Spatial Analysis of Humanoid Robot's Gait Using Dual-Quaternions

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Abstract

The most common approach used in modeling humanoid robots applies trigonometric expressions and Euler-angles for its description. Some problems of trajectory interpolation and singularity of motion representation are discussed and the dual quaternion representation is used as an alternative to avoid these problems and to obtain the models ensuring a general spatial analysis of the motion trajectory. The aim of this work is to present a different method to model kinematics of humanoid robots. Simulations are performed to evaluate the model numerically evaluating the humanoid robot dynamic equilibrium during the gait and the trajectory interpolation.

1 Introduction

In the last years the problem to construct a humanoid robot became relatively easier due to improvements of electronics. It is possible to use a cheap microcontroller and some servo actuators, which replace complex mechanical systems with a lot of gears and moving parts. The mechanism’s control also became relatively less complex in the implementation using standard languages for microcontroller

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programming and open sources codes, which have continuous improvements from different researches.

One of the challenges related to humanoid robots is still to obtain representative theoretical models due to a large number of parameters which have influence in its kinematics and dynamics.

The most common approach used in kinematic models of humanoid robots is done by a separation into two different planes, frontal and sagittal plane. Generally a model with 6 degrees of freedom in each leg is proposed: the hip joint with 3-DOF, arranged to move as a spherical joint and the knee joint with 1DOF and the ankle joint with 2-DOF. Similar approaches are presented in [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] and [11]. The inverse kinematics in general is obtained by trigonometric expressions in roll, pitch and yaw angles.

Some problems of trajectory interpolation and singularity of movement representation have been discussed and the dual quaternions appear to be an alternative to obtain the models without these problems and therefore ensuring a general analysis of the trajectory. The advantages of dual quaternions are mentioned in [12], [13] and [14], which in resume, can be just as efficient if not more efficient than using matrix methods.

The objective of this work is to obtain an inverse kinematics model for a humanoid robot in a spatial trajectory using algebraic geometry. The proposed method is based on the kinematic mapping as presented in [15]. The specific objectives are at first to define the spatial movement of feet and center of mass of a humanoid robot, with a 6R structure in each leg, based in the human gait, and secondly, to obtain an inverse kinematics model related to the structure using algebraic geometry methods, which result in polynomial expressions.

Kinematics of human gait has been studied in the last years by many authors. [16] presents the antero-posterior component of the spatial acceleration of the foot and [17] developed a biomechanical analysis of the spatial trajectory of the human body during the gait. The trajectory of the center of mass and the center of pressure position of the feet are analyzed with components in three directions, using Inertial Measurement Units (IMU) sensors.

In order to define the theoretical trajectory, the displacement curves proposed to the humanoid gait are twice differentiated to obtain the acceleration components. These components are compared with biomechanical curves based on IMU signals measured during human gait. Taking the theoretical components of displacement and acceleration, it is possible to analyze if the proposed movement has dynamics equilibrium during the gait. The proposed analysis is based on the Zero Moment Point Method presented in [18].

A gait cycle with 10 poses is proposed. For each pose, the inverse kinematics model yields 8 possible solutions of the joints. An algorithm is implemented to choose one of the 8 solutions which gives practical values applied to the humanoid robot.

At the end trajectory interpolation is discussed. One of the advantages using dual quaternions to represent a spatial trajectory is related to the possibility to interpolate the poses, ensuring that each new pose is represented by a quaternion that is contained in the Study Quadric. The problem is to define the control points, which can result in a trajectory with different acceleration. Therefore, an interpolation in the joint space is proposed.

2 Forward Kinematics

In the forward kinematics model, the base frame is attached to the foot, which is in contact with the ground. The end effector coordinate system is located in the center of mass, as shown in Fig. 1. So, taking the movement of the center of mass, the kinematics is represented by a 6R structure with D-H parameters displayed in Tab.1. Using these parameters, it is possible to obtain the position and orientation of the center of mass (CoM) related to the foot, represented by the base frame (B). The mathematical formulation of the forward kinematics is shown in Eq. (1).

It is also possible to compute the position and orientation of the other foot that is in movement, related to the base frame. In this case, taking into account the foot movement, the kinematics is represented by a 12R structure and the mathematical formulation is shown in Eq. (4). With this equation the position and orientation of the foot in movement related to the fixed foot is computed.

The overall kinematic model comprises a total of 12 revolute joints. In order to obtain the inverse kinematics model, the structure is divided into two parts, each with 6 revolute joints, and the center of this structure, the

Center of mass point, has its movement measured to enable the division proposed.

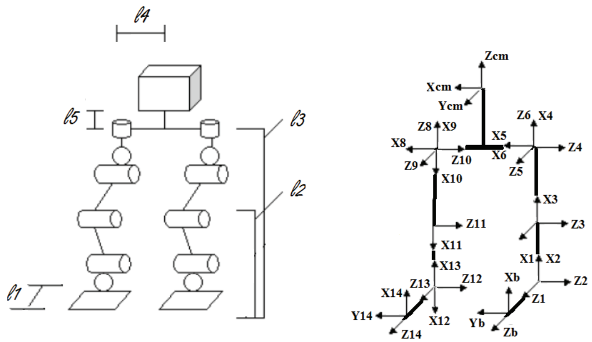

Fig. 1: Locations of coordinate frames.

Tab. 1: Denavit-Hartenberg parameters of both legs.

i	a_i	d_i	α_i	u_i
1	0	$-l_1$	0	0
2	0	0	$\pi/2$	u_1
3	l_2	0	0	u_2
4	l_3	0	0	u_3
5	0	0	$-\pi/2$	u_4
6	0	0	$\pi/2$	$u_5+\pi/2$
7	l_4	l_5	0	u_6
8	l_4	$-l_5$	0	0
9	0	0	$-\pi/2$	u_7
10	0	0	$\pi/2$	$u_8-\pi/2$
11	l_3	0	0	$u_9-\pi$
12	l_2	0	0	u_{10}
13	0	0	$-\pi/2$	$u_{11}+\pi$
14	0	l_1	0	u_{12}

$$T_B^{CoM} = M_1.G_1.M_2.G_2.M_3.G_3.M_4.G_4.M_5.G_5.M_6.G_6.M_7.G_7 \quad (1)$$

where,

$$\mathbf{G}_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_i & 1 & 0 & 0 \\ 0 & 0 & \cos(\alpha_i) & -\sin(\alpha_i) \\ d_i & 0 & \sin(\alpha_i) & \cos(\alpha_i) \end{pmatrix} \tag{2}$$

$$\mathbf{M}_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(u_i) & -\sin(u_i) & 0 \\ 0 & \sin(u_i) & \cos(u_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$T_B^{foot} = T_B^{CoM}.M_8.G_8.M_9.G_9.M_{10}.G_{10}.M_{11}.G_{11}.M_{12}.G_{12}.M_{13}.G_{13}.M_{14}.G_{14} \tag{4}$$

Eq. (1) and (4) were implemented in Matlab code in which it is possible to plot the poses of the structure for different joint values. Additionally, it is possible to check the humanoid model “walking” in a spatial scenario and to obtain the displacement of the feet and center of mass during the gait. To implement the full gait cycle the base frame in the foot was used, which is in contact with the ground. During the stance period of this foot, the displacement of 12 joints is calculated, which moves the center of mass and the swing foot forward. In the sequence, the base frame changes between the feet, since now the stance and swing phases change. The position of the base frame is updated in each step in a loop during the number of specified cycles. In order to implement the humanoid gait simulation, initially 10 poses during each gait cycle were prescribed (Fig. 2).

To implement the inverse kinematics model, a spatial trajectory as shown in Fig. 3 is proposed. The spatial displacement components of the trajectory are based on biomechanics curves presented in [16] and [17].

The proposed method can be applied for any general case. It is possible to prescribe a trajectory describing a general curve in space, which means that the trajectory must not be necessarily in a straight line.

The orientation of the center of mass and foot were taken as in Fig. 3. It describes the trajectory of the feet and center of mass during a complete gait cycle, described by 10 points. In this discrete gait cycle, there are 9 different points of the center of mass trajectory, since the 5th and 6th pose are the same. Related to each foot there are 5 points, since during the stance period the position is constant.

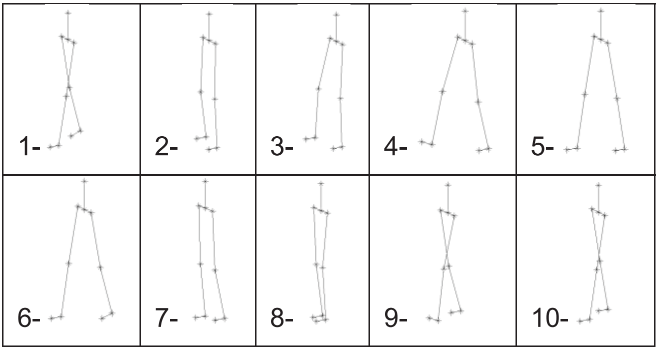

Fig. 2: Poses during gait cycle.

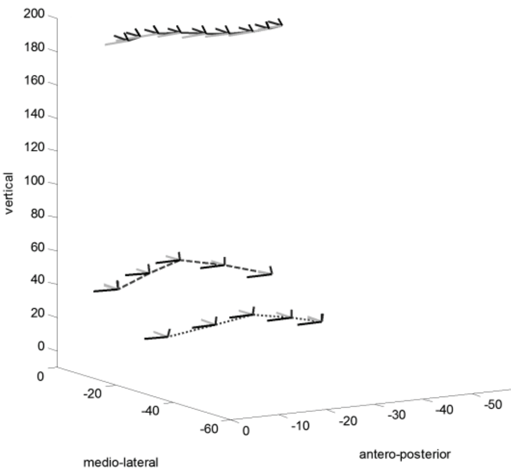

Fig. 3: Proposed trajectory and orientation of the foot and center of mass (values in mm).

3 Inverse Kinematics

According [19], both parametric and implicit representations, used to model mechanisms and robotic mechanical devices, have certain advantages compared to the other. Implicit polynomial methods are not as popular as parametric procedures because in kinematics it is generally difficult to obtain the polynomial representation of a kinematic chain. In [19] an implicitization algorithm is proposed to obtain the polynomial equations related to different kinematic chains. Techniques using polynomial representations of mechanism constraints have been very successful in solving the most challenging problems in kinematics. For example, the first algorithm published to solve the direct kinematics of the Stewart-Gough platform uses the polynomial approach [20]. Algebraic methods have been successfully used to obtain and to classify self-motions of Griffis-Duffy and Stewart-Gough manipulators, [21] and [22], or to synthesize planar, spherical or spatial four-bar mechanisms [23]. In addition, this technique has been used to completely analyze and solve the puzzling kinematics of the 3-UPU manipulator that has caused a lot of headache for many researchers [24].

The advantages one gains in describing the constraints by algebraic equations are manifold. The most important is that one obtains a global description of the mechanism behavior, as opposed to the local, which is obtained for example by screw theory.

Taking the implicitization algorithm and methods described in Husty's works, Pfurner developed in his PhD thesis [15] an "Analysis of spatial serial manipulators using kinematic mapping". Based on [15], the method is applied here to obtain the inverse kinematics of a humanoid robot.

The method proposed uses quaternions and dual-quaternions to represent rigid body orientations and translations in three spatial dimensions. W.R. Hamilton introduced quaternions in 1840, but [25] already discussed basic elements. For more details, [15] presents an introduction to the initial concepts.

A unit dual quaternion is used to represent any rigid transformation (position and rotation), as presented in [26]. It is possible to construct a unit dual-quaternion rigid transformation using Eq. (5) or (6),

$$P = q_r + \varepsilon \frac{1}{2} q_r q_t \quad \text{(Rotation than translation)} \tag{5}$$

or

$$P = q_r + \varepsilon \frac{1}{2} q_t q_r \quad \text{(Translation than rotation)} \tag{6}$$

with the dual unit ε having the property $\varepsilon^2 = 0$. The first part of the dual quaternion is called the real part and the term after ε the dual part.

Quaternions q_r and q_t denote the rotational and translational part, respectively, and $q_r q_t$ is the quaternion product in which each term can be found by relations:

$$q_r = \left[cos\left(\frac{\theta}{2}\right), n_x.sin\left(\frac{\theta}{2}\right), n_y.sin\left(\frac{\theta}{2}\right), n_z.sin\left(\frac{\theta}{2}\right)\right] \quad (7)$$

$$q_t = \left[0, t_x, t_y, t_z\right] , \quad (8)$$

where (n_x, n_y, n_z) is a unit-vector representing the axis of rotation and θ is the angle of rotation. The dual quaternion can represent a pure rotation the same as a quaternion by setting the dual part to $[0,0,0,0]$.

To represent a pure translation with no rotation, the real part can be set to identity $[1,0,0,0]$ and the dual part represents the translation $\left[0, \frac{t_x}{2}, \frac{t_y}{2}, \frac{t_z}{2}\right]$.

A dual quaternion is called normalized if the real part is normalized,

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 = 1 \quad (9)$$

The representation of the group of Euclidean displacements in dual-quaternion coordinates is sometimes also called Soma coordinates, Study coordinates or Study parameters (see [27] and [28]). Study's kinematic mapping κ maps an element α of SE(3) to a point $x \in P^7$ (seven dimensional projective space - see details about projective space in [29]). If the homogeneous coordinate vector of x is $[x_0 : x_1 : x_2 : x_3 : y_0 : y_1 : y_2 : y_3]^T$, the kinematic pre-image of x is the displacement α described by the transformation matrix:

$$\mathbf{A} = \frac{1}{\Delta}\begin{pmatrix} x_0^2+x_1^2+x_2^2+x_3^2 & 0 & 0 & 0 \\ l & x_0^2+x_1^2-x_2^2-x_3^2 & 2(x_1x_2-x_0x_3) & 2(x_1x_3+x_0x_2) \\ m & 2(x_1x_2+x_0x_3) & x_0^2-x_1^2+x_2^2-x_3^2 & 2(x_2x_3-x_0x_1) \\ n & 2(x_1x_3-x_0x_2) & 2(x_2x_3+x_0x_1) & x_0^2-x_1^2-x_2^2+x_3^2 \end{pmatrix} \quad (10)$$

where $\Delta = x_0^2 + x_1^2 + x_2^2 + x_3^2$, and

$$\begin{aligned} l &= 2(-x_0y_1 + x_1y_0 - x_2y_3 + x_3y_2), \\ m &= 2(-x_0y_2 + x_1y_3 + x_2y_0 - x_3y_1), \\ n &= 2(-x_0y_3 - x_1y_2 + x_2y_1 + x_3y_0). \end{aligned} \quad (11)$$

The lower three by three sub-matrix is a proper orthogonal matrix if and only if

$$x_0\, y_0 + x_1\, y_1 + x_2\, y_2 + x_3\, y_3 = 0; \quad (12)$$

and not all x_i are zero. When these conditions are fulfilled we call x= $[x_0 : ... : y_3]^T$ the Study parameters of the displacement α. The important Eq. (12) defines a quadric $S \subset P^7$ and the range of x is this quadric minus the three dimensional subspace defined by

$$E : x_0 = x_1 = x_2 = x_3 = 0; \quad (13)$$

We call S the Study quadric and E the exceptional or absolute generator (see more details about Study Quadric in [30]). For the description of a mechanical device in P^7 we usually need the inverse of the map given by Eq. (10) and (11), that is, we need to know how to compute the Study parameters from the entries of the rotation part of an orthogonal matrix $\mathbf{A} = [a_{ij}]_{i,j=1;...;3}$ and the translation vector $\mathbf{a} = [a_1; a_2; a_3]^T$. Mostly in kinematics literature a rather complicated and not singularity-free procedure, based on the Cayley transform of a skew symmetric matrix into an orthogonal matrix (see [31]) is used. The best way of doing this was however, already known to Study himself, [32] and [33]. He showed that the homogeneous quadruple $x_0 : x_1 : x_2 : x_3$ can be obtained from at least one of the following proportions:

$$\begin{aligned} x_0 : x_1 : x_2 : x_3 &= 1 + a_{11} + a_{22} + a_{33} : a_{32} - a_{23} : a_{13} - a_{31} : a_{21} - a_{12} \\ &= a_{32} - a_{23} : 1 + a_{11} - a_{22} - a_{33} : a_{12} + a_{21} : a_{31} + a_{13} \\ &= a_{13} - a_{31} : a_{12} + a_{21} : 1 - a_{11} + a_{22} - a_{33} : a_{23} + a_{32} \\ &= a_{21} - a_{12} : a_{31} + a_{13} : a_{23} - a_{32} : 1 - a_{11} - a_{22} + a_{33} \end{aligned} \quad (14)$$

In general, all four proportions of Eq. (14) yield the same result. If, however, $1+a_{11}+a_{22}+a_{33}=0$ the first proportion yields 0 : 0 : 0 : 0 and is invalid. We can use the second proportion instead as long as $a_{22}+a_{33}$ is different from zero. If this happens we can use the third proportion unless $a_{11}+a_{33}=0$. In this last case, we resort to the last proportion, which yields 0:0:0:1. Having computed the first four Study parameters the remaining four parameters $y_0:y_1:y_2:y_3$ can be computed from

$$\begin{aligned} 2y_0 &= a_1x_1 + a_2x_2 + a_3x_3, \\ 2y_1 &= -a_1x_0 + a_3x_2 - a_2x_3, \\ 2y_2 &= -a_2x_0 - a_3x_1 + a_1x_3, \\ 2y_3 &= -a_3x_0 + a_2x_1 - a_1x_2. \end{aligned} \quad (15)$$

To illustrate the application of the method, [34] presented some examples. In order to obtain the algebraic expressions, related to the constraint manifold of the structure, [15] states that the basic idea to analyze mechanisms with kinematic mapping is the following: the EE (end effector) of a mechanism is bound to move with the constraints given by the mechanism. Every pose of the EE coordinate system is mapped by kinematic mapping to a point in the kinematic image space. Therefore, every mechanism generates a certain set of points, curves, surfaces or higher dimensional algebraic varieties in the kinematic image space. The corresponding variety of a mechanism is called the constraint manifold. It fully describes the mobility of the EE of a manipulator. Essentially this constraint manifold is the kinematic image of the workspace. A parametric representation of the constraint manifold of an nR mechanism is given by computing the Study parameters of the forward transformation using the Denavit-Hartenberg parameters of the structure. The method of Denavit-Hartenberg (D-H) [35] is applied as described in [15].

Without loss of generality, a kinematic chain can be placed in the base frame in a suitable manner. A manipulator, where the first revolute axis coincides with the z-axis of the base frame and the DH parameter d_1 is equal to zero, is called a canonical manipulator.

Initially the parametric representation of the constraint manifold of a canonical 2R-mechanism is obtained, computed with Eq. (14) and (15), which reads:

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} (\cos(u_2)\cos(u_3) - \sin(u_2)\sin(u_3) + 1)(1 + \cos(\alpha_2)) \\ (\cos(u_2) + \cos(u_3))\sin(\alpha_2) \\ (\sin(u_2) - \sin(u_3))\sin(\alpha_2) \\ (\cos(u_2)\sin(u_3) + \sin(u_2)\cos(u_3))(1 + \cos(\alpha_2)) \\ \frac{1}{2}a_2(\cos(u_2)\cos(u_3) - \sin(u_2)\sin(u_3) + 1)(\sin\alpha_2) \\ -\frac{1}{2}a_2(\cos(u_2) + \cos(u_3))(1 + \cos(\alpha_2)) \\ -\frac{1}{2}a_2(\sin(u_2) - \sin(u_3))(1 + \cos(\alpha_2)) \\ \frac{1}{2}a_2(\cos(u_2)\sin(u_3) + \sin(u_2)\cos(u_3))(\sin(\alpha_2)) \end{pmatrix} \tag{16}$$

By inspection and direct substitution one can verify easily that these coordinates satisfy four independent linear equations, after applying half tangent substitution ($al_i = \tan\frac{\alpha_i}{2}$),

$$\begin{aligned} \overline{Hc}_{11}: &\quad 2a_2 al_2 x_0 - 4y_0 = 0, \\ \overline{Hc}_{12}: &\quad 2a_2 x_1 + 4al_2 y_1 = 0, \\ \overline{Hc}_{13}: &\quad 2a_2 x_2 + 4al_2 y_2 = 0, \\ \overline{Hc}_{14}: &\quad 2a_2 al_2 x_3 - 4y_3 = 0. \end{aligned} \tag{17}$$

It can be observed that these equations are just functions of D-H and Study parameters. In this case, these algebraic expressions were obtained by direct inspection of the Study parameters, but they also can be found using the implicitization algorithm proposed by [19].

From the idea of representing the mechanism using a set of linear equations, or hyperplanes, it is possible to move from a 2R canonical structure to a canonical serial 3R-chain. It results in a set of four hyperplanes depending linearly on the parameter $v_i = \tan\frac{u_i}{2}$, where u_i represents the angle of the joint “i”. It is shown in [15] that it is possible to take each one of the 3 joints of the mechanism to obtain its constraint manifold. Different choices result in different sets of hyperplanes.

In [36] a method is proposed for a general 6R mechanism, which is also described in [15]. By the representation for a 3R-chain, in the sequence, it is possible to divide a 6R structure in 2 structures of 3R chains. This results in a set of 8 hyperplanes in function of v_i (i=1, 2 or 3) and v_j (j=4, 5 or 6), which, together with the equation of the Study quadric and a normalizing condition, result in a set of equations which represents the structure.

The inverse kinematics problem becomes therefore an algebraic problem, since the values of the joint angles are obtained by solving this system of equations. In the design of the humanoid robot there is a spherical joint in the hip. This is a special case of a 3R-chain and this yields a set of 4 hyperplanes not dependent of any joint angle. The remaining 3R-chain can be expressed in function of one joint, resulting in a system of 8 equations, 4 equations are functions of v_i and 4 equations are functions just of the Denavit-Hartenberg and Study parameters. An arbitrary spherical 3R-chain (arbitrary means it is not necessary to be canonical) and the set of all hyperplane equations are described in [15].

For each of the 10 poses, the inverse kinematics model yields 8 possible solutions of the joints. It is possible to implement an algorithm to select one of the 8 solutions giving practical values applied to the humanoid robot. In this case, one of the asked conditions is to select solutions with $u_3 \leq 0$, since the knee cannot bend in the opposite direction. The second condition is that all 6 angles are between +/-120 degrees, since during gait, in general, any joint does not move with amplitudes outside of this range.

4 Trajectory Interpolation

The proposed trajectory is described by 10 discrete poses and the corresponding joint angles can be calculated using the inverse kinematics model. However is necessary to know the joint velocities and accelerations which move the joint between the desired poses, and which have direct influence to the spatial acceleration of the center of mass and to the dynamic equilibrium of the humanoid robot during gait. This movement depends on the joint angles values in the time of transition between the poses, which can be defined by the interpolation of the trajectory.

The problem to obtain a trajectory with a discretization as small as desired can be solved by different methods. In this section initially the interpolation of the poses is presented, which can be done by dual quaternion interpolation. This is an important advantage of models based on dual quaternions if compared with homogeneous matrices. A second possible method is the interpolation of the joint parameters, which results in a corresponding interpolation of the discrete poses of the spatial trajectory of the end effector, taking interpolated values of the joints angles and computing the forward kinematics.

One of the advantages using dual quaternions to represent a spatial trajectory is related to the possibility to interpolate the poses on the Study quadric. In [37] and [38] a complete analysis of the interpolation process is presented, with comparisons of different algorithms. Based on [39] these algorithms are implemented in Matlab and furthermore each of these methods is discussed with examples.

One of the discussed algorithms implements the QB-curves. This method interpolates 3 points on a quadric by changing two control points. One of the advantages of the QB-curves is to ensure the continuity of two segments by the same tangent in the last control point of the first segment and the first control point of the second segment. In [39] the influence of the design points in the resultant trajectory is discussed. Different values of the control points result in different trajectories, i.e., the solution of the problem is not unique. There are different combinations of control points which can interpolate the discrete poses of the trajectory resulting in different displacements and also in different corresponding velocity and acceleration curves.

The interpolation of the initial poses of the gait should result in the same acceleration curve, in order to compare the results of the acceleration of the foot and the simulated center of mass trajectory with those measured with the IMU sensors in a human gait, which have a different number of poses corresponding to different points in the image space. As there is more than one possibility to define the control points, the matching of the acceleration would be an optimization problem, in order to adjust the control points to minimize the difference between theoretical and experimental results. The results would be interpolated points contained in the Study quadric, but the problem to choose the best control points is not the method pursued in this work.

Here, the method of interpolating the joint values is used. This method can be applied under two different conditions. If the angles of the joints are already known at the desired poses, it is possible to move directly through the interpolation steps, but if just the poses are known, it is necessary to use the inverse kinematics model to obtain the joint angles corresponding to each pose.

After the process of interpolation, the angles are used in the forward kinematics model, in order to simulate the new poses. The advantage is that the code from forward kinematics used to simulate the robot gait can be applied to obtain the trajectory including the new joint angles, resulting in both, center of mass and foot interpolated poses.

A linear interpolation between the joint angles values can be taken since constant velocity is assumed during the joint movement between two poses. This means that acceleration is different from zero just in the instant in which the velocity changes between two constant values.

The Eq. (18) gives the expression to obtain the constant value of the joint velocity.

$$\omega_{i,p\rightarrow p+1} = \frac{u_{i,p+1} - u_{i,p}}{\Delta t}, \qquad (18)$$

where the values to be interpolated are between the poses “p” and “p+1”, $\omega_{i,p}$ is the angular velocity of the servo actuator “i” to move from the pose “p” to the pose “p+1”, $u_{i,p+1}$ is the angle of the joint “i” at the pose “p+1” and $u_{i,p}$ is the angle of the joint “i” at the pose “p”. The time to move from the pose “p” to the pose “p+1” is Δt. The assumption of constant velocity results in a small error as small as the values of Δt. For the gait analysis which divides the gait into 10 points, the value of Δt is assumed to be as small as possible to ensure this linear approach. The interpolated value of the joint angle “i” is:

$$u_{i,t} = u_{i,p} + t.\,\omega_{i,p\rightarrow p+1} \qquad (19)$$

Where “t” varies with the desired increment “dt” from 0 to $\Delta t - dt$. When “t” is equal 0 it results in $u_{i,t} = u_{i,p}$ which means that first point of interpolation is the value of the joint “i” at the pose “p”. If “t” is equal Δt it would result in the angle of the joint at the pose “p+1”, which is the same point of the first pose of the next segment. In order not to result in two equal angle values, “t” varies to $\Delta t - dt$, with the exception for the last two poses of the trajectory, in which “t” varies from 0 to Δt .

In real implementations, a simple set point for the actuator control can be the joint velocity. In this case, the value of the acceleration required to change the velocity during the time of transition is not specified, however a jerk condition can occur. This assumption is possible since experimental tests have demonstrated that for small time steps, the variation of the velocity (acceleration) is low. In addition, the velocity control is relatively simple to be implemented for the actuators.

Despite these assumptions of the joint velocity, the interpolated values of joint angles result in real poses for both center of mass and foot, ensuring the correct analysis of the trajectory and the comparison of the second derivative of the displacement with the experimental data of the accelerometers.

The joint values obtained for the proposed trajectory were interpolated and substituted into the forward kinematics model to simulate the displacement of the feet and center of mass. The second derivatives of these curves were compared with experimental results of human gait presented in literature, as mentioned in Introduction section.

If the analysis of the joints accelerations is desired, numerical differentiation of the joint velocities is applied.

5 Dynamic Stability

The second derivative of the proposed spatial displacement curves, after the interpolation process, yield the acceleration components of the motion. In order to define the theoretical trajectory, these components were compared with biomechanics curves based on IMU signals measured during human gait. The poses were adjusted to obtain the most similar behavior for the humanoid gait. The components are related to a global inertial frame. The cycle gait period of simulation was defined to be 0.38 seconds.

Using the theoretical components of displacement and acceleration makes it possible to analyze if the proposed movement is in dynamical equilibrium during the gait.

It is important to note that the analysis is done using the proposed movement of the center of mass and foot. The gait is analyzed without any perturbation, i.e. only the theoretical results of the spatial trajectory were taken into account.

The proposed equilibrium analysis is based on the Zero Moment Point Method presented in [18]. The model was assumed with concentrated mass as an inverted pendulum, as it was used in [1] and [40].

The trajectory of the Zero Moment Point lies in the horizontal plane. According to Vukobratovic the robot is in equilibrium positions during the movement, if the ZMP is positioned inside the polygon of the contact base.

Taking the proposed trajectory of the feet during the gate, it is possible to identify the double stance phase first, which is given by the period in which both feet are in contact with the ground. This information is obtained by analyzing the vertical component of the spatial trajectory with a zero amplitude for both feet.

When the single and double stance period are known, the polygon of contact base is observed in the lateral and frontal components of the spatial trajectory. During the single stance period, the polygon is given by the region of the single foot on the ground. In the lateral component it means the width of the foot, and in the frontal (antero/posterior) component, it means the length of the foot. During the double stance period, the contact base is defined as the region between both feet. Fig. 4 shows the curve of the ZMP and the polygon region of each foot when being in contact with the ground.

The numbers 1, 2 and 3 are related to the single stance period of the left foot, double stance period and single stance period of the right foot. In all cases, the position of the ZMP is inside the polygon of the base, represented by the grey square in each phase, which ensures that in the proposed trajectory the robot is in equilibrium according to the ZMP method.

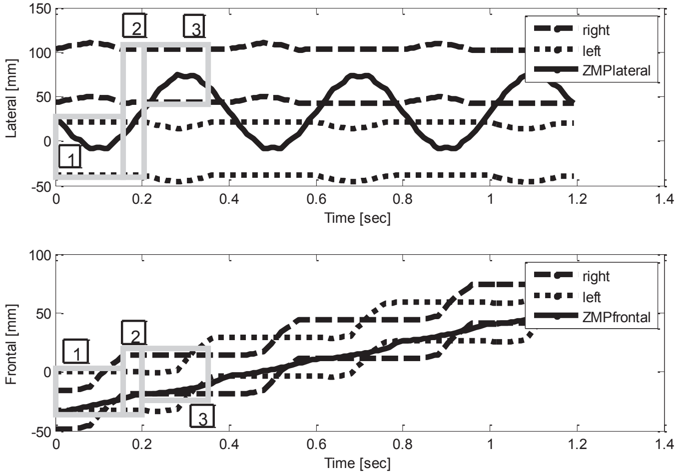

Fig. 4: ZMP during proposed trajectory.

6 Conclusion

In this work, an algebraic geometry analysis of the humanoid robot gait was proposed. Study parameters were employed instead of homogeneous matrix representation and trigonometric functions, in order to obtain a dual quaternion representation of the poses during a general spatial trajectory.

The biomechanical data of human gait were used to define the trajectory of the foot and center of mass of the robot. Taking the poses of the human gait, it was possible to define a trajectory for humanoid robots as similar as possible to the real. The dual quaternion formulation enabled to eliminate singularities of trajectory monitoring and modeling, which occur with the Euler angle representation. Numerical simulations were implemented to evaluate the dynamic stability of the proposed humanoid robot gait, taking the influence of the trajectory interpolation.

An important contribution of this work for future research is the possibility of to simulate the movement of the structure, calculating numerically the acceleration in each body in order to obtain the most optimized trajectory ensuring the equilibrium. The previous knowledge of the acceleration of the body can be used to add some small variation in amplitude due to dynamics efforts. In this case, it is possible to simulate the necessary trajectory in order to have equilibrium of the body also taking this additional acceleration in the resultant component. Different control strategies can be tested in the equilibrium simulation. A model based on acceleration data avoids measuring many parameters in real applications, since solely IMU sensors can be used. This results in cost reduction and also in reduction of the time to process the large number of data.

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