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Intermittent bursts presage the onset of instability in turbulent combustors

Vineeth Nair*[1], Gireeshkumaran Thampi[1] & R. I. Sujith[1]

[1] National Centre for Combustion Research and Development, Department of Aerospace Engineering,
Indian Institute of Technology Madras
Chennai, 600036, India
* Corresponding author: v.vineeth.nair@gmail.com

The dynamic transition from combustion noise to combustion instability was investigated experimentally in a laboratory scale turbulent combustor by systematically varying the flow Reynolds number. We observe that the onset of combustion-driven oscillations is always presaged by intermittent bursts of high-amplitude periodic oscillations that appear in a near random fashion amidst regions of aperiodic, low-amplitude fluctuations. These excursions to periodic oscillations become more and more frequent as operating conditions approach instability and finally the system transitions completely into periodic oscillations. A continuous measure to quantify this bifurcation in dynamics can be obtained by counting the number of peaks in a measured signal above a predefined threshold. The recurrence properties of the dynamics of such bursts were quantified using recurrence plots and the distribution of the aperiodic phases were examined. We conjecture that the intermittent oscillations emerge through the establishment of homoclinic orbits in the phase space of the global system which is composed of hydrodynamic and acoustic subsystems that operate over different time scales.

1 Introduction

The occurrence of combustion instabilities remains a challenging problem to the propulsion and power generation industry as they may be driven by a variety of flow and combustion processes usually coupled with one or more of the acoustic modes of the combustor [1]. Large amplitude pressure oscillations are easily established in confined, convecting combustion environments since only a small fraction of the available energy from heat release is sufficient to drive the oscillations with the attenuation in the confinement being weak. Such combustion-driven oscillations are detrimental and result in performance losses, reduced operational range and wear and tear and fatigue failure of the combustor walls due to increased heat transfer [2, 3].

Even in the absence of large amplitude oscillations, unsteady combustion tends to be a noisy process and this noise, termed ‘combustion noise’ in the literature is attributed mainly to two sources—direct combustion noise due to unsteady volumetric expansion in the reactive region and indirect combustion noise which is produced when the hot combustion products traverse a region with mean flow gradients [4]. Although there are a lot of studies that focus on the characteristics of combustion noise and combustion instabilities, relatively little attention has been received as regards the dynamics of transition from one to the other for changes in operational parameters. An understanding of the physical mechanisms behind such a change in behaviour of the global system comprising the flow, acoustics and the combustion processes is critical to identifying robust early warning signals to impending instabilities.

A systematic variation of operating conditions in bluff-body and backward-facing step combustors from stable to unstable operation was performed by Chakravarthy *et al.* [5, 6]. They found that at the regimes of stable operation characterized by low-amplitude broadband noise generation, the vortex shedding and duct acoustics do not lock-on. However, the broad band noise generation gives way to the excitation of high-amplitude discrete tones, at the onset of lock-on. In a recent experimental study, Gotoda *et al.* [7] reported that the transition to thermoacoustic oscillations happened from stochastic fluctuations to periodic oscillations through low dimensional chaotic oscillations, when the fuel equivalence ratio was varied.

The methodologies to prevent large-amplitude oscillations in combustors available in the literature largely focus on suppression of an incipient instability, i.e., an instability that has already begun. Understanding the dynamics of the transition and hence the route to instability, one has better chances of finding robust precursors that can forewarn the onset of an impending instability, thereby providing operators of fielded combustors with warning signals sufficiently in advance to take effective control action.

The dynamic transitions in a thermoacoustic system that consisted of a ducted, laminar, premixed flame was studied by Kabiraj *et al.* [8]. Transitions to chaos were identified to occur via quasiperiodic oscillations [9] as well as frequency-locked oscillations [10]. They also found that intermittent large-amplitude chaotic fluctuations were seen in unsteady pressure measurements prior to flame blowoff [8, 11]. To study the rich behaviour in dynamics of even such simple thermoacoustic systems, methods of nonlinear time series analysis were seen to be extremely useful. The present study follows similar lines of analysis in characterizing the dynamic transitions in turbulent combustors from low-amplitude combustion noise to high-amplitude combustion instability with the hopes of identifying robust precursors to detrimental oscillations in such combustors.

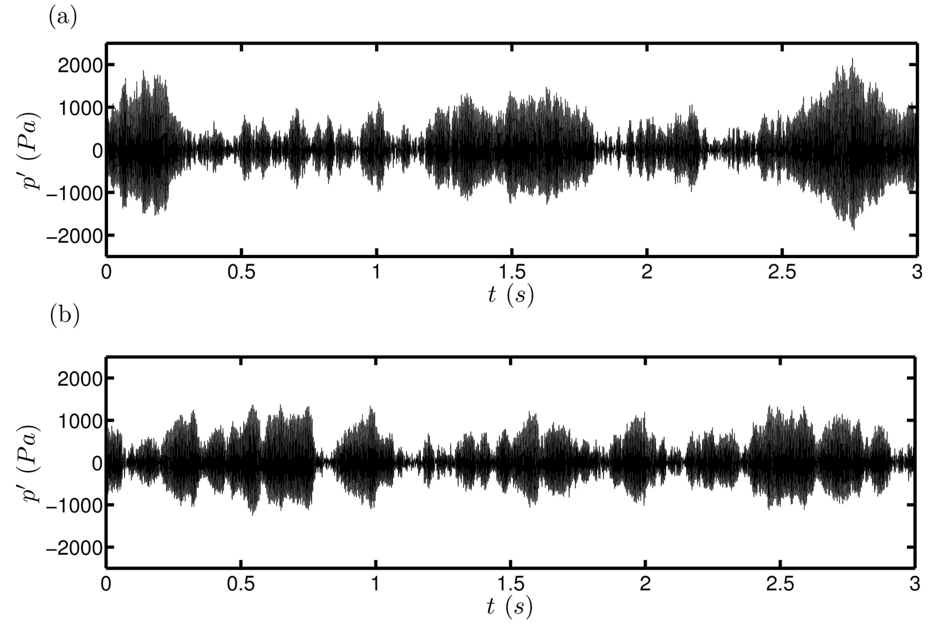

Figure 1: Intermittent signals obtained from various combustors. (a) Unsteady pressure signal from the bluff-body stabilized backward facing step combustor used in the study ($\dot{m}_a = 11.84 g/s$, $\dot{m}_f = 0.59 g/s$) and, (b) unsteady pressure measurement from a swirl-stabilized backward facing step combustor ($\dot{m}_a = 12.04 g/s$, $\dot{m}_f = 0.55 g/s$). The signal is composed of high amplitude periodic fluctuations interspersed amidst low amplitude chaotic fluctuations. Such intermittent burst oscillations are always observed prior to the onset of instabilities in turbulent combustors.

In the present study, we report the presence of intermittent dynamics in confined, compressible, turbulent flow environments prior to a transition to high-amplitude periodic oscillations from low-amplitude chaotic fluctuations. In a recent study, combustion noise was shown to be composed of chaotic fluctuations of moderately high dimensions ($d = 8 - 10$) by performing a series of tests for determinism and chaos [12]. Unsteady pressure measurements made at conditions immediately prior to oscillations were seen to be composed of bursts of high-amplitude periodic oscillations embedded amidst regions of chaotic fluctuations (see Fig. 1). Pressure measurements from both a swirl-stabilized combustor as well as a bluff-body stabilized combustor are shown to emphasize the fact that such bursts are observed prior to onset irrespective of the flame-stabilization mechanisms employed. This intermittency observed in combustors is different from the intermittency reported in measurements of energy dissipation in flow turbulence because what we have here are intermittent bursts of periodic oscillations that presage a transition in dynamics from aperiodic to periodic. The duration of such periodic bursts keep increasing as the flow parameters approach instability until the periodic oscillations take over.

One of the aims of the present study is to establish that such intermittent bursts always presage an impending instability in turbulent combustors. Continuous measures that rely on intermittency in a measured signal are sought for that can then serve as robust precursors to the onset of large-amplitude periodic oscillations. This involves studying the characteristics of the aperiodic states amidst the periodic bursts and the recurrence properties of such bursts. Also, since typical measures such as the amplitude of oscillations cannot serve as measures of bifurcation in such systems with varying amplitudes, we also have to identify suitable bifurcation measures to study intermittent transitions to instability in turbulent combustors.

The paper is outlined as follows. Section 2 outlines the setup used to investigate the intermittent oscillations in turbulent combustion. The methods of obtaining bifurcation diagrams for systems with intermittency and the recurrence properties of intermittent bursts are explored in Section 3. Finally, the conclusions are summarized in Section 4 and the procedure to obtain various measures that characterize intermittency are elaborated in the Appendix.

2 Experiments

Experiments were conducted on a bluff-body stabilized backward facing step combustor operating at high Reynolds numbers. The schematic of the setups used for the current study is shown in Fig. 2. It consists of a plenum chamber, a burner with a shaft (diameter 10 mm) for supporting the bluff-body and a combustion chamber with extension ducts. The bluff-body was a circular disk of diameter 47 mm and thickness 10 mm. It was located at a fixed position of 50 mm from the rearward facing step using a rack and pinion traverse with a least count of 1 mm. A disk of 2 mm thickness with 300 holes of diameter 1.7 mm was located 30 mm downstream of the fuel-injection location to act as a flashback arrestor. The combustion chamber consisted of a sudden expansion from the circular burner of diameter 40 mm into a square geometry of cross-section 90×90 mm^2. The length of the combustor chamber along with the extension ducts was set at 700 mm and pressure transducers and thermocouples were mounted at different locations along this length. A spark plug with a step-up transformer was mounted in the dump plane for ignition of the fuel-air mixture. A blow-down mechanism was used to supply air from high pressure tanks which then passed through a moisture separator before finally entering the plenum chamber. The central shaft was used to deliver fuel (LPG) into the chamber through four radial injection holes of diameter 1.7 mm.

To accurately control and measure the air and fuel flow rates, mass flow controllers with digital logging and monitoring capabilities (Alicat Scientific, MCR Series) were used which had an uncertainty of ± (0.8% of reading + 0.2% of full scale). Unsteady pressure measurements (p') was made 90 mm from the rearward facing step using a piezoelectric transducer (sensitivity 72.5 mV/kPa, 0.48 Pa resolution and $\pm 0.64\%$ uncertainty) mounted on a specially made pressure port flush mounted on the combustor wall. To ensure near-adiabatic boundary conditions, wall cooling was avoided and the operation of the combustor was restricted to short durations. Teflon adapters were used to protect the transducers from excess heating and the mounts were suitably designed to ensure integrity in the measured signals. The phase correction required with this arrangement was calculated to be less than 2^0. The signals from the transducers were acquired through a 16-bit

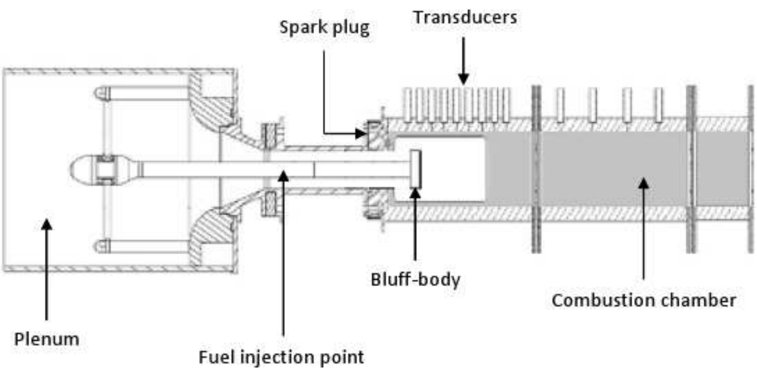

Figure 2: The bluff-body stabilized backward facing step combustor used in the current study. The fuel is injected through four radial holes in the central shaft and spark-ignited using an 11 kV ignition transformer. The fuel injection location was 160 mm upstream from the bluff-body. The length of the combustion chamber is 700 mm with three extension ducts, two of length 300 mm and one of length 100 mm. The design of combustor was adapted from [13].

A-D conversion card (NI-643) with an input voltage range of $\pm 5\ V$ and a resolution of $\pm 0.15\ mV$.

For all the experiments conducted, the ambient temperature was measured to be $(27 \pm 1)^0\ C$ using a dry bulb thermometer and the relative humidity was measured to be $(85 \pm 1)\%$ on a hygrometer. The measurements made by the mass flow controllers were in SLPM (standardized for air at ($25^0\ C$, 14.696 psi) which was then converted to g/s for computing the flow Reynolds number. The fuel used was LPG which is 60% C_4H_10 and 40% C_3H_8 by volume. The flow Reynolds number was obtained as $Re = 4\dot{m}D_1/\pi\mu {D_0}^2$, where $\dot{m}$ is the mass flow rate of the fuel-air mixture, D_1 is the diameter of the circular bluff-body, D_0 is the diameter of the burner and μ is the dynamic viscosity of the fuel-air at the experiment conditions. Corrections to viscosity were made for changes in the fuel-air ratio in the calculation of Reynolds numbers, the procedure for which can be found in [14].

3 Measures of intermittency

3.1 Burst count

One simple way to identify the onset of impending instabilities would be to count the number of peaks (N) in the signal $\phi(j)$ for a time duration (t) above a fixed threshold (ξ) which would correspond to acceptable levels of amplitude for the system. If N_{tot} is the total number of peaks that happen within that time one can then assign a probability of the system to attain instability as:

$$p = N/N_{tot} \tag{1}$$

This value of p is a measure of the proximity of the system to instability and forms a smooth measure that utilizes the intermittent nature of transitions to combustion instability. In Fig. 3, we have plotted values of p for various Reynolds number starting from low amplitude combustion noise to instability and back to stable operating regimes. The threshold was set at 600 Pa that corresponded to the levels of noise in the system during stable combustion. These values of p thus serves as an appropriate measure with which to draw the bifurcation diagram in systems exhibiting widely varying amplitudes in the signals for which an appropriate amplitude or norm cannot be defined. The value of p is seen to increase smoothly to 1 for increases in Reynolds number with the curve resembling a sigmoid. A hysteresis is clearly visible in this new bifurcation diagram that informs of the subcritical nature of the transition to combustion-driven oscillations. Thus,

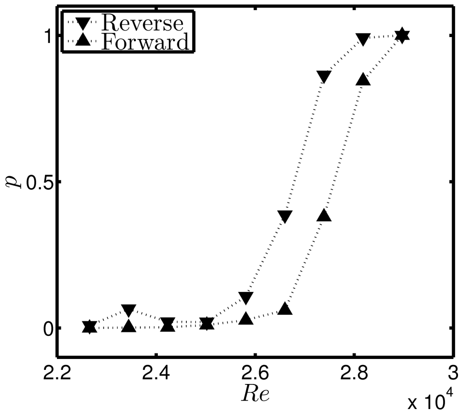

Figure 3: Bifurcation diagram for the transition from chaotic combustion noise to high amplitude combustion instability for the bluff-body stabilized backward facing step combustor ($\dot{m}_f = 0.59g/s$). The shape of the forward and return trajectories closely resembles a sigmoid curve. A hysteresis loop is clearly visible in the bifurcation diagram. This shows that the burst count with a set threshold forms an effective measure in the study of dynamics of systems exhibiting intermittent oscillations. The threshold was set at 600 Pa.

counting the bursts in the measured signal serves as an appropriate measure to obtain bifurcation diagrams.

The temporal features of the dynamics can be better understood by tracking the regularity of the trajectories using recurrence plots. Recurrence is a fundamental property of dynamical systems and recurrence plots allow one to visually identify the times at which the trajectory of the system visits roughly the same area in the phase space [15]. The technique requires reconstruction of the mathematical phase space of evolution of the pressure fluctuations, the procedure for which is outlined in [12]. In reconstructing an appropriate phase space, one requires knowledge of the appropriate embedding dimension d_0 as the optimum time lag τ_{opt} that is used to generate the delay vectors from the measured pressure time series (of length N_0). A recurrence plot is constructed by computing the pairwise distances between points in the phase space. Then, a matrix of recurrences may be obtained as:

$$R_{ij} = \Theta(\epsilon - \|\mathbf{p}'_i - \mathbf{p}'_j\|) \quad i, j = 1, 2, \ldots, N_0 - d_0\tau_{opt} \tag{2}$$

where Θ is the Heaviside step function and ϵ is a threshold or the upper limit of the distance between a pair of points in the phase space to consider them as close or recurrent. The indices represent the various time instances when the distances are computed and the boldface represents the vector of coordinates in the phase space. The recurrence matrix, which is a symmetric matrix composed of zeros and ones provides a visual method of exploring the dynamics of the system as it evolves in time. The zeros in the recurrence plot are marked with black points and represent those time instants when the pairwise distances are less than the threshold ϵ. White points in the recurrence plot correspond to the ones in the recurrence plot and correspond to those instants when the pairwise distances exceed the threshold.

3.2 Recurrence quantification

The recurrence plot for the signal shown in Fig. 1 a was plotted (Fig. 4 a) and displays black patches that are typical of intermittent signals. The white patches represent the times at which systems exhibits large-amplitude intermittent bursts and the black patches represent the times of low-amplitude chaotic oscillations. The nature of the intermittency shows features that could be isolated to those observed during type-II or type-III intermittency. Type-II intermittency is

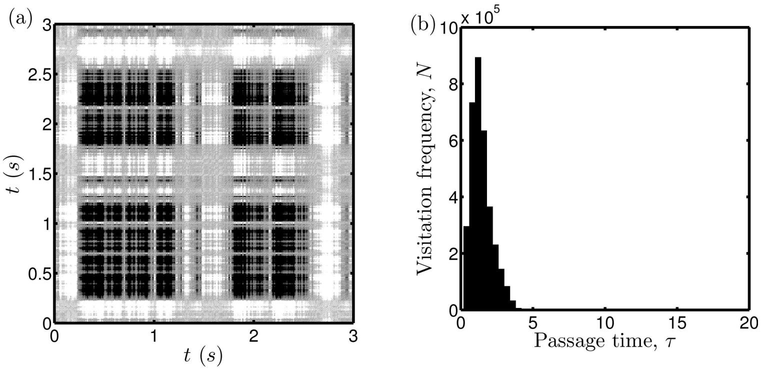

Figure 4: Recurrence characteristics for the signal shown in Fig. 1 a. (a) The recurrence plot displays black patches inside lighter shaded shapes which are typical of intermittent signals ($\epsilon = 0.2\lambda$ where λ is the size of the attractor). The black patches represent the times when the system exhibits low amplitude chaotic oscillations and white patches represent the bursts. (b) Histogram of the phases of low amplitude oscillations in the signal. This pattern with an exponential tail is seen only for intermittencies of type-II or type-III. Such a skewed distribution with the peak off the mean value and an exponential tail is a pattern that is only observed in systems with a homoclinic orbit in the phase space.

observed in systems close to a Hopf bifurcation and type-III is observed near a subcritical period doubling bifurcation. Although the subcritical nature of dynamics could be discerned from the bifurcation diagram (Fig. 2), identification of the exact nature of intermittency (type-II or type-III) was not possible from the recurrence plots. The nature of oscillations after the onset at times displays period-2 oscillations. Hence, even though present evidence points to a type-III intermittency, further investigation is required to clarify whether the bifurcation to instability is Hopf or whether it happens through a subcritical reversed period-doubling route from chaos.

The time spent by the dynamics in between two intermittent bursts is termed the passage time. The passage times at any intermittent operating condition can be estimated by obtaining the distributions of the vertical black lines (or horizontal black lines since the recurrence matrix is symmetric) in the recurrence plot. This is because the black points in the recurrence plot correspond to those instants of time when the dynamics is characterized by low-amplitude chaotic fluctuations. It is known that both type-II and type-III intermittencies have exponential tails in their distributions of passage times. Histogram of the aperiodic phases seen in the recurrence plot of Fig. 4 a is shown in Fig. 4 b. The histogram reveals a skewed distribution with its peak off the mean value and has an exponentially decaying tail. Such a distribution is a distinctive feature of homoclinic orbits in the underlying phase space [16]. A homoclinic orbit is one in which the unstable manifold of the hyperbolic fixed point of the system merges with its own stable manifold. In other words, such orbits take the system from a fixed point solution to an oscillatory state and back to the fixed point which one observes in the dynamics as intermittent bursts.

Such a homoclinic orbit is shown in the reconstructed phase space of the pressure signals in Fig. 5. The homoclinic orbit establishes connections between the irregular hydrodynamic fluctuations and the periodic acoustic fluctuations. There is a reinjection of the homoclinic orbit into the vicinity of the unstable periodic orbit every time there are concentrated heat release fluctuations from combustion in proper phase with the chamber acoustics. Since the periodic orbit is unstable, the system eventually returns to aperiodic oscillations unless sustained by a continuous heat release fluctuations. The onset of instability can then be deduced to occur when the heat release rate occurs in the proper phase range consistently enough so that the acoustic oscillations never die down; i.e., they become self-sustained. This can happen when the hydrodynamics locks-in to the acoustics resulting in a concentrated heat release through vortex formation and impingement at a

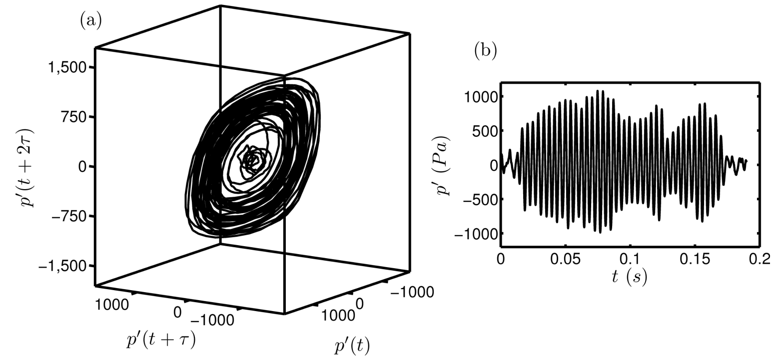

Figure 5: (a) A homoclinic orbit in the reconstructed phase space of the (b) unsteady pressure signal from the bluff-body stabilized combustor ($\dot{m}_a = 11.64 g/s$, $\dot{m}_f = 0.67 g/s$). The homoclinic orbit takes the system from low-amplitude aperiodic fluctuations to high amplitude periodic oscillations through an out of plane spiraling and then back to low-amplitude fluctuations through an in-plane spiraling. The orbit thus establishes connections between the hydrodynamic fluctuations and the confinement acoustics resulting in intermittent bursts.

location with appropriate phase matching.

In turbulent combustors, the route to instability from chaotic combustion noise is through an intermittent regime in operating conditions composed of high amplitude intermittent bursts amidst chaotic low-amplitude fluctuations. Several statistical measures may be constructed through a recurrence quantification analysis of a measured signal that could serve as useful measures of intermittent oscillations. These measures can further be used as precursors to an impending instability because they vary in a smooth fashion as the operating conditions traverse the intermittent regime into conditions of combustion instability. Some of these measures were obtained and plotted in Fig. 6. The definition of the various measures plotted and the procedure to compute them from a recurrence plot are detailed in the Appendix.

The density of points in the recurrence plot (which is termed RR) is seen to decrease on the approach of instability (Fig. 6 a). This is expected since the number of black points in the recurrence plot would come down as instability is reached because the pairwise distances now exceed the threshold more often. This decrease in the density of black points should then correspond to a decrease in the time spent by the system in aperiodic states (measured by τ_0, see Fig. 6 d). The quantity τ_0 also quantifies how long the system remains in a particular dynamical state (in this case, chaotic fluctuations). Hence we expect this quantity to tend towards 0 as the system transitions completely into periodic oscillations. The value of τ_0 will correspond to the duration of data acquisition ($3s$ in the present case) at conditions of combustion noise. The remaining two measures s and L_0 (Fig. 6 b, c) are quantifiers of the amount of order in the system. We see that the Shannon entropy of the signal (s) tends towards 0 at the onset of instability (Fig. 6 b). This indicates that the system is approaching a state of regularity or there is an emergence of order out of chaos. The average length of a diagonal line in the recurrence plot (L_0) should correspondingly show diminishing values towards the onset with more and more points exceeding the threshold in the recurrence plot. Thus, we see that the measures show a variation in behaviour well before the operating conditions approach combustion-driven oscillations. Hence, they can be used as robust precursors to an impending instability in fielded combustion systems, or more generally, in turbulent flow systems encountering periodic oscillations. We performed experiments in an aeroacoustic system and the measures predicted the onset of oscillations well before they occured.

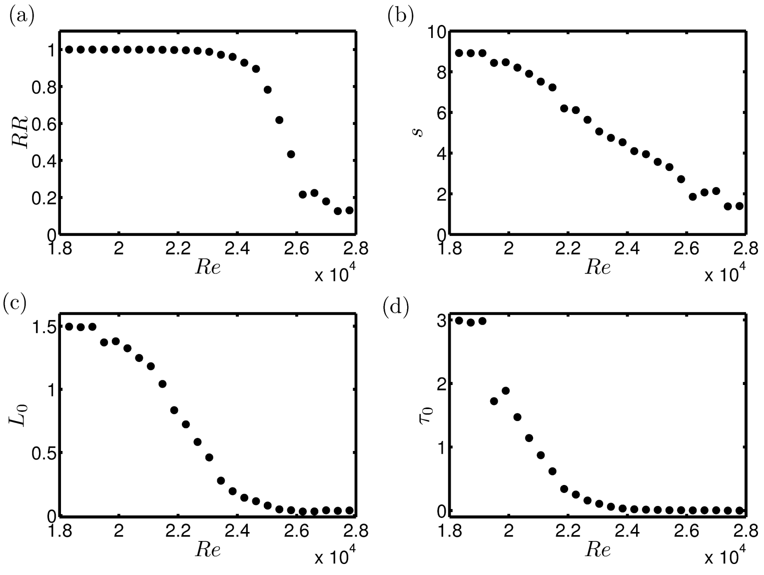

Figure 6: Smooth measures of intermittency that can serve as precursors to impending combustion instability; (a) the recurrence rate of dynamics which measure the density of points in the recurrence plot, (b) the entropy of the diagonal length distribution, (c) the average diagonal length, and (d) the average passage time spent by the dynamics in aperiodic fluctuations. Notice that all these quantities vary in a smooth fashion from chaotic low-amplitude combustion noise to high-amplitude periodic combustion instability. The definition of the quantities plotted is given in the Appendix ($\dot{m}_f = 0.59g/s$).

4 Concluding remarks

The transition from combustion noise to combustion instability in turbulent combustors was always seen to be presaged by intermittent bursts of high-amplitude periodic oscillations amidst regions of aperiodic, low-amplitude fluctuations. The intermittent oscillations were seen to emerge through the establishment of homoclinic orbits in the reconstructed phase space of unsteady pressure measurements. The reinjection of such homoclinic orbits near the vicinity of the unstable periodic orbit of acoustics was assisted through the unsteady heat release fluctuations. Such oscillations eventually die down to low-amplitude hydrodynamic fluctuations in the absence of self-sustaining heat release fluctuations from combustion established as a result of the feedback of the combustion process with the confinement acoustics.

A smooth and continuous measure to obtain bifurcation diagrams for turbulent combustors can be obtained by counting the number of peaks in a measured signal above a predefined threshold. Hysteresis was observed for variations in the flow Reynolds number using this measure. Further, precursors to an impending instability can be obtained through recurrence quantification that can warn an operator of fielded systems sufficiently in advance so that appropriate control action may be taken to prevent detrimental oscillations. The reason all these precursors work is because there will always be an intermittent regime amidst combustion noise and combustion instability. What the prescribed measures do is to simply act as quantifiers of the intermittency in a measured signal.

Appendix

A number of suitable markers that foretell an impending instability may be constructed by counting the number of black points in the recurrence plot. The density of black points in a recurrence plot measures the recurrence rate in the dynamics of the system and can be obtained as:

$$RR = \frac{1}{(N_0 - d_0\tau_{opt})^2} \sum_{i,j=1}^{N_0 - d_0\tau_{opt}} R_{ij} \tag{3}$$

where R_{ij} is 0 for a black point and 1 for a white point. The averaged diagonal length L_0 is obtained from the recurrence plot using the following relation:

$$L_0 = \frac{\sum_{l=l_{min}}^{N_0 - d_0\tau_{opt}} lP(l)}{\sum_{l=l_{min}}^{N_0 - d_0\tau_{opt}} P(l)} \tag{4}$$

where $P(l)$ is the frequency distribution of the black diagonal lines of length l, and l_{min} is a suitable lower limit for l to prevent noise corruption. The Shannon entropy, s of the signal can be obtained from the recurrence plot through the expression:

$$s = -\sum_{l=l_m}^{N_0 - d_0\tau_{opt}} p(l)\ln(p(l)) \tag{5}$$

where the probability that a diagonal line has length l, $p(l)$ is given by:

$$p(l) = \frac{P(l)}{\sum_{l=l_{min}}^{N_0 - d_0\tau_{opt}} P(l)} \tag{6}$$

Finally, the average passage time, τ_0 is evaluated as:

$$\tau_0 = \frac{\sum_{v=v_{min}}^{N_0 - d_0\tau_{opt}} vP(v)}{\sum_{v=v_{min}}^{N_0 - d_0\tau_{opt}} P(v)} \tag{7}$$

with $P(v)$ being the frequency distribution of the vertical (horizontal) black lines of length v in the recurrence plot, and $v_m in$ is a suitable lower limit for v to prevent noise corruption.

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