times10^6$ Euro. Figure 10 shows the total residual risk for all analysed regions that is achieved with different budget constraints, with the strategies identified using methods 1–5. In Table 4–Table 8, the selected strategies are listed separately for the five different methods, together with the corresponding total cost $C = \sum_i C_i(S_i)$ and total residual risk $R = \sum_i R_i(S_i)$ of the identified optimal solution and the sum of risk and cost $R + C$.

For the complete search (Table 4), not all possible solutions are listed, because these were evaluated by varying the available budget C_{max} in increments of 10^6€. For example, there are multiple solutions with total costs C in the interval (8,9]. With the selection of smaller increments, more solutions could have been found. On the other hand, for some levels of budget there is no solution that utilizes the entire available budget. For example, there is no combination of strategies that costs exactly $1\text{x}10^6$ Euro; the optimal solution for an available budget of $C_{max} = 10^6$ Euro costs $0.92\text{x}10^6$ Euro.

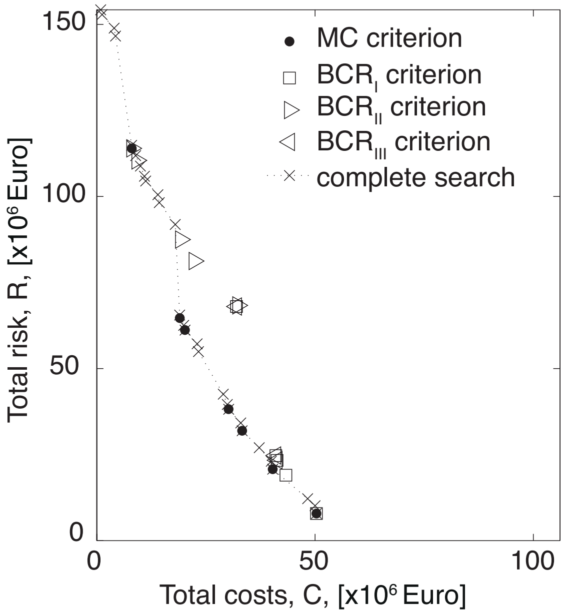

Figure 10. Total residual risk R versus cost C in all six regions, obtained by varying the available budget C_{max}. Comparison of the four investigated methods and the reference solution provided by the complete search.

When applying any of the four investigated methods with coordination parameters, the total number of solutions is reduced significantly relative to the complete search. With BCR_I five solutions are identified (Table 5) and with the MC approach nine solutions are found (Table 8). For most budget levels, the methods do not allow to fully exploit the available budget. For example, they are unable to identify a combination of strategies with total cost between 10×10^6 Euro and 19×10^6 Euro, even if these exist as seen from the complete search.

All solutions identified with the MC criterion are also solutions that can be found with the complete search. (Note that this is not evident from comparing Table 8 with Table 4, because Table 4 does not contain all solutions of the complete search but only those that are found for budget levels that are multiples of 10^3 Euro.) As discussed in Section 3.2.1, the solutions found with the MC criterion are a subset of the Pareto optimal solutions at the system level. Comparison of Table 4 with Table 8 shows which of the solutions are omitted with the MC criterion: With increasing budget, the protection level of the strategies identified in the individual regions with the MC criterion always rises. For example, in region 5, the Null option n is selected for budgets up to 20×10^6 Euro (for $\alpha \geq 3$) and strategy x for budgets of 21×10^6 Euro and higher (for $\alpha \leq 2.9$). In contrast, with the complete search, the protection level of the selected strategies varies with increasing budget. In region 5, the Null option n is selected for a budget of 1×10^6 Euro, the current state c for a budget of 2×10^6 Euro, strategy x for a budget of 3×10^6 Euro, and the current state is again selected for a budget of 4×10^6 Euro. In this region, strategies with higher protection level are selected to utilize the remaining budget. The solutions that are identified with the complete search but not with the MC criterion are thus rather unstable. In some regions (here regions 2, 4 and 5), a higher protection level is often selected to fully use the given budget, but if a slightly higher budget was available, the money should optimally be invested in other regions where measures are more expensive. In practice it might thus be beneficial not to implement such solutions found with the complete search. Instead, the non-allocated part of the budget may be saved for increasing the protection level through more efficient yet more expensive strategies in other regions when additional resources becomes available. The MC criterion allows identifying such solutions and the fact that it not always exploits the available budget completely may therefore be beneficial in practice.

In Figure 10 it can further be observed that all three BCR criteria partially identify suboptimal solutions, i.e. they find solutions for which there exist alternative solutions with simultaneously lower cost and lower risk. This is most evident for a maximum budget close to 35×10^6€, which is the case already presented in Section 4.2.

Table 4. Results of the parametric study – complete search: strategies selected in individual regions for different levels of budget. All values in [x10⁶ Euro].

Budget	Strategies	C	R	$R + \mathrm{C}$
0	n, n, n, n, n, n	0	157.25	157.25
1	n, z, n, n, c, n	0.92	154.26	155.18
2-3	n, z, n, n, x, n	1.22	152.87	154.09
4	n, n, n, c, c, n	4	148.9	152.90
5-7	n, z, n, c, x, n	4.32	146.62	150.94
8	n, n, x, n, n, n	8	114.85	122.85
9	n, z, x, n, c, n	8.92	111.86	120.78
10	n, n, c, n, n, n	10	108.85	118.85
11	n, z, c, n, c, n	10.92	105.86	116.78
12-13	n, z, c, n, x, n	11.22	104.47	115.69
14	n, n, c, c, c, n	14	100.5	114.50
15-17	n, z, c, c, x, n	14.32	98.22	112.54
18	c, n, x, n, n, n	18	91.85	109.85
19	n, n, y, n, n, n	19	65.54	84.54
20	n, z, y, n, c, n	19.92	62.55	82.47
21-22	n, z, y, n, x, n	20.22	61.16	81.38
23	n, n, y, c, c, n	23	57.19	80.19
24-28	n, z, y, c, x, n	23.32	54.91	78.23
29	c, n, y, n, n, n	29	42.54	71.54
30	c, z, y, n, c, n	29.92	39.55	69.47
31-32	c, z, y, n, x, n	30.22	38.16	68.38
33	c, n, y, c, c, n	33	34.19	67.19
34-37	c, z, y, c, x, n	33.32	31.91	65.23
38-39	x, z, y, n, x, n	37.22	27.01	64.23
40	x, n, y, c, c, n	40	23.04	63.04
41-48	x, z, y, c, x, n	40.32	20.76	61.08
49	x, z, y, c, x, c	48.32	12.16	60.48
50	x, n, y, c, c, x	50	10.14	60.14
51	x, z, y, c, x, x	50.32	7.861	58.18

Table 5. Results of the parametric study – $BCR_I \geq BCR_{req}$ criterion (BCR calculated with respect to the current state): strategies selected in individual regions for different values of BCR_{req}. All values in [x10⁶ Euro].

BCR_{req}	Budget	Strategies	C	R	$R + \mathrm{C}$
≥4.9	33-41	c, z, c, c, c, c	32.02	68.01	100.03
4.7-4.8	-	c, z, y, c, c, c	41.02	24.70	65.72
2.2-4.6	42-43	c, z, y, c, x, c	41.32	23.31	64.63
1.6-2.1	44-50	c, z, y, c, x, x	43.32	19.01	62.33
1.0-1.5	≥51	x, z, y, c, x, x	50.32	7.86	58.18

Table 6. Results of the parametric study – $BCR_{II} \geq BCR_{req}$ criterion (BCR calculated with respect to the Null option): strategies selected in individual regions for different values of BCR_{req}, All values in [x10^6 Euro].

BCR_{req}	Budget	Strategies	C	R	$R + \mathrm{C}$
≥5.3	1-8	n, y, n, n, n, n	0.01	156.38	156.39
3.0-5.2	9	n, y, x, n, n, n	8.01	113.98	121.99
2.4-2.9	10-19	n, y, x, n, x, n	9.21	110.49	119.70
2.1-2.3	20-22	n, y, x, n, x, n	19.21	87.49	106.70
1.3-2.0	23-32	c, y, x, c, x, n	22.31	81.24	103.55
1.0-1.2	≥33	c, y, x, c, x, x	32.31	68.34	100.65

Table 7. Results of the parametric study – $BCR_{III} \geq BCR_{req}$ criterion (BCR calculated with respect to the current state excluding the cost of the current state): strategies selected in individual regions for different values of BCR_{req}, All values in [x10^6 Euro].

BCR_{req}	Budget	Strategies	C	R	$R + \mathrm{C}$
≥2.3	33-41	c, y, c, c, c, c	32.01	68.03	100.04
1.2-2.2	-	c, y, y, c, c, c	41.01	24.72	65.73
1.0-1.1	≥42	c, y, y, c, x, c	41.31	23.33	64.64

Table 8. Results of the parametric study – MC criterion: strategies selected in individual regions for different required α. All values in [x10^6 Euro].

α	Budget	Strategies	C	R	$R + \mathrm{C}$
≥5.3	1-8	n, y, n, n, n, n	0.01	156.38	156.39
4.5-5.2	9-19	n, y, x, n, n, n	8.01	113.98	121.99
3.0-4.4	20	n, y, y, n, n, n	19.01	64.67	83.68
2.3-2.9	21-30	n, y, y, n, x, n	20.21	61.18	81.39
2.1-2.2	31-33	c, y, y, n, x, n	30.21	38.18	68.39
2.0	-	c, y, y, c, x, n	33.31	31.93	65.24
1.6-1.9	34-40	c, z, y, c, x, n	33.32	31.91	65.23
1.3-1.5	41-50	x, z, y, c, x, n	40.32	20.76	61.08
1.0-1.2	≥51	x, z, y, c, x, x	50.32	7.86	58.18

5 Discussion

We have described a quantitative framework for optimal allocation of resources for risk protection in a system where the actual measures are planned at the subsystem level, but the budget is limited at the system level (Figure 3). Using concepts from hierarchical optimization, we showed that Pareto optimal solutions can be obtained by selecting in each subsystem a strategy that complies with a required marginal cost (MC) criterion α (Section 3.2.1). Its value must be prescribed at the system level by a coordinator, prior to selecting the strategies in individual subsystems. However, determining the required α is not possible without prior knowledge of situation in the subsystems (i.e. the costs of protection measures and residual risk) and the available budget. In the classical theory of hierarchical optimization, the optimization is carried out iteratively (see Section 2.1.1): the coordinator prescribes an initial value of the coordination parameter, the optimizations in individual subsystems are carried out and the results are returned to the coordinator who then adjusts the coordination parameter; this process is repeated until an optimum at the system level is achieved. However, when planning risk protection strategies in practice, such an iterative process is typically infeasible, and decision criteria must be prescribed a-priori. Therefore, basic data on the system (approximate risk in individual subsystems, approximate cost of potential protection measures) must be gathered and an initial value of α should be selected based on these prior information. If results of risk analyses from at least some subsystems are available, they may be extrapolated to the whole system. Otherwise, representative subsystems should be selected and analysed prior to making any decision on α. Once the method is implemented, the value of α should be regularly adjusted based on the available results from risk analyses, accounting additionally for political and organizational constraints.

As discussed in Section 2.1.2, in practice only a limited number of strategies can be evaluated for most applications. This is because the identification of options and the assessment of their effects on the risk are typically time-consuming. They often require extensive modelling and numerical analysis efforts. Moreover, cost estimates for all proposed measures are required. One has therefore strong incentives for keeping the number of evaluated strategies low. Conversely, if only few strategies are considered, or if they are not selected appropriately, it is likely that the optimal one is not included in the set of investigated strategies. To deal with these contradicting goals, it is recommendable to use a two-phase process for identifying risk protection strategies in subsystems (Špačková et al., 2014). In a first phase, the strategies are evaluated in a less detailed, approximate manner and an optimal protection level is selected. In the second phase, a detailed assessment of the risk protection strategies around the optimal protection level is carried out.

In this paper the optimization is carried out based on expected values of risk (and cost). The attitude of the decision-maker to risk (e.g. risk aversion) should ideally be taken into account by means of the expected utility concept (see Section 2). The generalized marginal cost criterion should then be extended to a marginal utility criterion. In practice, however, utility functions for multiple decision-makers, societies and for varying types of decisions are not readily available, and may be difficult to obtain also in the long run. Societies preferences towards risk beyond the expected monetary costs may thus be better dealt by risk acceptance criteria, which may be added as additional constraints to the optimization.

In engineering risk assessments, commonly only tangible damages are taken into account and other consequences of hazard and failures events are disregarded. The risk reduction achieved with the protection strategies is thus likely to be underestimated (Messner and Meyer, 2005). Additionally, the dependence of benefits on the protection level is neglected in most analyses (see Section 2); a pioneering study taking into account this aspect for flood risk management has been published by Mori and Perrings (2012). The assumption of constant benefits can be unrealistic; e.g. the protection of a region against natural hazards entails benefits to society or the owner, because it enables societal and economic activity that would not be possible without the protection. Neglecting the dependence of benefits on the protection level can thus lead to an underestimation of the efficiency of the risk protection. The methodology for damage assessment and for including benefits of the risk protection beyond the risk reduction should be further investigated, especially for the field of public decisions.

In this paper, we considered the risk optimization problem from a global perspective of an overarching authority (state agency, company management). The problem of conflicting objectives of different stakeholders has not been discussed. This aspect can be addressed by the formulation of the objective function or it can be reflected in the way the damages, costs and benefits are estimated. What is considered as damage/cost by one stakeholder may not be relevant for other stakeholders. Additionally, the problem of equity, i.e. the fair distribution of resources and risks within the system, has not been considered in the presented solution. The equity issue is an important and relevant topic for public investments, where the solutions based on monetized costs and benefits tend to be advantageous for richer groups of the society.

Solutions for including equity considerations into the optimization of risk protection have been discussed for example in Shan and Zhuang (2013) and Zhou et al. (2012).

We considered risk protection measures in general terms, without distinguishing their specific types. Engineers typically design structural measures; exemplarily, for flood hazards these include flood barriers, reservoirs or enhanced protection of the elements at risk. Simultaneously, governments attempt to influence the development in the flood prone areas by zoning restrictions and land use planning (Surminski, 2009; Paudel et al., 2013). To stimulate an optimal land use, different economic instruments such as insurance schemes, taxes and compensations have been developed (Bräuninger et al., 2011). Engineering measures and land use policies relate closely to each other and they should be optimized jointly, but this is not commonly done in practice and has rarely been considered in research.

6 Conclusion

The problem of selection of optimal risk protection strategies under budget constraints was formulated as a discrete hierarchical optimization. The aim is to find an optimal combination of protection strategies in individual subsystems (e.g. administrative units) minimizing the sum of risk and cost at the system level (e.g. state agency). It was considered that the system cannot be optimized as a whole, because the planning of the protection measures is carried out independently in the individual subsystems (by different analysts and/or at different times). The problem is furthermore constrained by the limited budget available for risk protection that can be distributed amongst the subsystems.

It was shown that the optimal allocation of resources can be achieved by using the marginal cost (MC) criterion as the coordination criterion in the hierarchical optimization. By selecting strategies that have the same marginal cost in all subsystems, the optimum on the system level is ensured. We generalized the MC criterion to discrete problems, where it corresponds to the incremental benefit-cost ratio (BCR).

The average BCR criterion, which is broadly used in practice, was shown to be inappropriate as an optimization parameter. This was also demonstrated by a numerical example of six subsystems, representing individual catchments endangered by floods. The results of the optimization obtained using the BCR and MC criteria were compared to a reference solution. The results showed that the solutions obtained with the BCR criteria were suboptimal for some budget constraints, while all solutions identified with MC criterion corresponded to the reference solution. Additionally, the BCR is not defined uniquely in different studies, because the selected reference option utilized in BCR is not always the same. As a consequence, utilization of BCR for comparing the efficiency of investments against different risks (e.g. earthquake vs. flood protection) can be misleading, whereas the generalized MC criterion enables a consistent comparison of risk mitigation efficiency among different domains.

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