Efficiency of monolithic solvers for incompressible fluid-structure interaction problems

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Motivation

- Real world problems require accurate predictions at reasonable computational costs (accuracy vs. numerical effort)
- Parallel scalability for very large problem sizes

Problem Definition & Discretization

- Solid domain $\Omega^s$: nonlinear elastodynamics
- Fluid domain $\Omega^f$: incompressible Navier-Stokes equations with ALE observer
- Fluid-Structure Interface $\Gamma_{FSI}$: weak enforcement of coupling conditions using Lagrange Multiplier field $\lambda = \lambda_{FSI} = -\nabla \cdot \psi_{FSI}$ [1]
- Spatial discretization of structure and fluid field with finite elements
- Temporal discretization with single-step, single-stage, and fully implicit marching time integrators

Adaptive Time Stepping: Algorithm

- Individual error estimation in both fields via:
  - Comparison to auxiliary explicit scheme (e.g. Adams-Bashforth 2)
  - Error norms: length-scaled $L_2$-norms
- Deduce separate norm for FSI interface DOFs to account for central role of the interface
- Adapt step size based on estimated errors $\kappa_{est} = \frac{\text{tol error}}{\Delta t}$

- Compute $\Delta t_{new}$ for every subset of DOFs $\Delta t_{new} = \min (\Delta t_{max}, \min (\Delta t_{struct}, \Delta t_{fluid}))$
- Choose minimal time step size suggestion to guarantee accuracy everywhere in the domain $\Delta t_{new} = \min (\Delta t_{struct})$

Newton-Krylov with FSI-specific Preconditioning [4]

- Algebraic coarse level problem
- FSI coupling on all levels

Example: Pressure Wave

- Significant superiority in "difficult" FSI problems $\rho^s/\rho^f \approx 1$
- 3 Newton steps per time step (avg.)
- outperforms partitioned approaches by far
- Suitable for very large scale computations

Monolithic System of Equations

- Linear System of Equations
  - We exemplarily choose the structure field as master field $\rightarrow$ structure-handled interface motion
  - Spatial discretization of structure and fluid field
  - Deduce separate norm for FSI interface DOFs to
  - Compute $\tau_{FSI}$ for every subset of DOFs
  - Adapt time step size based on estimated errors
  - Choose minimal time step size suggestion to guarantee accuracy everywhere in the domain

Condensation of Lagrange Multipliers

- Use balance of linear momentum of slave interface DOFs for condensation
- Dual Mortar method leads to diagonal form of Mortar matrix $\mathbf{D}$
- Computationally cheap condensation of Lagrange multipliers and slave interface DOFs

Nonlinear GMRES with AMG-FAS Preconditioning

- FAS: Full Approximation Scheme
- Variational residual evaluation
- Motivation
  - Timings: residual evaluation vs. linear solve
  - Residual evaluation scales perfectly for fine grained parallelization
- Idea
  - nonlinear FSI-coupling on coarse levels using FSI-AMG hierarchy
  - acceleration by outer nonlinear Krylov-type solver [5]
- Multigrid library: Trilinos/MueLu [6]

Algorithm

- Algebraic coarse level problem
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Example: Pressure Wave

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AMG/BSG characteristics averaged over timesteps

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References