A Contribution to the Discussion on Thermoacoustic Energy from a Systemic Perspective

Ralf S. Blumenthal[*,1,4], Arun K. Tangirala[2], R.I. Sujith[3,4], Wolfgang Polifke[1]

[1] Lehrstuhl für Thermodynamik, TU München
Boltzmannstr. 15, D-85748 Garching, Germany
[2] Department of Chemical Engineering, IIT Madras
Chennai-600036, India
[3] Department of Aerospace Engineering, IIT Madras
Chennai-600036, India
[4] Institute for Advanced Study, TU München
Lichtenbergstr. 2a, D-85748 Garching, Germany
[*] Corresponding author: blumenthal@td.mw.tum.de

In using simplified time domain models for analyzing the dynamic and transient behavior as well as the stability of thermoacoustic systems, the choice of energy norm is an ongoing matter of debate. The present study intends to contribute to this discussion by adopting a systemic perspective on thermoacoustics. Independent of particular modeling choices, we only require the derivation of energy to be consistent with what we call a *benchmark derivation of energy* originating from first-principles governing equations. That is, in deriving the energy for a given model, we require no further assumptions to be introduced other than those already inherent to the model. For the case of a simplified Rijke tube setup as considered in the present study, the perturbation energy reduces to the classical acoustic energy with the Rayleigh term as driving source. With this metric, only marginal transient energy growth is observed. The corresponding optimal initial condition and the state at optimality are discussed and interpreted.

1 Introduction

Models are commonly used to represent a system of interest in an approximative yet realistic manner, thereby rendering it manageable to work with and to produce reasonable predictions of the true system behavior. In thermoacoustics, models are especially developed to describe the flame dynamics that may be in feedback with an acoustic and a flow field. The complexity of these models is usually dictated by the numerical resources available for simulations. Models built from first-principles can be extremely expensive in this respect. Therefore, simplified models are often deployed in thermoacoustics.

An important use of models in thermoacoustics is for stability analysis. How accurately a model can characterize the stability aspects not only depends on the quality of that model, but also importantly on the manner in which the energy of the system is quantified from the simplified model. Recent studies [1–3] have studied a relevant question – should an energy metric incorporate the effect of flame-related degrees of freedom in addition to the well-known classical acoustic energy, given that flame and acoustics are in feedback? In the present study, we address the core issue of deriving energy expressions from simplified models that are commensurate with the assumptions of the model, i.e., that the procedure to arrive at a given output is identical and independent of the particular modeling choice. The proposed approach is presented using a systemic perspective (systems theory framework) of a thermoacoustic system.

The most comprehensive models known are first-principles models based on governing conservation equations. Brear *et al.* [4] and Myers [5] have described a rigorous procedure based on first-principles to derive the perturbation energy of a system in which a flame interacts with an acoustic and a flow field. In the present study, we treat this procedure of deriving an energy norm as benchmark that we require to be met independent of which simplifications are chosen to reduce the model complexity of a thermoacoustic system. That is, if we choose to represent a thermoacoustic system by simpler models, it is important to stay consistent with the benchmark procedure to derive the perturbation energy. In particular, this amounts to ignoring the terms other than the dependent variables of the model. The resulting energy expression hence reflects whatever the model admits. We thus ensure that no assumptions other than those inherent to the model are introduced.

In particular, we investigate the transient energy behavior of a homentropic acoustic field in feedback with a flame approximated by a distributed time delay model [2, 6]. Heat conduction, viscous effects and temperature conductivity are neglected. The perturbation energy E as derived by Brear *et al.* then reduces to the well-known acoustic energy consisting of a kinetic and a potential energy term that are respectively proportional to the square of acoustic velocity and pressure. The source term S to the energy is the so-called Rayleigh term, i.e., the product of the acoustic pressure at the heat source and the heat release rate. Losses occur via the flux Φ and by volume losses represented by S_ζ:

$$\partial E/\partial t = -\nabla\Phi + S_\zeta + S \ .$$

The acoustic energy is hence the perturbation energy that is commensurate with the chosen model. In setting up the model, we require the model output y to represent the acoustic energy E. Only then can we ensure that the model predictions are free of assumptions other than those introduced in setting up the model itself. Use of the acoustic energy stands in contrast to previous studies [1,2] of such a simplified thermoacoustic system, in which other energy expressions are proposed that would not comply with the energy definition that we choose to employ in the present study.

The thermoacoustic system used in the present study is modeled as a network model in the time domain and is similar to the state space model introduced by Mangesius & Polifke [2]. Different elements (boundary conditions, straight ducts and a heat source) are connected to form a thermoacoustic system. We examine the thermoacoustic interaction of a flame enclosed in a duct open at both ends. The flame is modeled as a flame sheet in the G-equation framework [6, 7].

The present paper marks the beginning of further ongoing work. The results and conclusions drawn in this paper are therefore intentionally kept short. The main objective is to bring across a clear approach towards the modeling of a thermoacoustic system using simplified modeling assumptions but still staying consistent with benchmark procedures in deriving the perturbation energy. We argue that adopting a systemic perspective on thermoacoustic systems helps to establish a framework that admits any model, but does not introduce any additional assumptions. In addition, we believe that it will prove benefitial to conduct consistent and clear analyses of phenomena of interest such as transient growth, its role as to triggering, active and passive control and design optimization. A systems engineering approach has been followed by various authors in the past years (e.g., [8, 9]).

The remainder of the paper is organized as follows. We first review some basic elements of optimization in Sec. 2 to make the present paper as self-contained as possible. In Sec. 3, we present the model used in the present study, before using it to analyze the sensitivity and the receptivity of the thermoacoustic system in Sec. 4. A short outlook is given in the final Sec. 5.

2 A Systemic Approach to Thermoacoustics

In the present section, we consider a generic thermoacoustic system consisting of a flow, an acoustic field, and a heat source. It is formulated as an autonomous state space model

$$\begin{aligned} \dot{\mathbf{x}} &= \mathcal{A}\,\mathbf{x}\,, \\ \mathbf{y} &= \mathcal{C}\,\mathbf{x}\,. \end{aligned} \quad (1)$$

with state vector $\mathbf{x}$, output $\mathbf{y}$, system and output matrices $\mathcal{A}$ and $\mathcal{C}$, respectively. The state vector comprises all (discretized) variables of flow, acoustics and flame. $\mathcal{A}$ contains the system dynamics, i.e., the entire feedback between all three subsystems of flow, acoustics and flame. $\mathcal{C}$ is chosen such that $\mathbf{y}$ represents the output of interest that is commensurate with the modeling choice[1] As example, in the thermoacoustic model introduced later on in the paper, C only weights the states appearing in the definition of energy, but not the states related to the flame. For a linear system, the solution for the output is

$$\mathbf{y}(t) = \mathcal{C}\,\exp(\mathcal{A}(t-t_0))\,\mathbf{x}_0\;. \quad (2)$$

To optimize the output of the latter autonomous system, we are interested in finding the initial state $\mathbf{x}_0$ that leads to maximum output $\mathbf{y}$. We are thus seeking the sensitivity of the system to variations in the state vector $\mathbf{x}$. If N represents the maximum normalized output achievable at a given instant in time,

$$N(t) = \max_{\mathbf{x}_0} \frac{\mathbf{y}^T\,\mathbf{y}}{\mathbf{x}_o^T\mathbf{x}_0} = |\mathcal{C}\,\exp(\mathcal{A}(t-t_0))|^2_{L_2}\;, \quad (3)$$

the objective of the optimization procedure consists in finding the point in time for which N is maximized. This point corresponds to the maximum amplification of output that is possible for all times. For a nonlinear system, the optimization procedure can be done by adjoint optimization algorithms [10]. In case of a linear operator $\mathcal{A}$, it is possible to apply singular value decomposition (SVD) techniques to the matrix $\mathcal{C}\,\exp(\mathcal{A}(t_{max}-t_0))$. We thereby obtain the most sensitive base state $\mathbf{x}_0$ that leads to maximum amplification of the output $\mathbf{y}$ at $t = t_{max}$. The corresponding largest singular value expresses the amplification of state to output.

In practice, variations in the state vector can be achieved by passive control techniques. However, it may also be of interest to apply forcing to the system. In this case, the autonomous system (1) is extended by a forcing term[2] $\mathbf{u}$. The effect of $\mathbf{u}$ on $\mathbf{x}$ is governed by the input matrix $\mathcal{B}$, allowing for a flexible manner to force selected states:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathcal{A}\,\mathbf{x} + \mathcal{B}\,\mathbf{u}\,, \\ \mathbf{y} &= \mathcal{C}\,\mathbf{x}\,. \end{aligned} \quad (4)$$

For a linear system, the autonomous solution for the output given in Eq. (2) is extended by the forced solution,

$$\mathbf{y}(t) = \mathcal{C}\,\exp(\mathcal{A}(t-t_0))\,\mathbf{x}_0 + \mathcal{C}\int_{t_0}^{t} \exp(\mathcal{A}(\tau-t_0))\,\mathcal{B}\,\mathbf{u}\,\mathrm{d}\tau\;. \quad (5)$$

Say we have the possibility to force a linear system initially at rest (i.e., $\mathbf{x}_0 = 0$) with an impulsive forcing $\mathbf{u} = \hat{\mathbf{u}}\,\delta(t-t_0)$. In this case, the solution for the output reads

$$\mathbf{y}(t) = \mathcal{C}\,\exp(\mathcal{A}(t-t_0))\,\mathcal{B}\,\hat{\mathbf{u}}\;. \quad (6)$$

[1] .

[2] $\mathbf{u}$ represents the input and is not to be confounded with the velocity vector sharing the same notation.

In analogy to finding the OIC as shown above, it is possible to find the impulsive input that leads to maximum output in time. Define

$$P(t) = \max_{\hat{\mathbf{u}}} \frac{\mathbf{y}^T \mathbf{y}}{\hat{\mathbf{u}}^{\mathbf{T}} \hat{\mathbf{u}}} = |\mathcal{C} \exp(\mathcal{A}(t-t_0))\, \mathcal{B}|^2_{L_2} \ , \tag{7}$$

such that the maximum value of P is obtained from the optimal distribution of a given (prescribed) initial impulsive forcing. This optimal distribution tells us about the receptivity of the system to external forcing. Mind that the optimization is done such that it implicitly takes into account the choice of how the system is to be forced (via the input matrix $\mathcal{B}$).

In the following section, we present the model of a simple linear thermoacoustic system. We investigate its sensitivity and receptivity in the subsequent section 4.

3 A Thermoacoustic State Space Model

In the present study, we investigate a simple one-dimensional thermoacoustic system in the absence of flow. We assume ideal incompressible gas, homentropic conditions, and neglect viscosity and temperature conductivity. The heat source is treated as compact. In this case, the non-dimensional Euler equations to first order read (the dash to denote first-order quantities is omitted in the following for reasons of simplicity):

$$\begin{aligned} \frac{\partial u}{\partial t} &= -\beta_i^2 \frac{\partial p}{\partial x} + \zeta\, u \ , \\ \frac{\partial p}{\partial t} &= -\frac{\partial u}{\partial x} + \zeta\, p + K\, \dot{q}\, \delta(x - x_f) \ , \end{aligned} \tag{8}$$

that can be recast to a state space system as given in Eq. (4). u and p are the fluctuating quantities of velocity and pressure, that have been non-dimensionalized by a reference speed of sound c_{ref} and a reference dynamic pressure $\rho_{ref} c_{ref}^2$, respectively. x and t are space and time, non-dimensionalized by duct length $L_{ref} = L$ and the acoustic time scale L/c_{ref}, respectively. β_i is the ratio of speeds of sound of duct section i with respect to c_{ref}. The heat release rate $\dot{q}$ occurs compactly at the spatial position $x_f \in [0;1]$ and is weighted by a scalar measure of heat source strength K. An ad-hoc damping term $\zeta < 0$ is included to model boundary layer dissipation and acoustic duct end losses.

Transforming the system (8) to Riemann invariants f and g using

$$\begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ \beta_i^{-1} & \beta_i^{-1} \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix} \ , \tag{9}$$

we obtain

$$\begin{aligned} \frac{\partial f}{\partial t} &= -\beta_i \frac{\partial f}{\partial x} + \zeta\, f + \beta_i \frac{K}{2}\, \dot{q}\, \delta(x - x_f) \ , \\ \frac{\partial g}{\partial t} &= \beta_i \frac{\partial g}{\partial x} + \zeta\, g + \beta_i \frac{K}{2}\, \dot{q}\, \delta(x - x_f) \ . \end{aligned} \tag{10}$$

The governing equations are hence expressed in terms of rightward and leftward traveling acoustic waves f and g, respectively. The thermoacoustic system (10) is solved elementwise, in analogy to the network model approach in the frequency domain [11]. Two parts of the duct are divided by a heat source that results in a change in mean quantities of density ρ, temperature T and speed of sound c. The duct ends are closed by boundary condition elements. In the following, we describe the modeling of each of the elements.

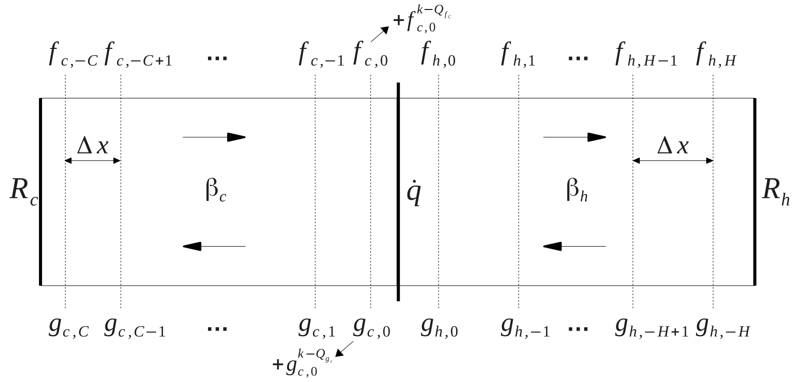

Figure 1: Schematic setup of the thermoacoustic model used in the present study. The duct sections possess constant values of density, temperature, speed of sound (all related to β_i), and constant cross-sectional area. The duct sections are enclosing a compact heat source producing heat release rate $\dot{q}$. The duct ends are closed by filter elements R_i at the boundaries

Duct section: Plane wave propagation

The duct sections enclosing the heat source are assumed to possess constant density, temperature and speed of sound. The solution to the system (10) for $x \in \{[0;1]\backslash x_f\}$, i.e., in the hot and cold duct section, respectively, can be written in terms of a characteristic ν:

$$\begin{aligned} f(x,t) &= \nu(t-\xi)\,\exp(\zeta\xi)\;, \\ f(x,t) &= \nu(t+\xi)\,\exp(-\zeta\xi)\;. \end{aligned} \tag{11}$$

The spatial coordinate x has been scaled according to $x = \beta_i \xi$ such that time t and space ξ lie on a single characteristic (i.e., $t = \pm x/\beta_i = \mp\xi$) that is invariant with respect to the speed of sound in each section. It can be seen from Fig. 1 that each duct section can be discretized into $N_i = l_i/\Delta x - 1 = l_i/(\beta_i \Delta\xi) - 1$ elements.

We now introduce the notation f_i^k, in which the index i and k denote the respective discrete positions in space ξ_i and in time t_k of a wave f. The same applies to g_i^k. The characteristic solution given in Eq. (11) is then fulfilled by the prescriptions $f_i^k\,\exp(\zeta\xi) = f_{i+1}^{k+1}\,\exp(\zeta\xi_{i+1})$ and $g_i^k\,\exp(-\zeta\xi) = g_{i+1}^{k+1}\,\exp(-\zeta\xi_{i+1})$. Taking into account the nomenclature of spatial indices as shown in Fig. 1, the waves propagate along the characteristic ν according to

$$f_{i+1}^{k+1} = f_i^k\,\exp(\zeta\,\Delta\xi)\;, \quad \text{and} \quad g_{i+1}^{k+1} = g_i^k\,\exp(\zeta\,\Delta\xi)\;. \tag{12}$$

Equation (12) has been described by Mangesius & Polifke [2] as “history update”, since the spatial coordinates can be interpreted as history in time. However, in the present formulation, we assign a discrete spatial position to each traveling wave and also take into account damping.

Compact heat source

The heat source is modeled as conical G-equation flame using a convective velocity model [6, 12]. Heat release fluctuations are produced in response to the history of fluctuations in velocity at the flame base,

$$\dot{q} = \int_0^{\tau_f} h_{\mathrm{CC}}\,u(t-\tau)\;\mathrm{d}\tau = \int_0^{\tau_f} h_{\mathrm{CC}}\,\eta(\tau)\;\mathrm{d}\tau\;, \tag{13}$$

where the impulse response (IR) filter h_{CC} is given in Eq. (18) in [6]. The flame states $\eta(\tau) = u(t-\tau)$ are hence the past velocity fluctuations at the flame base and represent additional states required for the flame model. They can also be interpreted as flame displacement along the flame surface [6, 12, 13]. In discrete time, the past states are $\eta_{0,n} = f^n_{c,0} - g^n_{c,0}$. τ_f is the characteristic time scale of flame response that is linked to the acoustic time scale by the Helmholtz number $\mathrm{He} = \tau_a/\tau_f$. In the present study, $\mathrm{He} = 1$, i.e., flame and acoustic time scale are equal.

The fluctuations in heat release rate $\dot{q}$ serve as compact source term to the acoustic field at the position of the flame x_f (see Eq. (8)). By integrating the governing equations Eq. (8) (or analog Eq. (10)) across the heat source from $x \in [x_f^- \ , \ x_f^+]$ and assuming negligible volume $\mathrm{d}V \to 0$, we obtain the jump conditions relating the outgoing and incoming waves with respect to the flame (Rankine-Hugoniot equations [14,15]. The outgoing waves $f_{h,0}$ and $g_{c,0}$ are hence functions of $f_{c,0}$, $g_{h,0}$ and of previous values of $f_{c,0}$ and $g_{c,0}$, i.e., $f^{k+1}_{h,0} = fct(f^k_{c,0}, g^k_{h,0}, f^{k-n}_{c,0}, g^{k-n}_{c,0})$ and $g^{k+1}_{c,0} = fct(f^k_{c,0}, g^k_{h,0}, f^{k-n}_{c,0}, g^{k-n}_{c,0})$.

Boundary elements

The boundary elements can be modeled as filters relating past values of incoming waves to the outgoing waves at the boundaries:

$$\begin{aligned} f^{k+1}_{c,-C} &= \mathbf{R_1}^T_j \, g^k_{c,C+j} \ , \\ g^{k+1}_{h,-H} &= \mathbf{R_2}^T_m \, f^k_{h,H+m} \ . \end{aligned} \tag{14}$$

Frequency-dependent boundary impedences can hence be accounted for in a straightforward manner [2]. In the present study, we choose optimal reflection at open duct ends, so the filters $\mathbf{R}_1 = \mathbf{R}_2 = -1$ reduce to a constant scalar value.

Energy

As mentioned in the introduction, the acoustic energy E contained in a duct with constant cross-sectional area non-dimensionalized by the same scales as used above reads

$$E = \frac{1}{2} \int\limits_{x=0}^{1} \left[\beta^{-2} u^2 + p^2\right] \ \mathrm{d}x = \frac{1}{2} \int\limits_{x=0}^{1} \left[f^2 + g^2\right] \beta^{-2} \ \mathrm{d}x \ . \tag{15}$$

Using the nomenclature as shown in Fig. 1, the energy contained in a slice i of the duct at instant k in time becomes $E^k_i = [(f^k_i)^2 + (g^k_i)^2] \, \beta_i^{-3} \ \mathrm{d}\xi_i$. Summing over all duct slices $x \in [0 \ ; \ 1]$ yields the acoustic energy as a function of time.

Summary

Combing all elements as described above, we obtain a discrete-time state space model analog to Eq. (4):

$$\begin{aligned} x^{k+1} &= A\, x^k + B\, u^k \ , \\ y^k &= C\, x^k \ , \end{aligned} \tag{16}$$

where the state vector at instant k in time reads

$$x^k = [\underbrace{f_{c,-C} \ \cdots \ f_{c,0} \ f_{h,0} \ \cdots \ f_{h,H}}_{f-\text{waves}} \ \underbrace{g_{h,-H} \ \cdots \ g_{h,0} \ g_{c,0} \ \cdots \ g_{c,C}}_{g-\text{waves}} \ \underbrace{g^{k-1}_{c,0} \ \cdots \ g^{k-Q}_{c,0} \ f^{k-1}_{c,0} \ \cdots \ f^{k-Q}_{c,0}}_{\text{history states}}]^T \ , \tag{17}$$

Flame Model	K	α	He	x_f	β_c	β_h	ζ	BC
CCO [6,12]	0.45	15°	1	0.3	1	1.4	−0.1	open-open

Table 1: Summary of parameters used in the present study

as visualized in Fig. 1. Due to the additional flame states needed for the flame model, the 2-norm of the state vector x does not correspond to the energy as given above. However, by choosing the output matrix C accordingly, we can ensure the 2-norm of the output y to correspond to the energy $E = y^T y$ as given in Eq. (15). We are thus able to isolate the interaction of flame and flow using a rather complex flame model with distributed delays and staying consistent with the benchmark procedure to derive the perturbation energy. E is the effect of the entire feedback loop between flame and acoustics. We therefore refer to E as the thermoacoustic energy, although it explicitly only contains acoustic quantities.

4 Some Results

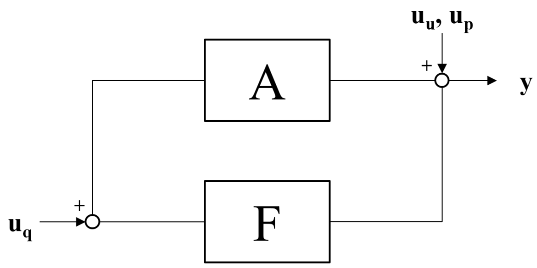

Figure 2: Systemic representation of the thermoacoustic system under investigation. The acoustic field A is in feedback with a flame F. Both subsystems can be forced by inputs u_i

In the present section, we study the transient behavior of a linear thermoacoustic system using the state space model introduced in the previous section. As there is no flow, the feedback loop reduces to the coupled system of two subsystems, that of the acoustics and that of the flame. The systemic setup is depicted in Fig. 2, the acoustic and the flame subsystem each illustrated by their linear system operator A and F, respectively. The state vector consists of acoustic and flame states (see Eq. (17)), each of which can be forced by an input u_i (see Fig. 2). All following results are obtained with the parameters as summarized in Tab. 1.

The solid line in Fig. 3 depicts the normalized evolution of the maximum energy norm $N(t)$ as given in Eq. (3). It can be seen that slight transient growth of of about 2% can be achieved peaking at $t \approx 0.6$. Performing SVD at $N = N_{max} = N(t \approx 0.6)$ yields the OIC x_{opt} for which the system actually reaches the maximum energy. The dashed line in Fig. 3 shows the evolution of thermoacoustic energy E (see Eq. (15)) normalized with respect to $N(0)$ of the autonomous system self-evolving from x_{opt}. As $E(0)/N(0) = 0.92$, the OIC has non-zero flame states that do not appear in the energy. The initial state leading to maximum transient energy growth thus requires the flame to be initially displaced, as can be seen from Fig. 5c. This finding corroborates the results of Subramanian & Sujith [1]. In the present case, transient growth of acoustic energy of merely $1.02/0.92 \approx 11\%$ can be achieved.

Figure 4 illustrates the shape of the optimal initial condition and the shape at optimality at $t \approx 0.6$ in terms of Riemann invariants. It can be seen that the OIC is such that the waves that will be approaching the flame from the left (i.e., the f waves and those that will become f waves

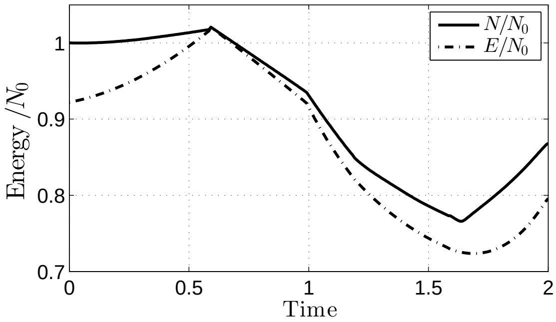

Figure 3: Maximum possible normalized energy N (full line) and energy of the autonomous thermoacoustic system evolving from the OIC reaching its maximum at $t \approx 0.6$ (dashed-dotted line)

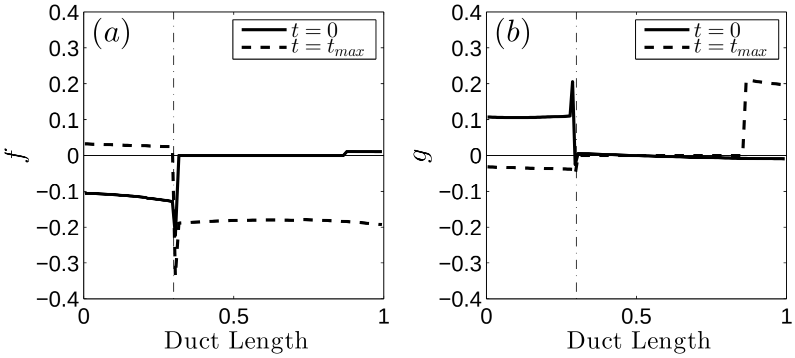

Figure 4: Spatial distribution of Riemann invariants f (a) and g (b) for the OIC at $t = 0$ (—) and at optimality $t \approx 0.6$ (– – –). The flame is located at $x_f = 0.3$ (–.–)

after reflection) are weighted preferentially. As mentioned before, the flame states are non-zero (not shown in the figure). At optimality, the largest displacement is on the hot side of the duct. The exact same results are shown in Fig. 5 in terms of acoustic velocity and acoustic pressure. At $t = 0$, there is a visible pressure wave at the location of the heat source, and the acoustic velocity is non-zero on the cold side of the duct. At optimality, the largest displacement of acoustic quantities is in the hot side of the duct. Figure 5c illustrates the flame states in addition to the IR filter (magnified by a factor of 10) that has been used to model the flame. $\dot{q}$ is obtained by convoluting the flame states with the IR filter (see Eq. (13)). The OIC is such that a strong pulse in velocity occurs at the flame base at $t = 0$.

In a next step, we assume that the system is initially at rest (i.e., $x_0 = 0$) and that it is perturbed by an impulsive forcing at $t = 0$. In an experimental setup, it may be only possible to force the acoustic states, e.g., by placing speakers along the length of the duct. In the following, we therefore set the input matrix B such that only the acoustic states are impulsively forced, but not the flame states. The resulting maximum possible response P in energy according to Eq. (7) is plotted in Fig. 6a. We observe that P is strictly decaying, which implies that it is not possible to obtain transient energy amplification. This strongly indicates that transient growth is indeed due to the acoustic-flame interaction and that it necessitates an initially displaced flame.

Figure 6b depicts the (rather theoretical) case of the initial impulsive forcing only being applied to the flame states, but not to the acoustic field. The system hence self-evolves from an initially displaced flame in a quiescent acoustic field. As the flame states do not appear in the thermoacoustic energy, the energy evolves from $y(0) = 0$.

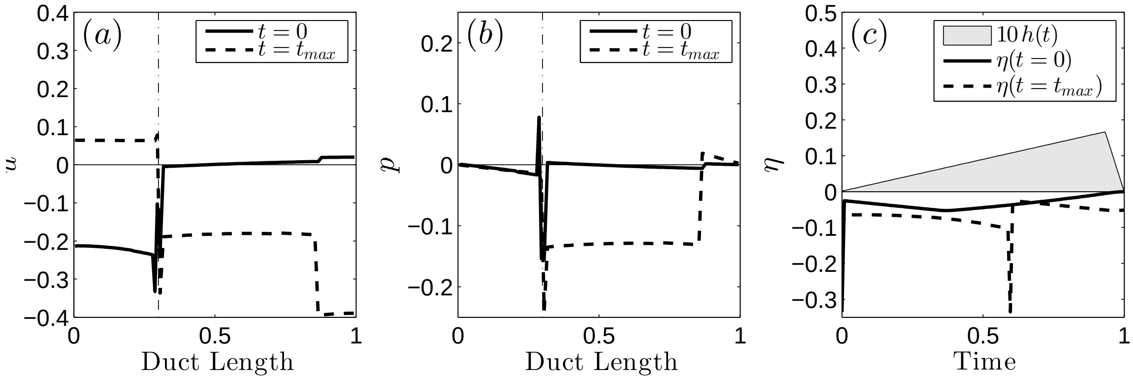

Figure 5: Spatial distribution of acoustic velocity (a), pressure (b) and distribution of flame states ν (c) for the OIC at $t = 0$ (—) and at optimality $t \approx 0.6$ $(- - -)$. The flame is located at $x_f = 0.3$ $(-.-)$ in (a) and (b). The shaded area in (c) corresponds to the magnified IR filter function of the flame onto which η is convoluted to produce $\dot{q}$

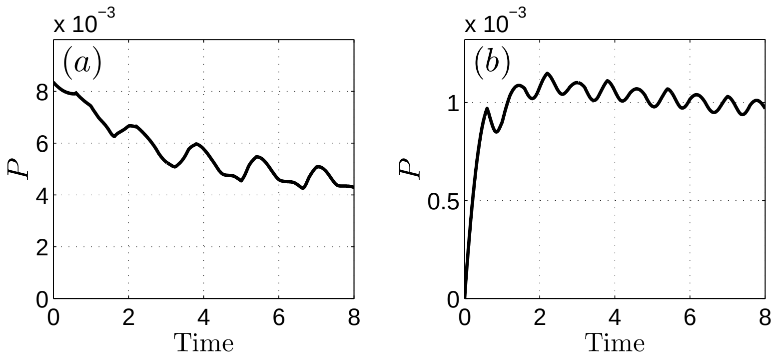

Figure 6: Maximum possible energy P achievable for the thermoacoustic system initially at rest and being impulsively perturbed at $t = 0$: Impulsive forcing of (a) acoustic states only and of (b) flame states only

5 Outlook

The present study is concerned with a physically consistent modeling approach for thermoacoustic systems. We show that a systemic perspective can be beneficially used to investigate a system in which a flow, an acoustic field, and a heat source interact with each other. Modeling assumptions for certain elements of the system can easily be taken into account without introducing additional assumptions other than those inherent to the models used.

The question of which energy norm to employ to investigate thermoacoustic configurations that make use of simplified modeling assumptions has been addressed in the present paper. The systemic perspective allows for a clear treatment of this issue. A benchmark procedure to derive the perturbation energy of a thermoacoustic system has been laid out in literature. This procedure must remain unaffected by particular modeling choices.

In the authors' opinion, the systemic approach yields strong potential to investigate interesting phenomena observed in thermoacoustic systems such as transient energy amplification and its role towards triggering a linearly stable system towards a nonlinear oscillating regime. It also provides for a straightforward inclusion of stochastic effects, i.e., the impact of noise onto the thermoacoustic system. Active and passive control as well as system design strategies are conceivable.

References

[1] P. Subramanian and R. I. Sujith. Non-normality and internal flame dynamics in premixed flame-acoustic interaction. *J. Fluid Mech.*, 679:315–342, 2011.

[2] H. Mangesius and W. Polifke. A discrete-time, state-space approach for modelling non-normal effects in thermoacoustic systems. *Int. J. Spray Comb. Dynamics*, 3(4):331–350, Dec 2011.

[3] F. Nicoud and T. Poinsot. Thermoacoustic instabilities: should the Rayleigh criterion be extended to include entropy changes? *Combustion and flame*, 142:153–159, 2005.

[4] Michael J. Brear, Frank Nicoud, Mohsen Talei, Alexis Giauque, and Evatt R. Hawkes. Disturbance energy transport and sound production in gaseous combustion. *Journal of Fluid Mechanics*, 707:53–73, 8 2012.

[5] M. K. Myers. Transport of energy by disturbances in arbitrary steady flows. *Journal of Fluid Mechanics*, 226:383–400, 1991.

[6] Ralf S. Blumenthal, Priya Subramanian, R.I. Sujith, and Wolfgang Polifke. Novel perspectives on the dynamics of premixed flames. *Combustion and Flame*, 160(7):1215 – 1224, 2013.

[7] L. Boyer and J. Quinard. On the dynamics of anchored flames. *Combust. Flame*, 82(1):51–65, 1990.

[8] B. Schuermans. *Modelling and Control of Thermoacoustic Instabilities*. PhD thesis, Ecole Polytechnique Federale De Lausanne, 2003.

[9] W. Polifke. System identification for aero- and thermo-acoustic applications. In C. Schram, editor, *Advances in Aero-Acoustics and Thermo-Acoustics*, number ISBN-13 978-2-87516-012-6 in VKI LS 2011-01, pages 1–46. Von Karman Institute, Rhode-St-Genèse, BE, 2011.

[10] M. P. Juniper. Triggering in the horizontal Rijke tube: non-normality, transient growth and bypass transition. *J. Fluid Mech.*, 667:272–308, January 2011.

[11] S. R. Stow and A. P. Dowling. Low-order modelling of thermoacoustic limit cycles. Number GT2004-54245 in Proc. of ASME Turbo Expo 2004 Power for Land, Sea and Air, Vienna, Austria, June 14-17 2004. ASME.

[12] T. Schuller, D. Durox, and S. Candel. A unified model for the prediction of laminar flame transfer functions: comparisons between conical and V-flame dynamics. *Combust. and Flame*, 134:21–34, 2003.

[13] Preetham, H. Santosh, and T. Lieuwen. Dynamics of laminar premixed flames forced by harmonic velocity disturbances. *J. Propul. Power*, 24(6):1390–1402, 2008.

[14] J. J Keller. Thermoacoustic oscillations in combustion chambers of gas turbines. *AIAA Journal*, volume 33, number 12:2280–2287, 1995.

[15] W. Polifke, C. O. Paschereit, and K. Döbbeling. Constructive and Destructive Interference of Acoustic and Entropy Waves in a Premixed Combustor with a Choked Exit. *Int. J. of Acoustics and Vibration*, 6(3):135–146, 2001.