Nonlinear effects saturating wave amplitude growth in thermoacoustic prime movers : a review.

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Int'l Summer School & Workshop on
Non-Normal and Nonlinear Effects in Aero- and Thermoacoustics
Technische Universität München
June 18 - 21, 2013

Nonlinear effects saturating wave amplitude growth in thermoacoustic prime movers : ~~a review~~ an overview.

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Plan

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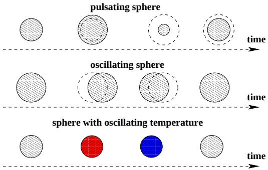
Elementary sources of sound emission, unsteady heat release

- The *linearized* equations of motion for an *inviscid* and *non-heat-conducting* ideal gas

$\partial_t \rho' + \rho_0 \mathrm{div}\mathbf{v} = Q(\mathbf{r}, t),$ Q :mass addition

$\rho_0 \partial_t \mathbf{v} + \mathbf{grad} p = \mathbf{f}(\mathbf{r}, t),$ $\mathbf{f}$: force

$\rho_0 C_p \partial_t \tau - \partial_t p = \dot{q}(\mathbf{r}, t),$ $\dot{q}$: Heat release

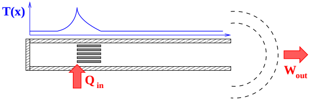
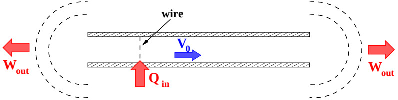


- The resulting inhomogeneous wave equation (using $p = RT_0\rho' + R\rho_0\tau$) is

$$\Delta p - \frac{1}{c_0^2}\partial_{tt}^2 p = -\frac{\partial_t \dot{q}}{T_0 C_p} - \rho_0 \partial_t Q + \mathrm{div}\mathbf{f}$$

- Contrarily to Q and f, $\dot{q}$ is difficult to derive in realistic cases because
  - ♭ The heat conductivity of the gas should be considered
  - ♭ Generally, $\dot{q} = \dot{q}(p, \mathbf{v})$, which may lead to self-sustained oscillations (e.g. Rijke tube etc ...)

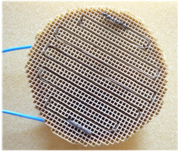
Autonomous thermoacoustic oscillators : two examples

TA Laser	**Rijke tube**
	
$\dot{q} = f(d_x T, p, v_x, stack \dots)$	$\dot{q} = f(T_{wire}, v_x, V_0 \dots)$
⋆ not acoustically compact	⋆ acoustically compact ($\dot{q} \approx \dot{q}(t)\delta(x - x_{wire})$)
⋆ Paradigmatic of TA engines (energetic applications)	⋆ Paradigmatic of combustion instabilities

Common features

⋆ Autonomous oscillators driven by heat (which both fulfill the "Rayleigh criterion" $\int p\dot{q}.dt > 0$)
⋆ Simple in terms of geometry, but complicated operation above threshold . . .

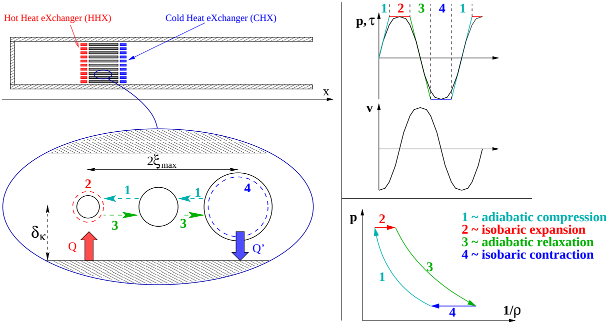
The ThermoAcoustic Laser : a prototypical example



- Heat input => sound output (self-sustained acoustic oscillations at the frequency of the most unstable mode(s))
- Keywords : heat engine (prime-movers and heat pumps), autonomous oscillator
- Paradigmatic of the challenges that need to be taken up to understand more deeply the processes controlling the operation of TA engines

The TA Laser : a conceptual, very simplified approach . . .

⋆ Simplistic description => Lagrangian approach, follow a gas parcel submitted to a standing wave through a channel with an axial temperature gradient $d_x T_0$



=> Work production if $|d_x T|\xi_{max} > \tau_{max}$ and $d_x T < 0$

⋆ But actual processes are actually more complex (heat exchange occurs continuously)

⋆ A key point : need of an imperfect thermal contact

=>"efficient" gas parcels should be at about δ_κ from the wall, where $\delta_\kappa = \sqrt{2\kappa/\omega}$ is the (frequency-dependent) thermal boundary layer thickness

⋆ Threshold of thermoacoustic instability (for a given mode) ?

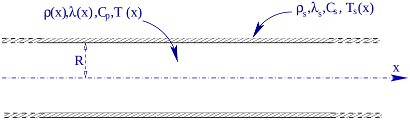
=> Sound amplification must be larger than losses

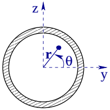
Basic equations (1)

⋆ Plane wave propagation through a viscous and heat-conducting gas along a duct submitted to a temperature gradient.

⋆ Both the production of acoustic work and the "bucket brigade" heat transport by sound along the duct are then described by second-order quantities.

the "bucket brigade"





Main assumptions		
⋆ **Low amplitudes**	⋆ **Typical wavelength » R**	⋆ **No mean flow**
$P(x,r,t) = P_0(x) + p(x,r,t)$	=> planes waves	⋆ **ideal gas**
$\rho(x,r,t) = \rho_0(x) + \rho'(x,r,t)$	$p(x,r,t) = p(x,t)$	⋆ $\rho_s C_s \gg \rho_0 C_p$
$T(x,r,t) = T_0(x) + \tau(x,r,t)$	=>"boundary layer approximation"	⋆ $\lambda_s \gg \lambda$
$\mathbf{v}(x,r,t) \ll c_0$	$\lvert\partial_r\zeta\rvert \gg \lvert\partial_x\zeta\rvert$	
$S(x,r,t) = S_0(x) + s(x,r,t)$	$(\zeta = p, \rho', \tau, \mathbf{v}, s)$	

Basic equations (2)

⋆ From the governing equations (with $\partial_r \gg \partial_x$) and the boundary conditions $\tau|_{r=R} = 0$ and $v_x|_{r=R} = 0$, all variables are expressed in terms of $\tilde{p}$ and $\partial_x \tilde{p}$, and averaged over the duct's cross-section, which leads to :

$$\begin{aligned}
\langle \tilde{v}_x \rangle &= \frac{i}{\rho_0 \omega} \partial_x \tilde{p} \,(1 - f_\nu) \\
\langle \tilde{\tau} \rangle &= \frac{\tilde{p}}{\rho_0 C_p} (1 - f_\kappa) - \frac{d_x T_0}{\rho_0 \omega^2} \partial_x \tilde{p} \left(1 - \frac{\mathrm{Pr} f_\nu - f_\kappa}{\mathrm{Pr} - 1} \right) \\
\langle \tilde{\rho}' \rangle &= \frac{1 + (\gamma - 1) f_\kappa}{c_0^2} \tilde{p} - \frac{d_x T_0}{T_0 \omega^2} \partial_x \tilde{p} \left(1 - \frac{\mathrm{Pr} f_\nu - f_\kappa}{\mathrm{Pr} - 1} \right)
\end{aligned}$$

where $\zeta(x, r, t) = \Re\left(\tilde{\zeta}(x, r) e^{i\omega t},\right)$, $\nu = \mu/\rho_0$, $\kappa = \lambda/(\rho_0 C_p)$, $\mathrm{Pr} = \nu/\kappa$, $\gamma = C_p/C_v$ and $f_{\kappa,\nu}$ depends on both $\delta_{\kappa,\nu}/R$ and the waveguide geometry[1,2]

⋆ Eliminating $\tilde{\rho}'$, $\tilde{v}_x$ and $\tilde{\tau}$ leads to the thermoacoustic wave equation in the Fourier domain* :

$$\boxed{\rho_0 \partial_x \left(\frac{1 - f_\nu}{\rho_0} \partial_x \tilde{p} \right) - \frac{f_\kappa - f_\nu}{1 - \mathrm{Pr}} \frac{d_x T_0}{T_0} \partial_x \tilde{p} + \left(\frac{\omega}{c_0} \right)^2 (1 + (\gamma - 1) f_\kappa)\, \tilde{p} = 0}$$

[1] N. Rott, Adv. Appl. Mech., 1980 ; [2] G.W. Swift, J. Acoust. Soc. Am., 1988.

* For the wave equation in the time domain, see for instance [N. Sugimoto, J. Fluid. Mech., 2010]

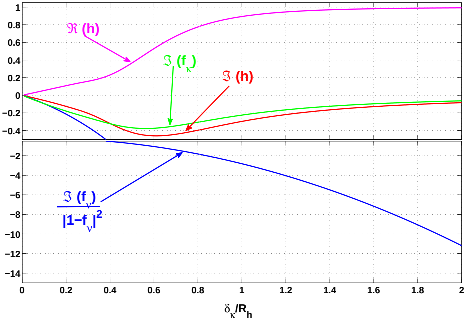
Sound amplification

⋆ From the knowledge of the acoustic variables, it is possible to calculate the time-average second order acoustic power produced (or absorbed) per unit volume $\mathfrak{w}_2$ ($[\mathfrak{w}_2] = \mathrm{W.m^{-3}}$) :

$$\mathfrak{w}_2 = \partial_x(\overline{p < v_x >}).$$

⋆ After some calculations[3],

$$\begin{aligned}
\mathfrak{w}_2 &= \mathfrak{w}_\kappa + \mathfrak{w}_\nu + \mathfrak{w}_{sw} + \mathfrak{w}_{tw}\\
\mathfrak{w}_\kappa &= \frac{1}{2}\frac{(\gamma-1)\omega}{\rho_0 c_0^2}\Im\left(f_\kappa\right)|\tilde{p}|^2\\
\mathfrak{w}_\nu &= \frac{1}{2}\omega\rho_0\frac{\Im(f_\nu)}{|1-f_\nu|^2}|\langle\tilde{v}_x\rangle|^2\\
\mathfrak{w}_{sw} &= -\Im(h)\frac{d_x T_0}{T_0}\frac{1}{2}\Im\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right)\\
\mathfrak{w}_{tw} &= \Re(h)\frac{d_x T_0}{T_0}\frac{1}{2}\Re\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right)\\
h &= \frac{f_\kappa - f_\nu}{(1-\mathrm{Pr})(1-f_\nu)}
\end{aligned}$$



[3] A. Tominaga, Cryogenics, 1995.

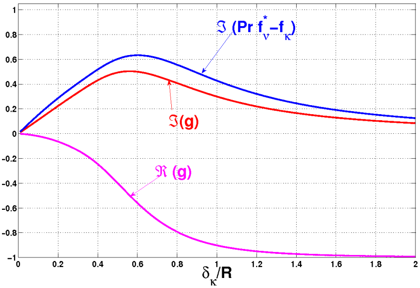
Heat transport by sound

⋆ From the knowledge of the acoustic variables, it is also possible to calculate the thermoacoustic heat flux q_2 ($[q_2] = \mathrm{W.m}^{-2}$) :

$$q_2 = \rho_0 T_0 (\overline{s < v_x >}), \quad s = \frac{C_p}{T_0}\tau - \frac{1}{\rho_0 T_0} p$$

⋆ After some calculations,

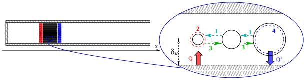
$$\begin{aligned} q_2 &= q_{sw} + q_{tw} + \lambda_{ac} d_x T_0 \\ q_{sw} &= -\Im(g)\frac{1}{2}\Im\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right) \\ q_{tw} &= \Re(g)\frac{1}{2}\Re\left(\tilde{p}\langle\tilde{v}_x^*\rangle\right) \\ \lambda_{ac} &= \frac{\rho_0 C_p}{2\omega(1-\mathrm{Pr}^2)}\frac{\Im\left(\mathrm{Pr} f_\nu^* - f_\kappa\right)}{|1-f_\nu|^2}|\langle\tilde{v}_x\rangle|^2 \\ g &= \frac{f_\nu^* - f_\kappa}{(1+\mathrm{Pr})(1-f_\nu^*)} \end{aligned}$$

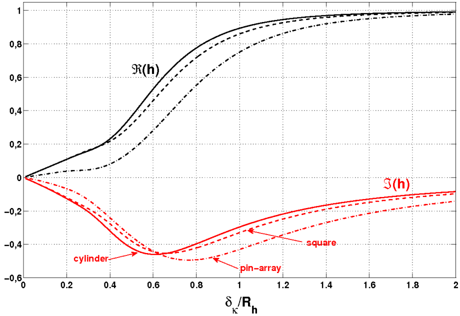


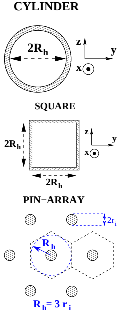
Amplification of a standing wave

$$\mathfrak{w}_2 = \underbrace{\mathfrak{w}_\kappa + \mathfrak{w}_\nu}_{\text{losses}} + \underbrace{\overbrace{\mathfrak{w}_{sw}}^{\propto -\Im(h)} + \overbrace{\mathfrak{w}_{tw}}^{\propto \Re(h)}}_{\text{production}}$$

$$h = (f_\kappa - f_\nu)/[(1 - \mathrm{Pr})(1 - f_\nu)]$$





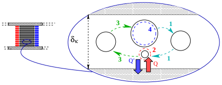


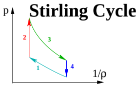
⋆ In case of a standing wave, one need $\delta_\kappa \leq R_h$ (stack) and always have $|\Im(h)| < 1$
=> intrinsic irreversibility[4] due to the need of an imperfect thermal contact
⋆ But in case of a travelling wave phasing , we have $|\Im(h)| \approx 1$ if $\delta_\kappa \geq R_h$
=> a travelling wave phasing combined with a regenerator ($\delta_\kappa \geq R_h$) should be better !

[4] J. C. Wheatley et al, J. Acoust. Soc. Am., 1983.

How to build an intrinsically reversible Thermoacoustic engine ?

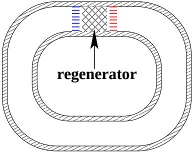
$$\mathfrak{w}_2 = \underbrace{\mathfrak{w}_\kappa + \mathfrak{w}_\nu}_{\text{losses}} + \underbrace{\overbrace{\mathfrak{w}_{sw}}^{\propto -\Im(h)} + \overbrace{\mathfrak{w}_{tw}}^{\propto \Re(h)}}_{\text{production}}$$





=> use a **regenerator**$(\delta_\kappa \geq R_h)$ and a **travelling wave** phasing between p and v_x

⋆ [Ceperley,JASA,1979]



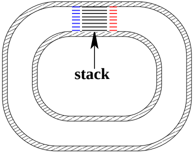
=> closed-loop resonator

- $\tilde{p}, \tilde{v}_x \propto e^{-i(kx-\omega t)}$,
- $Z = \frac{\tilde{p}}{\tilde{v}_x} = \rho_0 c_0$
- $f \approx c_0/L$

=> does not sing...

because $\mathfrak{w}_{tw} < \mathfrak{w}_\nu$

⋆ [Yazaki et al., PRL, 1998]



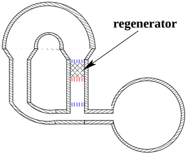
=> closed-loop resonator

- $\tilde{p}, \tilde{v}_x \propto e^{-i(kx-\omega t)}$,
- $Z = \frac{\tilde{p}}{\tilde{v}_x} = \rho_0 c_0$
- $f \approx c_0/L$

=> does sing...

...but employs a stack

⋆ [Backhaus et al., Nature,1999]



=> acoustic feedback loop

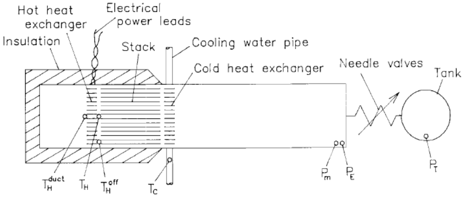
- **local** TW phasing,
- $Z >> \rho_0 c_0$ **locally**
- $f << c_0/L$

=> does sing...

...quite well

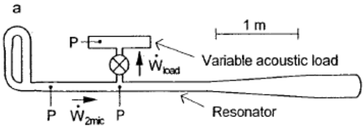
Typical examples (from G.W. Swift and co-workers, LANL, USA)

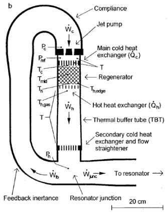
A Standing wave engine [Swift, JASA, 1992]



- fluid† : 13.8 bar Helium
- frequency : 120 Hz
- drive ratio p/P_0 : 6%
- T_H : 1000 K
- W_{load} : 630 W
- Q_H : 7 kW
- $\eta = 0.09$ ($0.13\eta_c$)

A thermoacoustic-Stirling engine [Backhaus & Swift, Nature, 1999]





- fluid† : 30 bar Helium
- frequency : 80 Hz
- drive ratio p/P_0 : 6-10%
- T_H : 1000 K
- W_{load} : 710 W
- Q_H : 2.4 kW
- $\eta = 0.3$ ($0.42\eta_c$)

† Need a large γ, low Pr, and high P_0 (in 1-bar air at 120 $\mathrm{dB_{SPL}}$, $W_{ac} \approx 10^{-2}\mathrm{W.m}^{-2}$ ...)

Advantages of TA engines (and heat pumps)

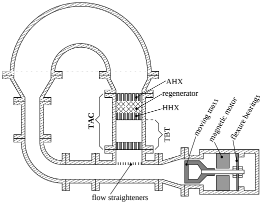
⋆ Simple, no or not much moving parts => potential reliability
⋆ Working gas = pressurized inert gas => harmless, environmental friendly
⋆ Good efficiencies (currently up to 32% in thermo-acoustic conversion for engines[5]), with potential improvements.

Limitations

⋆ Moderate output powers => up to a few kW
⋆ Good efficiencies => to be improved.
⋆ Development limited to a few research labs and start-up companies

Potential applications

⋆ Waste heat recovery, micro-cogeneration
⋆ Thermoacoustically driven pulse-tube refrigeration (cryogeny . . .)
⋆ Domestic refrigeration ? electronics cooling ?
. . .

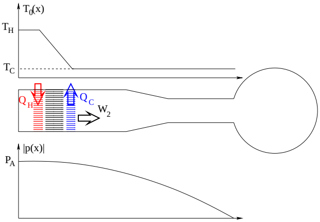


Sketch of a thermo-acousto-electric generator. Such kind of engine has been demonstrated to achieve a global efficiency of 18 % [6]

[5] Tijani et al., J. Appl. Phys., 2011 ; [6] Backhaus et al., Appl. Phys. Lett., 2004.

Basic principles of design simulation tools

- The basic principles of design tools may be summarized as
  - A **two-port** modeling of acoustic wave propagation under an assigned $T_0(x)$ through the device, leading to a **characteristic equation**
  - An **energy balance** under an assigned heat input Q_H, which accounts for the **thermoacoustic heat transport by sound** Q_2.
- A simplistic illustration would be, for instance, as follows



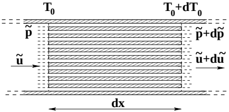
1. Assign Q_H and set $P_A = 0$ (and thus $Q_2 = 0$)
2. Solve heat transfer => get $T_0(x)$
3. Solve acoustics => get spatial distribution $\tilde{p}(x)$ and working frequency f
4. Fix arbitrary amplitude P_A
5. Calculate thermoacoustic heat transport Q_2
6. Solve heat transfer => get $T_0(x)$ (accounting for Q_2)
7. Repeat steps 3-6 until equilibrium, i.e. $Q_H = Q_C + W_2$, with $Q_C = Q_{C_0} + Q_2$.

Two-ports modeling of wave propagation

⋆ Back to the governing equations, expressed in terms of pressure $\tilde{p}$ and volume velocity $\tilde{u} = S\langle \tilde{v}_x \rangle$ (S :cross-sectional area of the duct)

$$d\tilde{p} = -\frac{i\omega\rho_0 dx}{S}\frac{1}{1-f_\nu}\tilde{u}$$

$$d\tilde{u} = -\frac{i\omega S dx}{\gamma P_0}\left[1+(\gamma-1)f_\kappa\right]\tilde{p} + \frac{(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)}\frac{dT_0}{T_0}\tilde{u}$$



⋆ Then make use of electroacoustic analogy[7]

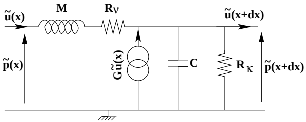
$$d\tilde{p} = -(i\omega M + R_\nu)\tilde{u}$$

$$d\tilde{u} = -\left(i\omega C + 1/R_\kappa\right)\tilde{p} + G\tilde{u}$$

$$M = \frac{\rho_0 dx}{S}\frac{1-\Re(f_\nu)}{|1-f_\nu|^2} \quad C = \frac{S dx}{\gamma P_0}\left(1+(\gamma-1)\Re(f_\kappa)\right)$$

$$R_\nu = \frac{\omega\rho_0 dx}{S}\frac{\Im(-f_\nu)}{|1-f_\nu|^2} \quad \frac{1}{R_\kappa} = \frac{\gamma-1}{\gamma}\frac{\omega S dx \Im(-f_\kappa)}{P_0}$$

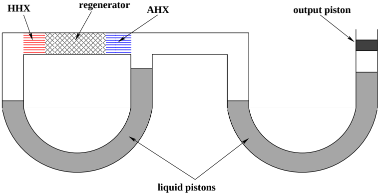
$$G = \frac{f_\kappa - f_\nu}{(1-f_\nu)(1-\sigma)}\frac{dT_0}{T_0}$$

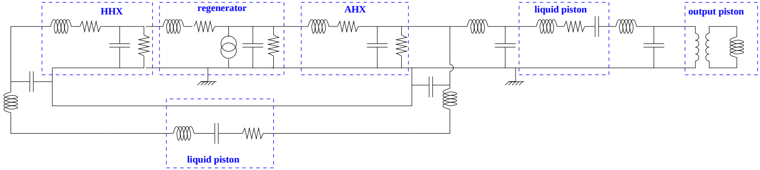


[7] G.W. Swift, « Thermoacoustics : a unifying perspective for some engines and refrigerators », 2002.

Two-ports modeling of wave propagation (2)

⋆ A simple example using **lumped elements** : the fluydine engine[8]



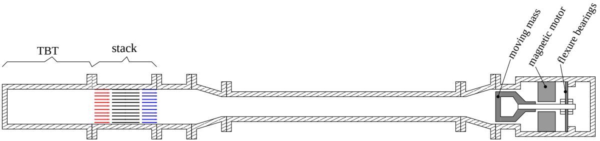


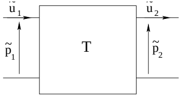
$$M_{liq} = \frac{\rho_{liq} L_{liq}}{S}, \quad R_{liq} = \frac{\omega \rho_{liq} L_{liq}}{S} \frac{\Im\left(-f_{\nu_{liq}}\right)}{|1 - f_{\nu_{liq}}|^2}, \quad C_{liq} = \frac{S}{2\rho_{liq} g}$$

[8] C.D. West, Liquid Piston Stirling Engines, V. N. Reinhold, New-York,1983

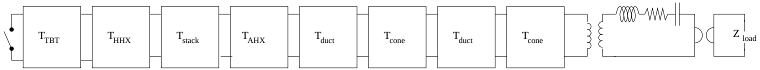
Two-ports modeling of wave propagation (3)

⋆ When the lumped elements assumption cannot be retained, use of the T-matrices formalism (see example below)





$$\begin{bmatrix}\tilde{p}_2\\ \tilde{u}_2\end{bmatrix} = \begin{bmatrix}T_{pp} & T_{pu}\\ T_{up} & T_{uu}\end{bmatrix}\begin{bmatrix}\tilde{p}_1\\ \tilde{u}_1\end{bmatrix}$$



† T_{TBT} and T_{stack} calculated numerically from the thermoacoustic wave equation ...

‡ The two-port model, which leads to a characteristic equation, need to be coupled with the energy balance to determine steady-state conditions.

Limitations of the existing design tools

⋆ Up to now, the design of TA engines is always realized from the linear TA theory.

⋆ A famous, free-downloadable software developed at Los Alamos : DELTA-EC[9]

⋆ Efficient tool for design purposes, which works quite-well for the prediction of steady-state operation of Low-Amplitude TA engines

But . . .

⋆ These tools are restricted to the linear (or weakly nonlinear) regime

⋆ These tools are restricted to a 1-D description of the phenomena

⋆ They predict performance in steady-state

⋆ The user need to be experienced in the field[†]

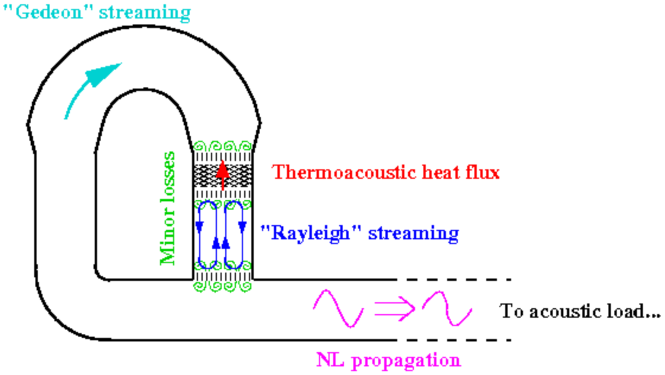
[9] W.C. Ward et al., J. Acoust. Soc. Am., 1994

[†] « Intuition is required from the start and successful solutions yield intuition », W.C. Ward, Acoustics'08, Paris, 2008

Plan

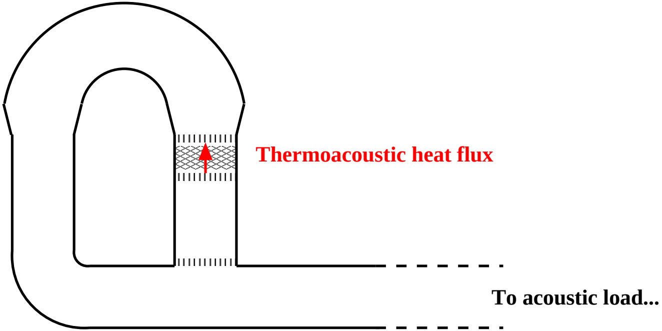
Autonomous oscillators => nonlinear saturation processes . . .

⋆ Thermoacoustic engines = **self-sustained oscillators**
⋆ Most of the **nonlinear** processes controlling wave saturation are **well identified . . .**



⋆ Some of them are well-predicted by theory (at least for moderate pressure levels) . . .
⋆ . . .but most of them are poorly described

Heat transport by sound . . .



Heat transport by sound ...

⋆ As in any heat engine, work production is accompanied with heat flow from hot to cold ...
⋆ A second-order (and therefore nonlinear) effect which is taken into account in the linear thermoacoustic theory ...
⋆ Its definition is unambiguous (for low amplitudes) :

$$q_2 = \frac{1}{2}\Re\left[\frac{f_\nu^* - f_\kappa}{(1+\mathrm{Pr})(1-f_\nu^*)}\tilde{p}\langle\tilde{v}_x^*\rangle\right] + \frac{\rho_0 C_p}{2\omega(1-\mathrm{Pr}^2)}\frac{\Im\left(\mathrm{Pr}f_\nu^* - f_\kappa\right)}{|1-f_\nu|^2}|\langle\tilde{v}_x\rangle|^2 d_x T_0$$

But ...

⋆ The thermal and viscous functions are only known for simple geometries like parallel plates, circular tubes[10] , pin-array[11] ...

⋆ Actual stack/regenerators employ more complex materials


mesh grids


NiCr foam

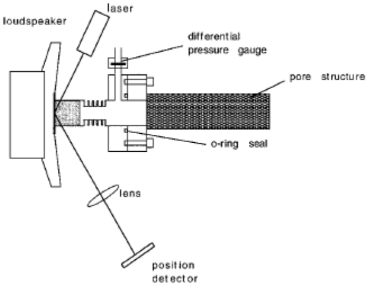

RVC foam

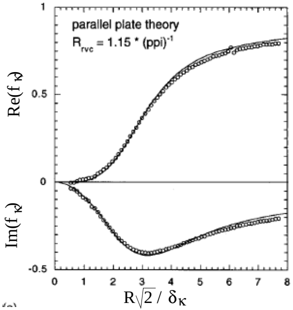
[10] Rott, Adv. Appl. Mech., 1980 ; [11] Swift et al., JASA, 1993.

Determination of $f_{\nu,\kappa}$ for tortuous media ...

⋆ Measure the complex compressibility (related to f_κ) of different materials[13–15]

$$C(\omega) = -\frac{1}{V_0}\frac{\tilde{V}_1(\omega)}{\tilde{P}_1(\omega)}$$





⋆ Other approaches => use results established for steady flow

♭ Use experimental data of viscous drag and heat exchange coefficient for steady flow within heat exchangers (e.g. in ref. [16]), and proceed to conversion for oscillating flows[17]

♭ Shortcoming => validity of this "quasi-steady" flow assumption ?

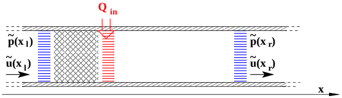
[12] Roh et al.,JASA,2007 ; [13] Hayden et al., JASA, 1997 ; [14] Wilen, JASA, 2001 ; [15] Petculescu et al., JASA, 2001 ; [16] Kays and London, *Compact Heat exchangers*, Mc Graw Hill, NY, 1964 ; [17] Swift et al., J. Thermophys. Heat Transf., 1996

Digression : description of stack/regenerators...

⋆ The knowledge of $f_{\nu,\kappa}(\omega)$ is

♭ required for the evaluation of both $\mathfrak{q}_2$ and $\mathfrak{w}_2$,

♭ not sufficient , since we need to know the thermophysical properties of the stack



⋆ How to know $T_0(x)$ from Q_{in} ?
=> not trivial (anisotropy ...)
⋆ What about the 1-D assumption ?
Is T_0 uniform through a section ? => no ...

⋆ A solution explored at LAUM : get information from the measured T-matrix of the ThermoAcoustic Core (TAC) under various heating conditions

♭ Measure $T_{TAC}(\omega, Q_{in})$ using a two-load method[18],

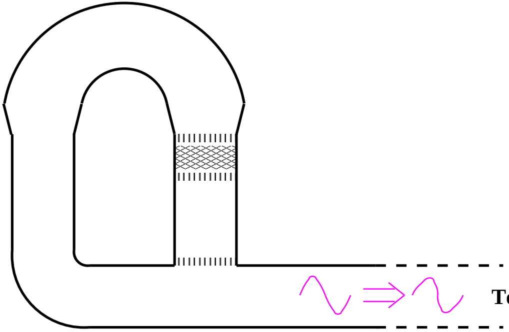
♭ Use the experimental T-matrix to design a TA engine that uses the TAC characterized beforehand[19], or to calculate the thermophysical properties if the stack[20].

$$\begin{pmatrix} \tilde{p}(x_r) \\ \tilde{u}(x_r) \end{pmatrix} = \begin{bmatrix} \mathcal{T}_{pp}(\omega, Q_{in}) & \mathcal{T}_{pu}(\omega, Q_{in}) \\ \mathcal{T}_{up}(\omega, Q_{in}) & \mathcal{T}_{uu}(\omega, Q_{in}) \end{bmatrix} \times \begin{pmatrix} \tilde{p}(x_l) \\ \tilde{u}(x_l) \end{pmatrix} = T_{TAC}(\omega, Q_{in}) \times \begin{pmatrix} \tilde{p}(x_l) \\ \tilde{u}(x_l) \end{pmatrix}$$

Advantages = "black box" approach, no need to know $f_{\nu,\kappa}$ or T_0
Shortcoming = linear approach, only valid for low amplitudes

[18] M. Guedra et al., JASA, 2011 ; [19] F.C. Bannwart et al., JASA, 2013 ; [20] M. Guedra et al., submitted to J. Appl. Therm. Eng., 2013.

Cascade process of higher harmonics generation



Higher harmonics generation : basic concepts (1)

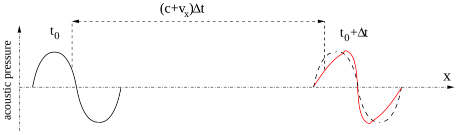
⋆ Starting with the 1-D governing equations in the time domain, without dissipation :

$$\begin{aligned} \rho\left(\partial_t v_x + v_x \partial_x v_x\right) &= -\partial_x P \\ \partial_t \rho + \partial_x\left(\rho v_x\right) &= 0 \\ P &= K\rho^{\gamma} \end{aligned}$$

⋆ Assuming $v_x = v_x(p)$, a particular solution analogous to a plane wave travelling along $x \uparrow$ can be found[21] :

$$\partial_t p + \left(c + v_x\right)\partial_x p = 0$$

where $1/c^2 = d_\rho p$.



wave distorsion $\sim$ cascade process of higher harmonics generation

⋆ Impact on the operation of TA engines ?

- Increase of viscothermal losses, since $\langle \varpi_{\nu,\kappa} \rangle \propto \sqrt{\omega}$
- Decrease of thermoacoustic amplification, since δ_κ depends on ω
- An effect which can be promoted if higher harmonics coincide with resonance frequencies of the TA engine . . .

[21] A.D. Pierce, *Acoustics*, Acoustical Society of America, NY, 1991

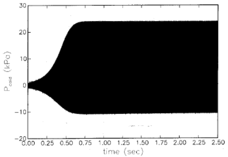
NL acoustics : a brief review

An extensively studied topic . . .

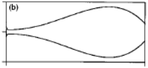
⋆ Discussed in several reference textbooks[22,23]
⋆ Also studied both analytically (Multiple Scale Method[24] , Burgers equation[25]) and numerically[26,27] in the frame of thermoacoustics
⋆ Proposal for optimum resonator's shape[28] or for active control methods[29]
⋆ but a lack of quantitative comparison with experiments

. . .which however seems not to be predominant in thermoacoustics

⋆ High-power thermoacoustic engines employ inharmonic resonators, anyway.
⋆ Waveform distorsion may be observed, but no efforts to include NL propagation in design tools (not trivial and too tedious . . .)



Transient behavior in a SW TA engine under an assigned ΔT, from [26]

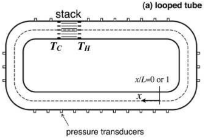


An optimum resonator's shape that maximize the Q of a resonator, from [28]

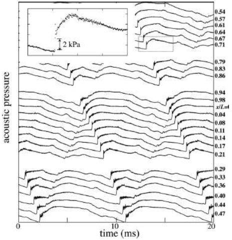
[22] Rudenko & Soluyan, consultant bureau, NY, 1977 ; [23] Hamilton & Blackstock, Ac. Soc. Am., 1998 ; [24] Karpov et al., JASA, 2000 ; [25] Gusev et al., Acust. Acta Acust., 2000 ; [26] Karpov et al., JASA, 2002 ; [27] hamilton et al., JASA, 2002 ; [28] Ilinskii et al., JASA, 2001 ; [29] Gusev et al., JASA, 1998.

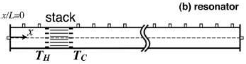
Higher harmonics generation : shock waves

⋆ Still, the study of NL propagation in TA engines is an interesting topic . . .
⋆ Recently, shock waves reported in a closed-loop, stack based TA engine [Biwa et al., JASA, 2011], and acoustic intensity measurements were performed.

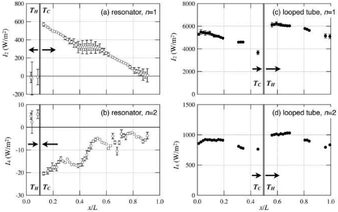


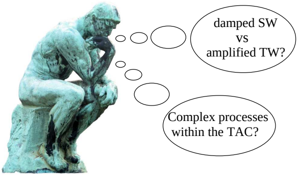
TW engine
=> shock wave along x ↑



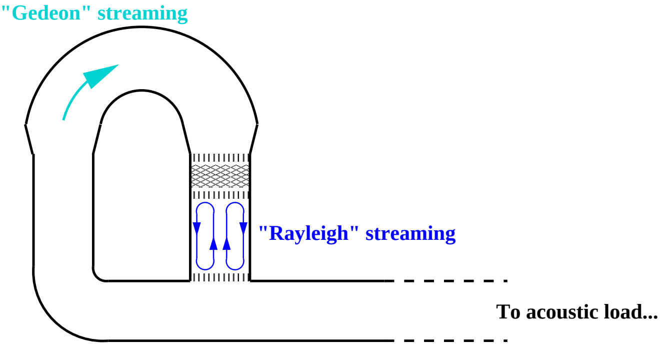


SW engine
=> no shock waves





Acoustic streaming



Acoustic streaming : basic concepts

⋆ Start with the Navier-Stokes equation, with $\partial_y >> \partial_x$, then make successive approximations ($\zeta = \zeta_0 + \zeta_1 + \zeta_2 \ldots$) up to second order, and after time averaging, one gets :

$$\boxed{\nu_0 \partial^2_{yy} \overline{v_{x_2}} = 1/\rho_0 \partial_x \overline{p_2} + \partial_x \left(\overline{v^2_{x_1}}\right) + \partial_y \left(\overline{v_{x_1} v_{y_1}}\right) - \partial_y \left(\overline{\nu_1 \partial_y v_{x_1}}\right)}$$

⋆ A steady velocity $\overline{v_{x_2}}$ generated by acoustic oscillations :

- because of the **convective derivative** :

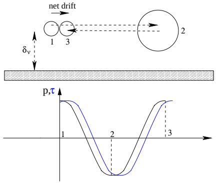
$$\partial_x \left(\overline{v^2_{x_1}}\right) + \partial_y \left(\overline{v_{x_1} v_{y_1}}\right)$$

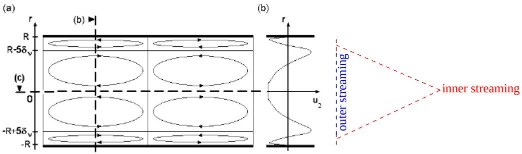
- because ν depends on temperature ($\nu = \nu_0(T_0) + \nu_1(\tau)$)

$$\partial_y \left(\overline{\nu_1 \partial_y v_{x_1}}\right)$$

- ...and actually **many other sources** of streaming

$$\nu \propto T^{1+\beta}$$
$$\int_1^2 \nu.dt > \int_2^3 \nu.dt$$

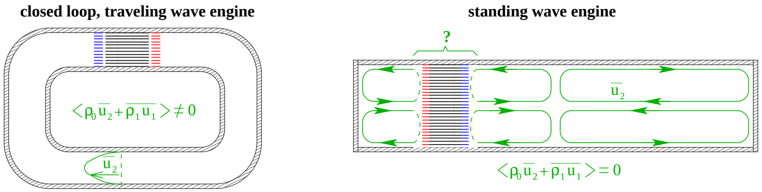




Acoustic streaming, theoretical works : a brief review (not exhaustive)

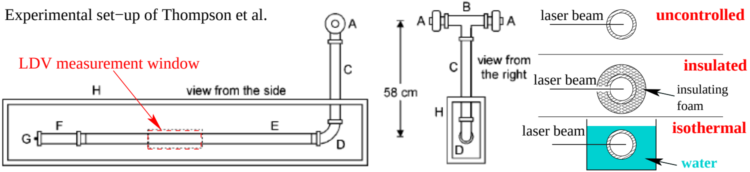
- ⋆ Faraday (1831) : first experimental observation
- ⋆ Rayleigh (1883) : outer streaming, large ducts ($R >> \delta_\nu$)
- ⋆ Schlichting (1932) : inner streaming
- ⋆ Rott (Z.A.M.P., 1974) : outer streaming, large ducts, thermal effects ($d_x T_0$, $\mu = \mu(T)$)
- ⋆ Gedeon (Cryocoolers, 1997), Gusev (JASA, 2000) : closed-loop, travelling wave devices
- ⋆ Menguy et al. (JASA, 2000) : inertia effects at larger amplitudes (as $Re_{nl} = \sqrt{\mathrm{M/Sh}} \approx 1$).
- ⋆ Waxler, Bailliet et al. (JASA, 2001) : arbitrary duct radius and thermal effects
- ⋆ Since around 2000, many numerical studies

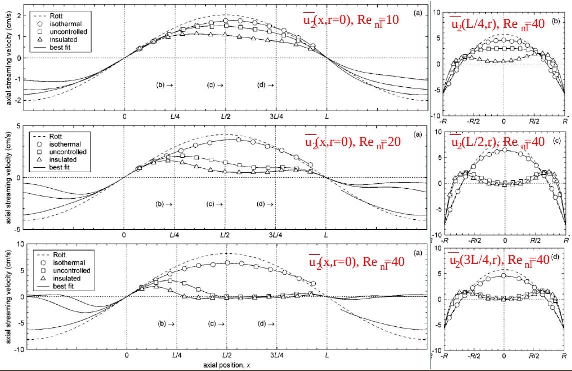
Up to now, many experimental and numerical studies of $\overline{\mathbf{v_2}}$ in empty resonators, but only a very few in TA engines (even simple ones).



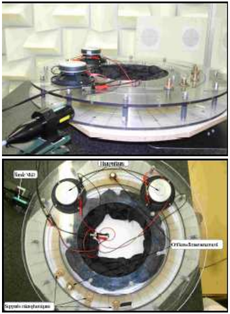
Acoustic streaming in SW resonators

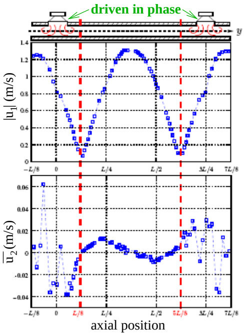
⋆ Thompson et al. (JASA, 2004) : outer streaming at high $\mathrm{Re}_{\mathrm{nl}}$ ($\mathrm{Re}_{\mathrm{nl}} = \sqrt{M/Sh}$, impact of temperature distribution and fluid inertia

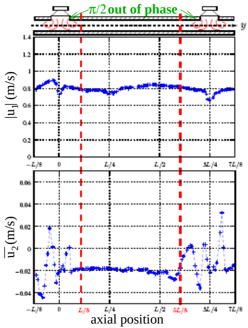




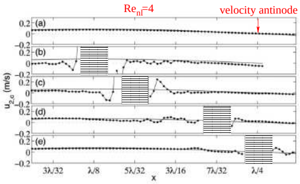
Acoustic streaming in a closed-loop resonator (Desjouy et al.,JASA,2009)

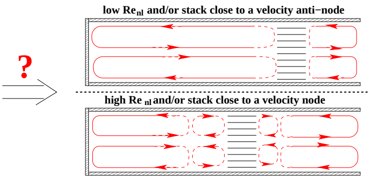






Acoustic streaming in SW resonators with a stack (Moreau et al., JASA, 2009)



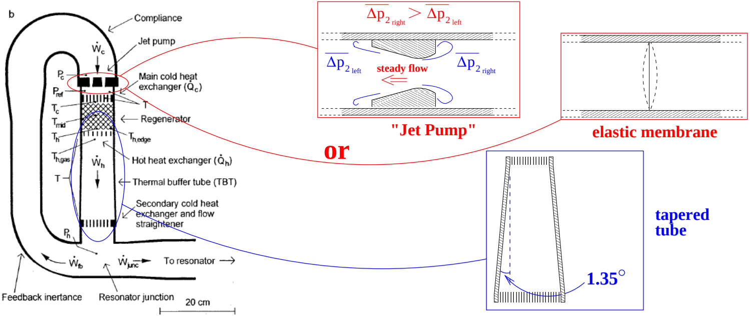


Acoustic streaming from the engineering standpoint

⋆ Acoustic streaming well-described only in simple devices at low amplitudes .

=> Empirical solutions to remove acoustic streaming

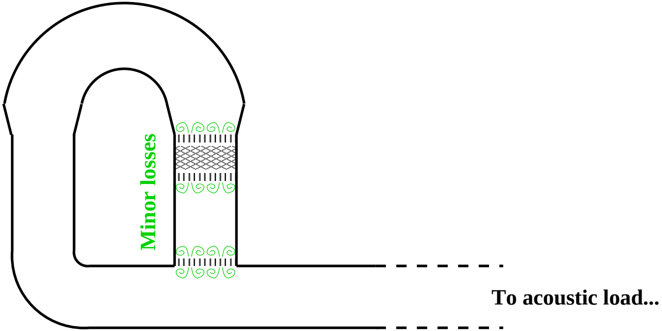
=>"jet pumps"[30] (makes $\rho_0 \overline{u_2} + \overline{\rho_1 u_1} = 0$) or membranes[31] (makes $\overline{u_2} = 0$) to suppress Gedeon streaming
=>"tapered" tube[30,32] to diminish Rayleigh streaming in the TBT



⋆ Open questions : Heat transport by $\rho_0 \overline{\mathbf{v_2}} + \overline{\rho_1 \mathbf{v_1}}$? Is acoustic streaming always undesirable for TA engines ?

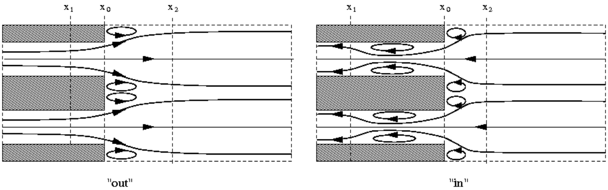
[30] Backhaus et al., JASA, 99 ; [31] Tijani et al, JASA, 2011 ; [32] Olson et al., Cryogenics, 1997

Aerodynamical and thermal edge effects



Basic concepts

⋆ Geometrical singularities => Vorticity and minor losses‡



Specificity of acoustic flow ?

⋆ Memory effects ? => are established results for steady flows applicable to oscillating flows (quasi-steady assumption)

Impact on thermoacoustics : only losses or more complex processes ?

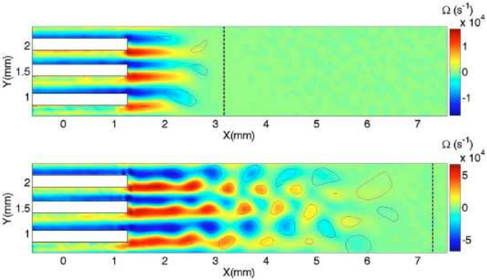
⋆ Not only losses, e.g. effect of vortex shedding in oscillating flow ?
⋆ Temperature oscillations also affected by the singularity

=> Impact on the aerodynamical heat exchange between stack and HX ?

‡ **Minor losses does not mean negligible losses**

Aerodynamical edge effects : academic studies

- ⋆ Many computational and experimental studies since around 2000, among which :
  - ♭ Pressure drop[33] or nonlinear acoustic impedance measurements[34]
  - ♭ Flow vizualization[35−37] compared with numerical modeling[38,39]
- ⋆ Main outcomes
  - ♭ Estimated Δp from steady flow (quasi-steady assumption) are in the same order of magnitude than those measured (to be confirmed however ...)
  - ♭ At high amplitudes, observation of vortex shedding[37], possibly responsible for oscillatory heat transfer between stack and HX[38]

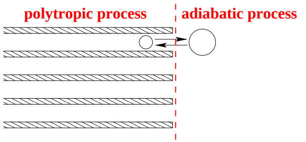


Vorticity field at the edge of a stack, for $p_{ac} = 1000$ Pa and $p_{ac} = 4000$ Pa (from [37])

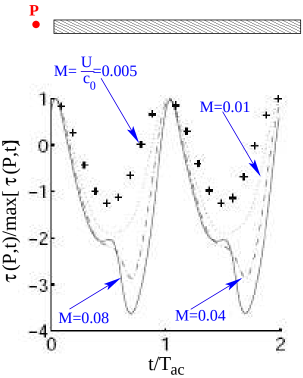
[33] Wakeland et al., JASA, 2004 ; [34] Petculescu et al., JASA, 2003 ; [35] Blanc-Benon et al., C.R. Meca. 2003 ; [36] Berson et al. Heat Mass Transf.,2008 ; [37] Berson et al.,JASA, 2008 ; [38] Besnoin et al., Acust. Acta Acust. 2004 ; [39] Marx et al., JASA, 2003

Thermal edge effects, also

⋆ Stack termination is also a strong singularity in terms of heat transfer
=> τ highly NL at the edges of the stack [Gusev et al., JSV, 2000 ; Gusev et al., JASA, 2001]



Abrupt transition in terms of heat transfer when the gas parcel passes through the stack's termination



Computation of temperature fluctuations τ at point P along two acoustic cycles, for various Mach numbers (from [D. Marx, PhD, 2003])

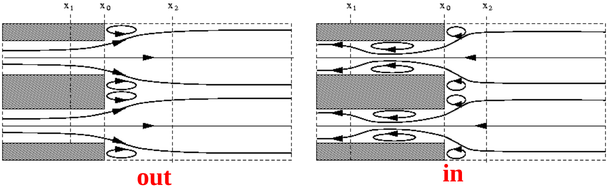
NB : Results also confirmed experimentally using Cold wire Anemometry ([Berson et al., Int. Journ. Heat Mass Transf., 2011])

Aerodynamical and thermal edge effects : open questions

⋆ Impact of vortex shedding and highly NL temperature oscillations on heat transfer between stach and HX ?
⋆ Optimum distance between stack and HX (specifically for TA heat pumps) ?

Estimate of minor losses (from the engineering standpoint)

=> Evaluation of $\overline{\Delta p}$, and therefore of additional NL losses from the time-averaging of well-known results of fully developped steady flows (e.g. in [Idelchik, handbook of hydraulic resitances, 1986])



$$\omega t \leq \pi,\ \Delta p_{out}(t) = -\frac{1}{2} K_{out} \rho_0 U^2 \sin(\omega t)$$

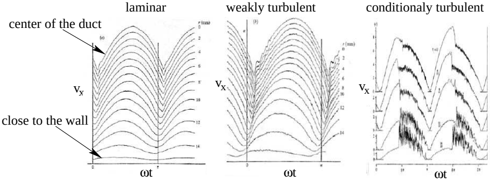
$$\omega t \geq \pi,\ \Delta p_{in}(t) = \frac{1}{2} K_{in} \rho_0 U^2 \sin(\omega t)$$

$$W_{minor} = S\overline{\Delta p(t) U \sin(\omega t)} \propto U^3$$

Turbulence ?

Specificity of turbulent oscillating (acoustic) flows ?

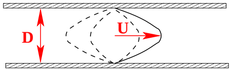
⋆ Extensively studied in steady flows
⋆ Poorly studied in oscillating flows
⋆ Often pointed out as responsible for discrepancies between experience and theory.



⋆ Reference works : Merkli and Thomann[40] (**1975**) and a few others[41−47]

⋆ Specificity of turbulent oscillating (acoustic) flows ? => Turbulence and subsequent losses[§] are controlled by 2 dimensionless numbers (vs 1 for steady flows), e.g.

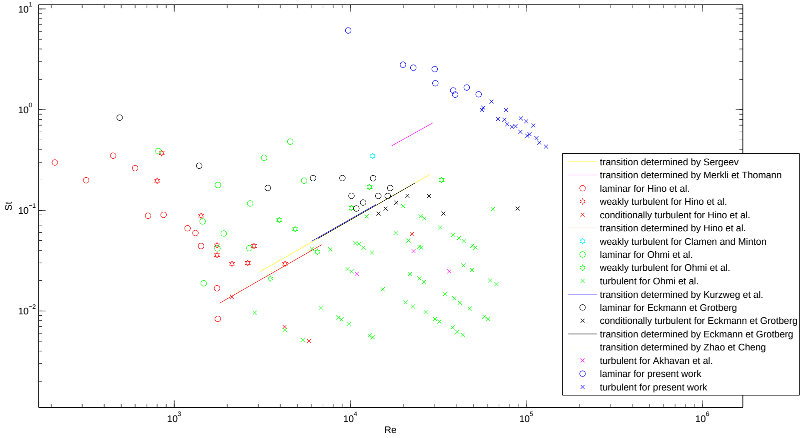
$$\mathrm{St} = \frac{\omega D}{U} \;\&\; \mathrm{Re} = \frac{UD}{\nu}$$



[40] Merkli & Thomann, JFM, 1975 ; [41] Sergeev, Fluid Dyn., 1966 ; [42] Hino et al., JFM, 1976 ; [43] Ohmi et al.,JSME, 1982 ; [44] Kurzweg et al., Phys. Fluid A, 1989 ; [45] Eckman et al.,JFM, 1991 ; [46] Zhao et al. Int. J. Heat Fluid Flow, 1996 ; [47] S. Moreau, Phd, Poitiers, 2006.

§ Wall's roughness being left apart

Experiments : transition laminar → conditionally turbulent (1)

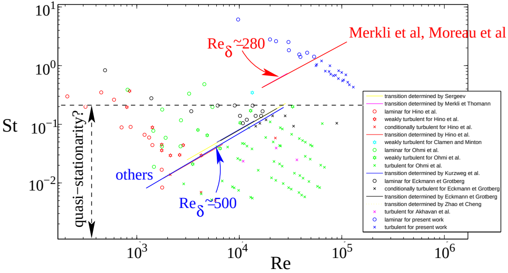


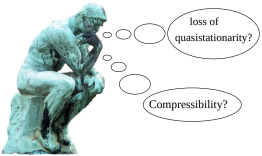
Transition laminar—→conditionnally turbulent as function of Re and St (from [S. Moreau, Phd, Poitiers, 2006])

Experiments : transition laminar → conditionally turbulent (2)

Authors	year	Fluid	Method	Freq. (Hz)	Ampl. param.	Freq. param.
Sergeev	1966	water	V	0.6-4	Re	$\mathrm{Wo} = R\sqrt{\omega/\nu}$
Merkli et al.	1975	air	HW + V	50-150	$Re_\delta = U\delta/\nu$	
Hino et al.	1976	air	HW	0.2-8	Re_δ	$\mathrm{Sto} = R/\delta$
Ohmi et al.	1982	air	HW	0.05-6	Re	$\sqrt{\omega'} = R\sqrt{\omega/\delta}$
Kurzweg et al.	1989	water	V	0.6-6	Re_δ	Wo
Eckman et al.	1991	air	HW+LDV	0.1-1.5	$\mathrm{Re}_\delta/\mathrm{Wo}$	Wo
Zhao et al.	1996	air	HW	0.1-10	x_{max}/D	$D^2\omega/\nu$
Moreau et al.	2006	air	LDV	80-120	Re	Wo

V=visualization, HW=Hot Wire, LDV=Laser Doppler Velocimetry



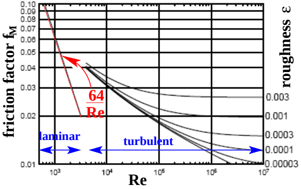


Evaluation of losses due to turbulence (DELTA-EC)

=> modification of $\langle \mathfrak{w}_\nu \rangle$ from the time-averaging of well-known results of fully developped steady flows :

- **steady flows**

$$\Delta p = f_M \frac{L}{D} \frac{1}{2} \rho_0 \langle v_x \rangle^2$$
$$\langle \mathfrak{w}_{\nu,turb} \rangle = \frac{\Delta p \langle v_x \rangle}{L} = \frac{\rho_0 f_M \langle v_x \rangle^3}{2D}$$



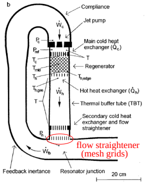
- **oscillating flows** $\left(\mathrm{Re(t)} = \frac{\langle v_x(t) \rangle D}{\nu}\right)$

$$\langle \mathfrak{w}_{\nu,turb} \rangle = \frac{\rho_0 \overline{f_M |\langle v_x \rangle|^3}}{2D},$$
$$f_M(\mathrm{Re(t)}) \approx f_M + \frac{df_M}{d\mathrm{Re}} \left(|\mathrm{Re(t)}| - \mathrm{Re}_{\max}\right)$$

where both f_M and $d_{\mathrm{Re}} f_M$ are evaluated at $\mathrm{Re}_{\max}$ from Moody chart, knowing ϵ and Re (approach considered unsatisfactory by the authors themselves . . .)

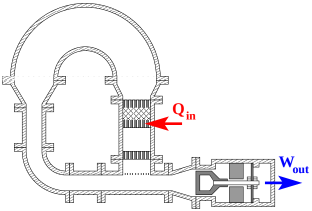
Avoiding turbulence from the engineering standpoint

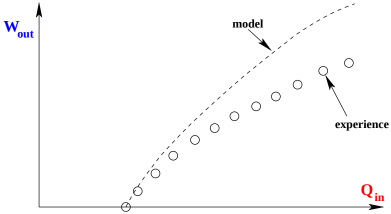
- polish internal surfaces of ducts
- use of "flow straighteners"



Plan

Introduction





⋆ How to fit experiments and theory ? => adjust any poorly known parameter ?

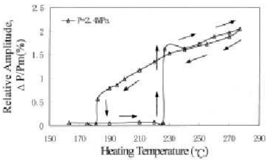
⋆ Is it satisfactory ? Not at all. . .

⋆ How to dissociate the role of each NL process ?

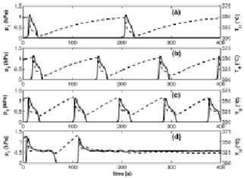
=> study the transient regime

Why studying the dynamics of thermoacoustic oscillators

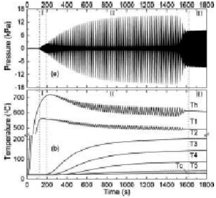
- ⋆ Much more information in the transient regime than in steady-state
- ⋆ TA Oscillators can exhibit complicated dynamics[48–53]



Hysteresis[53]



Switch on/off[51]



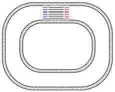
"Fishbone-like instability"[54]

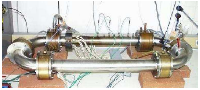
> A key point : each of the NL processes operates with its own time-scale.

=> Back to basics :

Are we able to predict, even qualitatively, the transient regime of wave amplitude growth/saturation in simple thermoacoustic devices ?

A closed loop, stacked−based, travelling wave engine

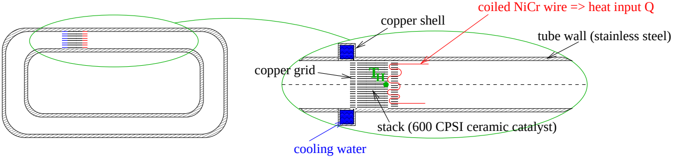




A quarter wavelength, standing wave engine

[48] Penelet et al., Cryogenics, 2002 ; [49] Swift, JASA, 1992, [50] Zhou et al., Cryogenics, 1998 ; [51] Penelet et al., Phys. Let. A, 2006 ; [52] Penelet et al., Int. J. Heat Mass Trans., 2012 ; [53] Chen et al., Cryogenics, 1999 ; [54] Yu et al. JASA, 2010.

The dynamics of a closed-loop TAO : experiments



⋆ **Experimental protocol**

♭ Set $Q = Q_0$ slightly below $Q_{onset} \approx 55$W

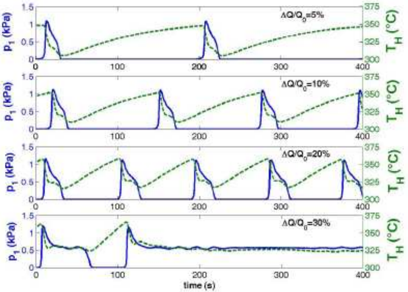
♭ At $t = t_0$, set Q to $Q_0 + \Delta Q$ => onset.

⋆ **Experimental results**[51]

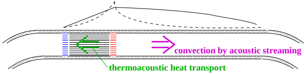
♭ Low ΔQ, periodic switch on/off

♭ Larger ΔQ, stabilization after overshoot

NB : $T_H|_{t\to\infty} < T_H|_{t=0}$
=> the details of $T_0(x)$ impacts TA amplification !



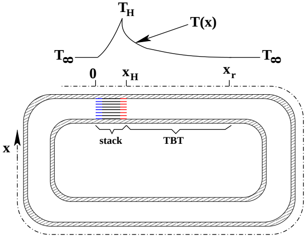
=> NL effects involving heat transport by sound seem to be predominant !



The dynamics of a closed-loop TAO : modeling (1)

⋆ **Description of wave amplitude growth/attenuation.**

♭ For a given temperature distribution $T(x)$ through the TAC, each element is described by its T-matrix, leading to a characteristic equation[55]



$$\begin{pmatrix} \tilde{p}(x_r) \\ \tilde{u}(x_r) \end{pmatrix} = M_{stack} \times M_{TBT} \times M_{loop} \times \begin{pmatrix} \tilde{p}(x_r) \\ \tilde{u}(x_r) \end{pmatrix}$$

$$= \begin{bmatrix} \mathcal{M}_{pp} & \mathcal{M}_{pu} \\ \mathcal{M}_{up} & \mathcal{M}_{uu} \end{bmatrix} \times \begin{pmatrix} \tilde{p}(x_r) \\ \tilde{u}(x_r) \end{pmatrix}$$

$$=> \det\left(M_{stack} \times M_{TBT} \times M_{loop} - I_2\right) = 0$$

$$=> f(\omega, T(x)) = 1 + \mathcal{M}_{pp}\mathcal{M}_{uu} - \mathcal{M}_{pu}\mathcal{M}_{up} - (\mathcal{M}_{pp} + \mathcal{M}_{uu}) = 0$$

♭ Allow the angular frequency ω to be complex[56], and find $\omega = \Omega + i\epsilon$ so that $f(\omega, T(x)) = 0$.

♭ Quasi-steady state assumption (i.e. $\epsilon \ll \Omega$) :

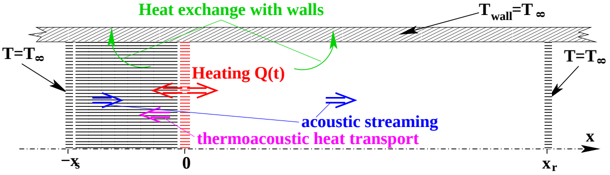
$$\frac{dp_{rms}}{dt}(x_0, t) \approx \epsilon\,[T(x)]\, p_{rms}(x_0, t)$$

where $\epsilon = \Im(\omega)$ is the amplification rate ($\epsilon > 0$: amplification, $\epsilon < 0$: attenuation)

[55] Penelet et al., Acust. Acta Acust., 2005 ; [56] Guedra et al., Acust. Acta Acust. 2012.

The dynamics of a closed-loop TAO : modeling (1)

⋆ **Unsteady heat transfer[57] (simplified and summarized).**



$$\text{Stack } (x<0): \quad \frac{\partial T}{\partial t} + \frac{\Phi \rho_0 C_p}{\rho_s C_s}\frac{\dot m}{\rho_0}\frac{\partial T}{\partial x} = \frac{\lambda_s + \Gamma_\lambda P^2}{\rho_s C_s}\frac{\partial^2 T}{\partial x^2} - \gamma^{s\leftrightarrow w}(T - T_\infty),$$

$$\text{TBT } (x>0): \quad \frac{\partial T}{\partial t} + \frac{\dot m}{\rho_0}\frac{\partial T}{\partial x} = \kappa_f \frac{\partial^2 T}{\partial x^2} - \gamma^{f\leftrightarrow w}(T - T_\infty),$$

$$\text{HHX } (x=0): \quad (\lambda_s + \Gamma_\lambda P^2)\left.\frac{\partial T}{\partial x}\right|_{x=0^-} = \lambda_f \left.\frac{\partial T}{\partial x}\right|_{x=0^+} + \frac{Q(t)}{\pi R^2}$$

$$\text{Delay for streaming}: \quad \frac{d\dot m}{dt} + \frac{\dot m}{\tau_v} = \frac{\Gamma_{str} P^2}{\tau_v}$$

where Γ_λ, Γ_{str} and $\tau_v \approx 0.9s$ are evaluated from refs. [58], [59], and [60],respectively.

Nomenclature	
$\dot m = \rho_0 \overline{v_{x_2}} + \overline{\rho_1 v_{x_1}} \propto P^2$	steady mass flow rate due to acoustic streaming
$\rho_0 C_p$ (resp. $\rho_s C_s$)	volumetric heat capacity of fluid (resp. of stack)
λ_f (resp. λ_s)	thermal conductivity of fluid (resp. of stack)
P	peak amplitude of acoustic pressure at some reference point
Φ	porosity of stack

[57] G. Penelet et al., Phys. Rev E, 2005 ; [58] T. Yazaki, J. Heat Transf., 1983 ; [59] V. Gusev et al., JASA, 2000 ; [60] m. Amari et al., Acust. Acta Acust., 2003.

The dynamics of a closed-loop TAO : modeling (2)

⋆ **Modeling of the transient regime : summary.**

Equations describing unsteady heat transfer (including heat transport by sound) . . .

$$\begin{aligned}
\frac{\partial T}{\partial t} + \frac{\Phi \rho_0 C_p}{\rho_s C_s} \frac{\dot{m}}{\rho_0} \frac{\partial T}{\partial x} &= \frac{\lambda_s + \Gamma_\lambda P^2}{\rho_s C_s} \frac{\partial^2 T}{\partial x^2} - \gamma^{s \leftrightarrow w} (T - T_\infty) , \\
\frac{\partial T}{\partial t} + \frac{\dot{m}}{\rho_0} \frac{\partial T}{\partial x} &= \kappa_f \frac{\partial^2 T}{\partial x^2} - \gamma^{f \leftrightarrow w} (T - T_\infty) , \\
\left(\lambda_s + \Gamma_\lambda P^2\right) \left.\frac{\partial T}{\partial x}\right|_{x=0^-} &= \lambda_f \left.\frac{\partial T}{\partial x}\right|_{x=0^+} + \frac{Q(t)}{\pi R^2} \\
\frac{d\dot{m}}{dt} + \frac{\dot{m}}{\tau_v} &= \frac{\Gamma_{str} P^2}{\tau_v}
\end{aligned}$$

. . .coupled to that describing wave amplification[¶] (possibly with additional dissipation due to minor losses) :

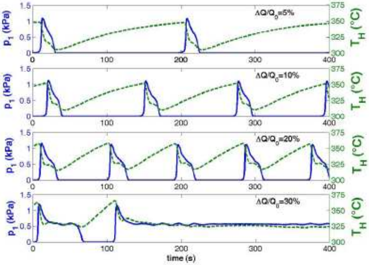
$$\frac{dP}{dt} = \epsilon \left[T(x,t)\right] P - \epsilon_{minor} P^2$$

=> Nonlinear set of differential and partial differential equations, solved using a finite difference scheme

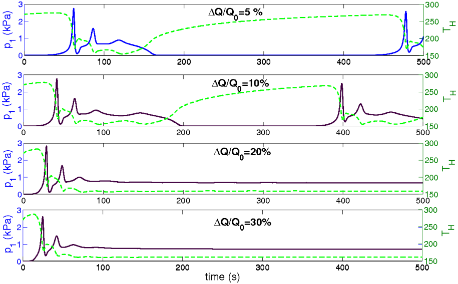
[¶] The variations of $T(x)$ being assumed negligible at the time-scale of a few acoustic periods

The dynamics of a closed-loop TAO : Theoretical results (1)

Experiments



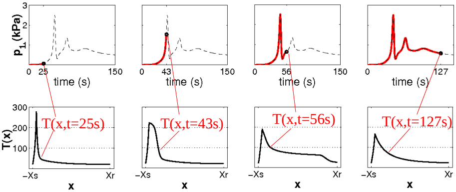
Simulations



=> The spontaneous and periodic onset damping is reproduced qualitatively

From the results of simulation this complicated behavior is due to acoustic streaming

The dynamics of a closed-loop TAO : Theoretical results (2)

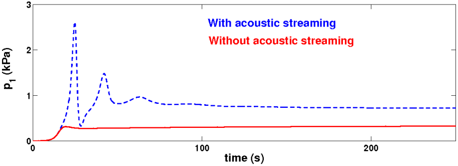


⋆ T_H ↘ while P ↗ => the shape of the temperature field, notably in the TBT, strongly impacts TA amplification within the stack

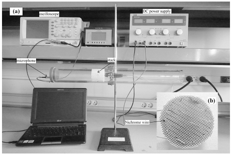
⋆ Acoustic streaming plays a crucial role.

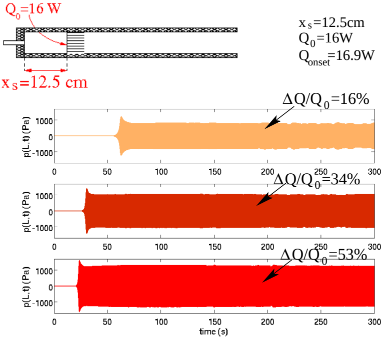
⋆ Impact of acoustic streaming ? not obvious . . .

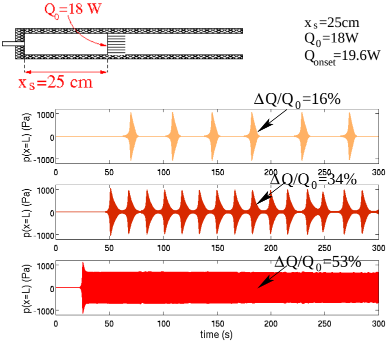
. . .does not necessarily lowers the engine's efficiency



The transient regime of a TA Laser : experiments



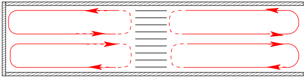




=> observation of periodic switch on/off while

$\rho_0 \langle \overline{v_{x_2}} \rangle + \langle \overline{\rho_1 v_{x_1}} \rangle = 0\,!\,!\,!$

=> "Rayleigh" streaming ?



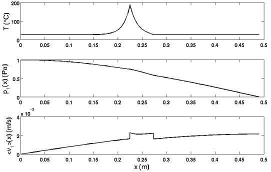
The transient regime of a TA Laser : simplistic modeling (1)

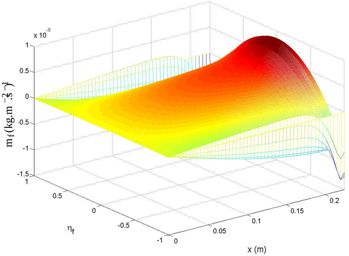
⋆ Spatial distribution of acoustic streaming

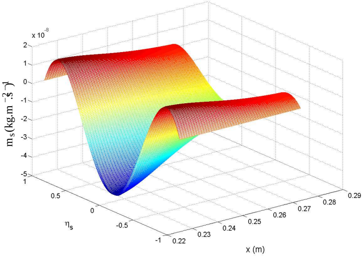
♭ Fix x_s, fix Q_0 to Q_{onset}, and calculate $T(x)$

♭ Calculate the spatial distribution of acoustic variables, and set $p_1(x=0) = P = 1$ Pa.

♭ Calculate the spatial distribution of the time average mass flow $\dot{m} = \rho_0 \overline{v_{x_2}} + \overline{\rho_1 v_{x_1}}$ within the stack and the waveguide







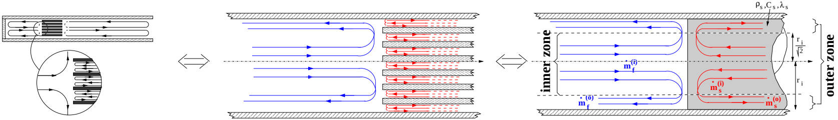
Spatial distributions of $\dot{m}$ within the waveguide ($\dot{m}_f$, left) and within one channel of the stack ($\dot{m}_s$, right), for x_s=22.5 cm, and $Q_0 = 20.17$W. (Calculated from [Bailliet et al.,JASA,2001])

⋆ $\dot{m}_{f,s}$ is 2D => Rough simplifications are needed

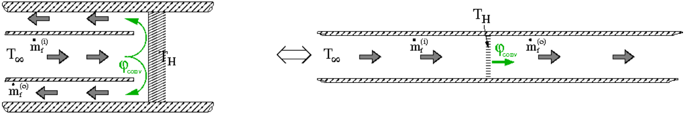
⋆ Stack & waveguide treated as isolated systems to compute $\dot{m}$ => How is $\dot{m}$ at the interface ?

The transient regime of a TA Laser : simplistic modeling (2)

⋆ Simplistic account of heat convection by acoustic streaming, **without equations**



=> Separate the waveguide and stack cross-sections into an inner zone with temperature $T^{(i)}$, and an outer zone with temperature $T^{(o)}$



=> Account for the heat removed by acoustic streaming at the stack's interface

The transient regime of a TA Laser : simplistic modeling (3)

⋆ Simplistic account of heat convection by acoustic streaming, **with equations**[61]

Waveguide

$$x \leq x_s,\ \frac{\partial T^{(i)}}{\partial t} + \frac{\langle|\dot{m}_f|\rangle}{2\rho_f}\frac{\partial T^{(i)}}{\partial x} = \kappa_f \frac{\partial^2 T^{(i)}}{\partial x^2} - \frac{T^{(i)} - T_\infty}{\tau_f},$$

$$x \leq x_s,\ \frac{\partial T^{(o)}}{\partial t} - \frac{\langle|\dot{m}_f|\rangle}{2\rho_f}\frac{\partial T^{(i)}}{\partial x} = \kappa_f \frac{\partial^2 T^{(o)}}{\partial x^2} - \frac{T^{(o)} - T_\infty}{\tau_f},$$

Stack

$$x \geq x_s,\ \frac{\partial T^{(i)}}{\partial t} - \frac{\Phi\rho_f C_f}{\rho_s C_s}\frac{\langle|\dot{m}_s|\rangle}{2\rho_f}\frac{\partial T^{(i)}}{\partial x} = \left(\kappa_s + \Gamma_\kappa P^2\right)\frac{\partial^2 T^{(i)}}{\partial x^2} - \frac{T^{(i)} - T_\infty}{\tau_s},$$

$$x \geq x_s,\ \frac{\partial T^{(o)}}{\partial t} + \frac{\Phi\rho_f C_f}{\rho_s C_s}\frac{\langle|\dot{m}_s|\rangle}{2\rho_f}\frac{\partial T^{(i)}}{\partial x} = \left(\kappa_s + \Gamma_\kappa P^2\right)\frac{\partial^2 T^{(o)}}{\partial x^2} - \frac{T^{(o)} - T_\infty}{\tau_s},$$

Interface

$$\frac{Q}{\pi r_i^2} = \lambda_f \left.\frac{\partial T}{\partial x}\right|_{x_s^-} - \left(\lambda_s + \Gamma_\lambda P^2\right)\left.\frac{\partial T}{\partial x}\right|_{x_s^+} + \frac{C_f}{2}\left(\langle|\dot{m}_f|\rangle + \Phi\langle|\dot{m}_s|\rangle\right)\left(T|_{x_s} - T_\infty\right),$$

Amplification $\frac{dP}{dt} = \epsilon\left\{T(x,t)\right\}P,$

Streaming

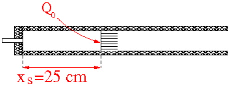
$$\frac{\partial \dot{m}_f}{\partial t} = \frac{\Gamma_f P^2}{\theta_f} - \frac{\dot{m}_f}{\theta_f},\ \frac{\partial \dot{m}_s}{\partial t} = \frac{\Gamma_s P^2}{\theta_f} - \frac{\dot{m}_s}{\theta_s},$$

- ∼ streaming, - ∼ TA heat pumping

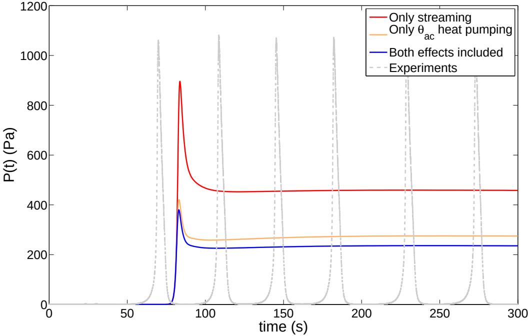
with $T(x,t) = \frac{T^{(i)}(x,t) + T^{(o)}(x,t)}{2}$ and $T|_{x=0} = T|_{x=x_s+d_s} = T_\infty.,$

[61] Penelet et al., Int. Journ. Heat Mass Transf., 2012

The transient regime of a TA Laser : Theoretical results

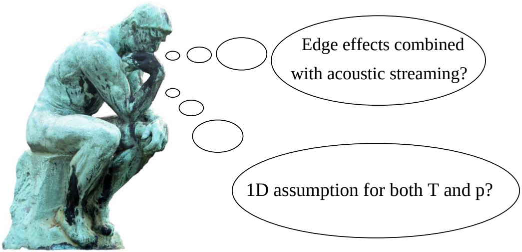


x_s =25 cm, Q_0 =19.5 W, (Q_{onset} =21.2 W), $\Delta Q/Q_0$ =16 %



The transient regime of a TA Oscillators : the big deal . . .

What is the mechanism responsible for the switch on/off process ? . . .



. . .and how can we claim understanding TA engines if we cannot reproduce such an effect ?

The ThermoAcoustic Oscillator as an interesting dynamical system

⋆ An autonomous oscillator driven by heat

- which is almost out of control above threshold
- and which exhibits complicated dynamical behaviors (overshoot, "integrate and fire regime", . . .)

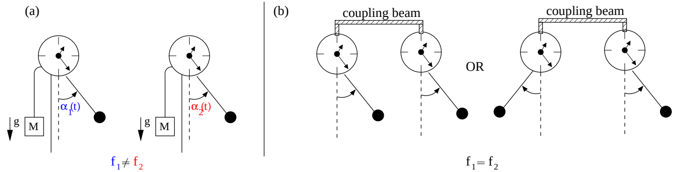
⋆ Investigate the nonlinear coupling between a TAO and an external sound source, with two objectives

- revisit universal aspects of synchronization[62] phenomena in the frame of thermoacoustics (notably for teaching purpose).
- investigate the active control of TA engine to $\nearrow$ their efficiency

[62] A. Pitkovsky, M. Rosenblum, J. Kurths, « Synchronization : A Universal Concept in Nonlinear Science », Cambridge University Press, NY, 2001.

Synchronization phenomena

⋆ First observations : Huygens (1665, « sympathy of two pendulum clocks »)

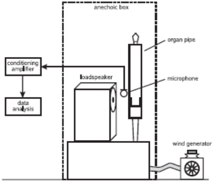


⋆ Use or observation of synchronization phenomena are nowadays abundant

- ♭ biology, medicine(singing crickets, circadian rythm, cardiac pacemaker . . .),
- ♭ electronics engineering(triode generators for radio communications . . .),
- ♭ mechanics(clocks, organ pipes . . .), physics/chemistry(B–Z reaction, . . .),
- ♭ social life(clapping audience).

⋆ An example in the field of acoustics : synchronization of organ pipes

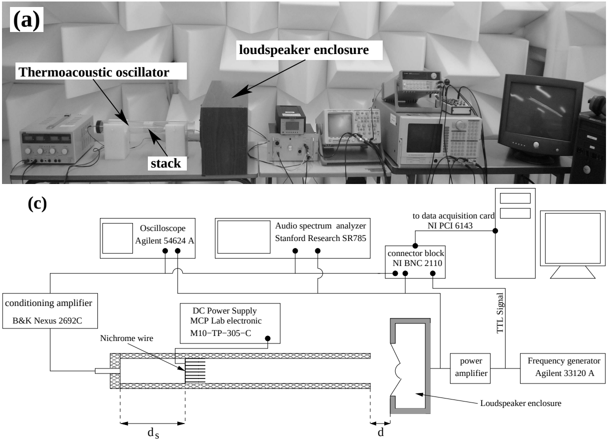
- ♭ Former study by Lord Rayleigh : mutual synchronization of two organ pipes and the quenching effect (oscillation death)
- ♭ . . .but even recent studies[63,64] :



Sketch of the exp. by Abel et al.[64]

[63] Abel et al, J. Acoust. Soc. Am. 119 :2467, 2006 ; [64] Abel et al. Phys. Rev. Let., 103 :114301, 2009.

Synchronization of a TAO by an external sound source[65]



(a) Photograph of the complete experimental set-up. (c) Sketch of the experimental set-up.

[65] Penelet & Biwa, Am. Journ. Phys., 2013

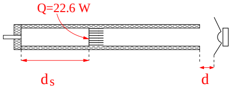
Synchronization by a sound source : experimental protocol

⋆ Experimental protocol

♭ Fix heat power Q=22.6 W ($Q > Q_{onset}$)

♭ Fix both d and d_s

♭ Proceed to measurements‖ by varying the driving frequency f and the loudspeaker voltage U_{rms}

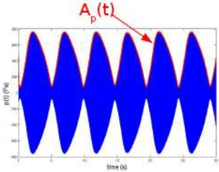


⋆ Signal processing

♭ Make both FFT and Hilbert transforms of $p(t)$ and $U(t)$

♭ Quantities of interest for data analysis :

- Frequency spectra $p(f)$ and $U(f)$
- Amplitude modulation $A_p(t)$
- Instantaneous phase difference $\Psi(t) = \Phi_p(t) - \Phi_U(t)$



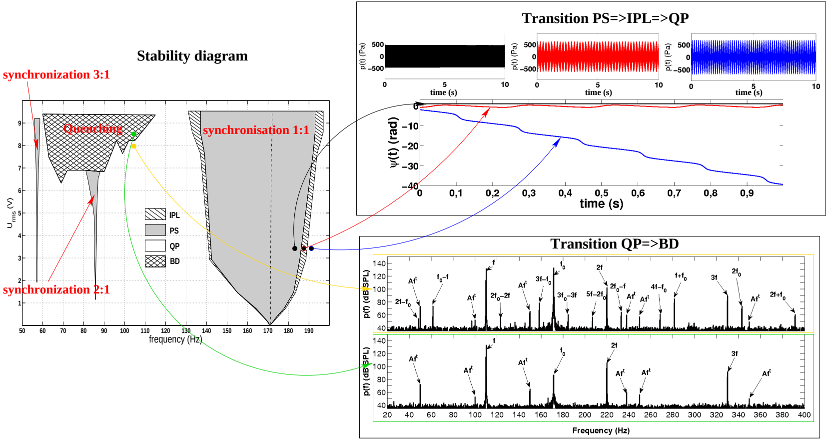
⋆ Different possible states

(PS) Perfect Synchronization	(IPL) Imperfect Phase Locking	(QP) QuasiPeriodicity	(BD) Beating Death
$f_{TAO} = f$ (or $f_{TAO} = \frac{f}{n}$) $A_p(t) = c^{te}$ $\Psi(t) = c^{te}$	$f_{TAO} \neq f$ (or $f_{TAO} \neq \frac{f}{n}$) $A_p(t) \neq c^{te}$ $\Psi(t) \neq c^{te}$ but bounded	$f_{TAO} \neq f$ (or $f_{TAO} \neq \frac{f}{n}$) $A_p(t) \neq c^{te}$ $\Psi(t) \neq c^{te}$ (unbounded)	$A_p(t) = c^{te}$

‖ Each set of measurements ∼ 12-24 hours of total duration !!

Synchronization by a sound source : the stability diagram

⋆ Stability diagram as a function of $U_{loudspeaker}$ and f, for d=1 mm, d_s=8 cm and Q=22.6 W



♭ Some universal aspects of synchronization are retrieved by this experiment

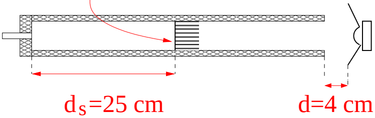
♭ Some aspects are intrinsic to the TAO itself :

=> why are the Arnold tongues asymetric ?

=> why does quenching occurs around $f \approx f_{TAO}/2$?

Other phenomena related to the external forcing of a TAO

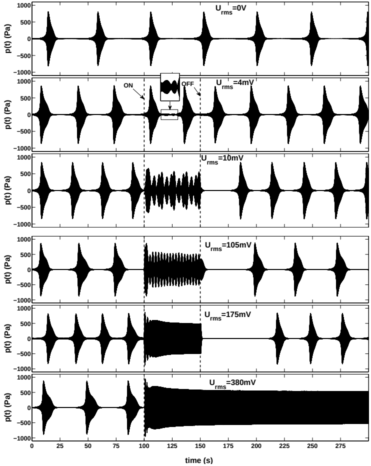
⋆ **What if the TAO is operated as a relaxational oscillator ?**



Loudspeaker off => "integrate and fire regime", with $f_{TAO} \approx 177.1$ Hz

⋆ **Experimental procedure**

1.- Settle f to 176.9 Hz ($\neq f_{TAO}$)
2.- Switch the loudspeaker on at t=100 s
3.- Switch the loudspeaker off at t=150 s
4.- Repeat steps 1-3 with another driving voltage

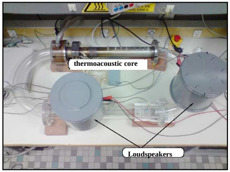


interpretation ? => Simply quenching or more complicated processes ?
=> does external forcing lead to the transformation of a bistable regime (with periodic switch between two stable states) into a stable one ?

And to conclude . . .what about Active Control of a TAO to increases its efficiency

Basic ideas

- Above threshold (and at fixed heat input Q_0), TA engines are "out of control"
- Is there a possibility to ↗ the efficiency of a TA engine with auxiliary acoustic sources ?

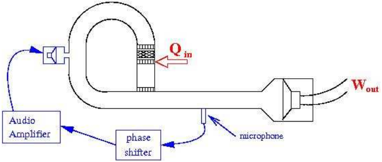


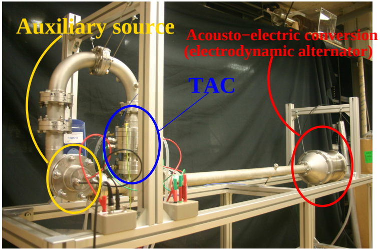
A proof of concept study performed succesfully[66]

- Closed-loop , stack based, TA engine
- Active control by 2 sources controlled in both amplitude and phase

A new proof of concept study which is being performed . . .

- Thermo-Acousto-Electric engine
- Active control by 1 source and a feedback loop





. . .see the presentation by C. Olivier et al. to the next workshop !

C. Desjouy et al., J. Appl. Phys., 2010.

Plan

Conclusion

⋆ Conclusion of part 1 (linear theory, design tools)

Design tools are available, but

- they are based on the linearization to 1^{st} order of the governing equations,
- they are based on a 1-D description both acoustics and heat transport,
- they predict steady-state operation.

⋆ Conclusion of part 2 (nonlinear processes)

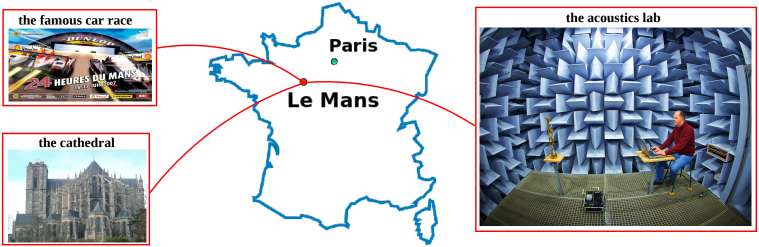
NL process	Academic understanding	Appropriate modeling	Impact on TA Engines
TA Heat pumping	☺	☺	significant
NL acoustics	☺		not much
Streaming	☺	☹	significant
Edge effects	☺	☺	significant
Turbulence	☹	☹	?

⋆ Conclusion of part 3 (dynamics of TA Oscillators)

♭ Even simplest TA devices exhibits complicated behaviors which are not very well understood

♭ The study of the transient regime may provide a deeper physical insight on the mechanisms responsible for sound saturation . . .

♭ and may therefore provide new opportunities to increase the engine's efficiencies



Thank you for your attention . . .

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