Improving Energy-efficiency of Multi-antenna Receivers via Adaptation of ADC Resolutions

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Abstract—The energy-efficiency of a receiver, which we measure in this work by the number of information bits conveyed per Joule of energy consumption at the receiver, is greatly influenced by the design and bit resolution of the analog-to-digital converter (ADC) it employs. For a given ADC, using higher resolution leads to less quantization error in the digitized receive signal, yet requires a larger power dissipation. This trade-off can be utilized to maximize the energy-efficiency by adapting the ADC resolution according to the state of the communication channel. For a receiver with multiple antennas, the resolution of ADC associated with each antenna can be jointly adapted, leading to a multivariate integer programming problem which is difficult to solve to optimality. We propose several adaptation strategies which differ in complexity and the amount of online computations, and compare their performances based on the simulation results. Furthermore, we show that equipping the receiver with more antennas is beneficial to its energy-efficiency when only a subset of antennas associated with relatively good channel conditions is chosen for reception, and an appropriate ADC resolution is employed for the quantization of receive signals. As the improvement in energy-efficiency falls off with increasing number of antennas, a cost-effective antenna number can be determined by the receiver, depending on the application scenario and the physical limitations on the device.

I. INTRODUCTION

In modern receiver design, more and more receive functions are implemented by digital hardware due to its high speed and low cost, which requires the analog receive signal to be converted into digital format as early as possible [1]. The analog-to-digital converter (ADC), which is connected directly to the output of receive antenna in this case, is expected to be a limiting factor of the system as it consumes a significant amount of power when operating at high sampling rate and resolution. It was reported in [2] that the power dissipation of an 8-bit ADC with sampling rate 20 GS/s reaches as much as 10 Watt, which is obviously impractical for most mobile devices. From a communication point of view, this motivates the employment of low ADC resolution, as high sampling frequency is required by many applications such as cognitive radio. In recent years, there have been various works investigating the performance limit of communications over quantized channel, e.g., [3][4], where the focus was on the loss of capacity when low ADC resolution is used, as well as design of the quantizer. In our previous work [5], we treat the ADC resolutions in a multi-antenna receiver as adaptable parameters and attempt to maximize the energy-efficiency of the system, which is quantified as the number of information bits conveyed per Joule of energy consumption. The optimization is based on the trade-off between power dissipation of the receiver and quantization error introduced by the A/D conversion, and significant gain over systems employing fixed, although low, ADC resolutions has been observed with the simulation results provided in the paper. In this work, we further pursue the idea of ADC resolution adaptation based on available channel state information (CSI), with the emphasis on methods with low complexity and less online computations, and discuss also the impact of the number of antennas on system energy-efficiency.

At the receiver side, diversity can be achieved via the deployment of multiple antennas. With independent signal paths and maximal ratio combining (MRC), the gain in receive signal-to-noise ratio (SNR) equals the number of receive antennas [6]. However, the hardware complexity and power consumption of the receiver also scales with the number of antennas. A common technique to compensate for this drawback, known as antenna selection (AS) [7], is to choose signals received by a subset of antennas based on the SNR values for further processing, thus reducing the necessary RF chains and the associated power consumption. When optimizing the ADC resolution for each antenna, we implicitly perform AS since the receive signal of an antenna with 0 bit of resolution will not be processed. This motivates the proposal of suboptimal algorithms which aim at finding the optimal number of active antennas while their ADC resolutions are kept identical. The important role of AS can be understood when we compare the performance of the optimal ADC adaptation scheme and suboptimal adaptation schemes with or without AS via numerical simulations.

The rest of the paper is organized as follows. In Section II, we introduce the system model and in particular, give expressions of a lower bound on channel capacity and the total power dissipation as functions of the ADC resolution vector. The problem of maximizing energy-efficiency is then formally given. We propose several static and adaptive schemes for the adaptation of ADC resolutions in Section III, followed by simulation results and analysis exhibited in Section IV. Section V concludes the paper with a summary of ideas, methods, and results which have been presented, and shortly discusses related open issues as well as possible future works.

Notations: we use boldfaced letters to represent vectors and matrices throughout the paper. The operator $(\cdot)^{\text{H}}$ stands for the Hermitian of a matrix, the symbol $1_M$ denotes the identity matrix of dimension $M \times M$, and $\text{diag}(A)$ denotes the diagonal matrix with the same diagonal elements as $A$. 
II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a multiple-input multiple-output (MIMO) system with $N$ transmit antennas and $M$ receive antennas. The vector channel output $y \in \mathbb{C}^M$ before quantization is given by

$$y = \sqrt{\alpha} H x + \eta,$$

where the channel matrix $H \in \mathbb{C}^{M \times N}$ contains i.i.d. Gaussian distributed channel coefficients with zero mean and unit variance, $x \in \mathbb{C}^N$ is the vector of transmitted symbols, and $\eta \in \mathbb{C}^M$ is the i.i.d. zero-mean complex Gaussian noise vector. The positive scalar $\alpha$ stands for the average combined gain of transmit power per antenna and the communication channel. Assuming uniform power allocation at the transmitter and uncorrelated transmit symbols, we have that the covariance matrix of the transmit signal $R_{xx} = E[x x^\text{H}]$ is equal to the identity matrix. The covariance matrix of the unquantized channel output $y$ is then computed as

$$R_{yy} = E[y y^\text{H}] = R_{\eta \eta} + \alpha H H^\text{H},$$

where $R_{\eta \eta} = E[\eta \eta^\text{H}] = \sigma^2 \cdot 1_M$ is the covariance matrix of the noise vector with $\sigma^2$ denoting the noise power at each receive antenna. Defining the ratio $\gamma = \alpha/\sigma^2$, we have

$$R_{yy} = \sigma^2 \left(1_M + \gamma H H^\text{H}\right).$$

The receiver is equipped with a number of A/D converters, possibly less than the number of antennas $M$ due to the limitation in size and power. For the ease of derivation, we assume that there are $M$ identical A/D converters at the receiver, all of which act as scalar quantizers. The continuous-time receive signal at each antenna is first sampled at rate $f_s$, and then quantized to a certain level represented by a finite number of bits. Let $b \in \{0, 1, \ldots, b_{\text{max}}\}^M$ be the vector of resolutions employed by each ADC, where $b_{\text{max}}$ is the maximal number of bits that an ADC could use for a single sample. Antennas whose receive signals are not selected to be processed can be seen as connected to an ADC with resolution 0. The vector of quantized output $r \in \mathbb{C}^M$ can be written as

$$r = y + q,$$

where $q$ stands for the quantization noise which is usually correlated with $y$. We depict the system model of the multi-antenna receiver in Figure 1, where the quantization process is represented by the $Q(\cdot)$ operator. Given the channel matrix $H$, which can be obtained via training and channel estimation, the ADC resolution $b$ can be adapted accordingly in order to optimize certain performance metrics which include the power or energy consumption of the receiver circuitry. The assumption of perfect knowledge of $H$ is of course unrealistic, especially when the case of data access with finite precision is under consideration. We make such an assumption here to evaluate the performance limit of ADC resolution adaptation exploiting the diversity gain, and leave the problem of channel estimation with quantized receive pilots and ADC resolution optimization with imperfect CSI for future work.

We have derived in [5] a capacity lower bound of quantized MIMO channels as dependent on the resolution vector $b$, which generalizes the result in [8] where uniform ADC resolution across the antennas is employed. A simple power consumption model was also established therein to enable the formulation of the energy-efficiency maximization problem. Since the system model used in this work remains in large part the same, a review of the framework in [5] will be given in the sequel without elaboration on the technical details.

A. Capacity lower bound of the quantized channel

According to the Bussgang theorem [9][8], the output of the nonlinear quantizer can be decomposed into a desired signal part and an uncorrelated distortion as

$$r = Fy + e,$$

where the noise vector $e$ is uncorrelated with $y$, and the linear operator $F$ is taken as the MMSE estimator of $r$ from $y$:

$$F = E[ry^\text{H}] E[y y^\text{H}]^{-1} = R_{ry} R_{yy}^{-1}. $$

Consequently, we can define an effective channel $H' = FH$ and an effective noise vector $\eta' = F\eta + e$, such that $r = H'x + \eta'$. Now, if we define a new MIMO Gaussian channel with the input-output relation $r_{\text{G}} = H'x + \eta_{\text{G}}$ and assume that the covariance matrix of the noise vector $\eta_{\text{G}}$ is the same as that of $\eta'$, the capacity of the new channel provides a lower bound on that of the quantized channel, for Gaussian distributed noise minimizes the mutual information. Based on this observation and assuming that the channel input $x$ is Gaussian distributed and the transmission bandwidth is $B$, we have

$$I(x; r) \geq B \log_2 \left|1_M + R_{\eta' \eta'}^{-1} H' R_{\eta' \eta'} H'^\text{H}\right| .$$

For scalar distortion-minimizing quantizers, we are able to arrive at the achievable rate expression in bit/sec as

$$R(b) \triangleq B \log_2 \left|1_M + R_{\eta' \eta'}^{-1} H' R_{\eta' \eta'} H'^\text{H}\right| = B \log_2 \left| \frac{R_{yy}(1_M - \rho) + \rho \text{diag}(R_{yy})}{R_{\eta' \eta'}(1_M - \rho) + \rho \text{diag}(R_{\eta' \eta'})} \right|,$$

where $\rho$ is a diagonal matrix containing the distortion factors

$$\rho = \begin{bmatrix} \rho_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \rho_M \end{bmatrix} \approx 2^{-2b_1} \cdots 2^{-2b_M} .$$

The approximation of distortion factor $\rho_i$ with $2^{-2b_i}$ is in the asymptotic sense and is valid for Gaussian quantization source. Note that $R$ as a function of the resolution vector $b$ depends also on $B$, $\gamma$ and the channel matrix $H$. 

![Figure 1. Quantization of the MIMO channel and ADC resolution adaptation](image-url)
B. Power consumption model

Power dissipation of the ADC associated with the $i$-th antenna can be calculated as [10]

$$P_{ADC,i} = \begin{cases} c_0 \cdot \sigma^2 \cdot 2^{b_i}, & b_i > 0, \\ 0, & b_i = 0, \end{cases}$$

(2)

where $c_0$ is a constant determined by the specific design of the ADC. When positive resolution is employed, power dissipation of the ADC grows exponentially with $b_i$. Otherwise, the ADC does not incur any power consumption. We model the total power consumption of the receiver by

$$P(b) = c_0 \cdot \sigma^2 \sum_{i \cdot b_i > 0} 2^{b_i} + c_1,$$

(3)

where $c_1$ is a constant accounting for the power consumption of other components of the receiver. In practice, the specific design of the receiver, especially the complexity of decoding, determines whether the power consumption of A/D converters is the dominant part in the total power consumption [11]. For all simulations we take $B = 1$ MHz, $b_{max} = 8$, $c_0 = 2 \times 10^{-4}/\sigma^2$ Watt, and $c_1 = 0.02$ Watt with reference to [12].

C. Maximization of Energy-efficiency

Motivated by the demand to increase the lifetime of mobile terminals and other communication devices powered by battery, as well as by the desire to conduct green communications, energy-efficiency has become another important performance metric for communication systems over the past years. For different applications, one might want to minimize the energy needed to transmit/receive a certain amount of data, or to maximize the operation time given fixed available energy while a constant data rate is provided. In this work, we focus on the unconstrained maximization of the bit per Joule metric at the receiver, with the ADC resolution vector $b$ as the optimization variable, which is defined as

$$\max_{b \in \{0, \ldots, b_{max}\}^M} \frac{R(b)}{P(b)} \triangleq \eta$$

(4)

where the expressions for $R(b)$ and $P(b)$ are given by (1) and (3), respectively. This problem is of particular interest as it potentially provides insight to the solutions of the constrained optimizations we mentioned above. We denote the optimization objective, i.e., energy-efficiency of the receiver, with $\eta$ and subsequently denote its optimal value with $\eta^*$. The optimization in (4) with respect to integer-valued bit resolutions is a combinatorial problem with a search space growing exponentially with $M$ and $b_{max} + 1$, leading to prohibited computational complexity. For a practical ADC, the maximal bit resolution $b_{max}$ is typically a rather small number. Therefore, when the number of receive antennas $M$ is also small, an exhaustive search for the optimal $b$ is possible. Yet for a large receive antenna array we would need more effective search algorithms. Common techniques for tackling integer programming, such as the branch and bound method, often requires the computation of upper bounds on the optimal objective, which is usually achieved via solving a relaxation of the original problem [15]. In our case, this does not seem an option due to the non-linearity and discontinuity of the objective function, i.e., even if we allow $b_i$ to take real values on $[0, b_{max}]$, it is still very hard to solve (4) to optimality. We propose in [5] to apply the Particle Swarm Optimization (PSO) technique, which has been shown to produce near-optimal solutions for systems of small-to-medium scale. In the next section, we discuss a few more methods for solving (4) suboptimally, but require a lot less online computations.

III. Static and Adaptive Receive Strategies

Bit resolution of the A/D converter is conventionally a fixed parameter and is not included in the design optimization. Exploiting the trade-off between data access precision and power consumption of the ADC, gain in energy-efficiency of the system can be expected from the optimization on ADC resolution. We touched upon this issue first in [14], where the ADC resolution as a real-valued parameter is optimized for single- and multi-antenna receivers. Therein it has been assumed that uniform ADC resolution is employed for all receive antennas, which is also the common practice in real systems. According to the result of [5], this is obviously suboptimal given the availability of channel state information, not only because higher or lower resolution can be chosen for antennas with good or bad channel conditions, but also because of the fact that the receive signal from an antenna with relatively bad channel condition can be completely ignored, i.e., processed with 0 bit resolution of the ADC. This observation motivates some of the methods we propose in the following, which combine the idea of ADC resolution adaptation together with antenna selection, and achieve, to different extents, balance between performance and computational complexity.

A. Uniform ADC resolution adaptation without AS

The simplest way of performing ADC resolution adaptation is to allow a single, positive resolution for all antennas, and optimize this value based on the instantaneous CSI. This can be done via the enumeration of all feasible resolutions $1, \ldots, b_{max}$, hence requires $b_{max}$ times of evaluations of the objective. Another even simpler method is to optimize the single resolution offline, and apply the obtained optimal value irrespective of the CSI which means no online computation is needed. For the offline optimization, one could generate independently a large number of channel realizations, enumerate all feasible ADC resolutions for each realization, and finally find the resolution that gives the maximal average energy-efficiency. In fact, we have maximized the ergodic energy-efficiency with this process. The results of the optimization can be recorded in a look-up table and used directly online.

B. Uniform ADC resolution adaptation with AS

With this method, we try to distinguish the antennas according to their channel conditions and abandon the signals from the bad ones. For the remaining selected antennas, uniform ADC resolution is applied so that the complexity of the algorithm does not increase too much. Similar as before, an offline and an online version of this method can be proposed. For the offline method, we optimize over a large number of channel realizations, the ADC resolution as well as the number of active antennas. During the online operation, one only needs to find the corresponding number of antennas with the best channel conditions, and apply the optimized ADC resolution. For the online method, the optimal number of active antennas...
and the optimal ADC resolution with respect to the given CSI are to be found via enumerations over both parameters, rendering much heavier computational burden to receiver.

C. Non-uniform ADC resolution adaptation based on PSO

When we do not restrict the ADC resolutions employed by the active antennas to be identical but attempt to find the energy-efficiency maximizing, non-uniform resolution vector $b$, more effective integer programming technique is necessary if $M$ and $b_{\text{max}}$ are not small enough for an exhaustive search to be feasible. The PSO method [15] serves as a good candidate here since it is rather independent of the problem structure and can be implemented easily. Although originally proposed for solving unconstrained optimizations with real-valued variables, the method can be applied to tackle integer programming without any complicated modification [16].

The PSO method is a stochastic optimization technique based on the social behaviour of a population of individuals. Each individual, termed as a particle, moves in the feasible region of the optimization problem probing for good solutions, and shares information with the whole set of particles, called the swarm. The movement of the particles in each generation of the algorithm is random, but also depends on the memory of the individual particles as well as of the swarm. Let the swarm contain $S$ particles. For initialization of the algorithm, $S$ feasible solutions of the optimization are generated at random, which are denoted with $b_1^0, b_2^0, \ldots, b_S^0$. In the $(k+1)$-th generation, the particle $s$ evolves according to the formula

$$
\begin{align*}
    v^{k+1}_s &= w v^k_s + s_1 r_1 (p^k_s - b^k_s) + s_2 r_2 (p^k_s - b^k), \\
    b^{k+1}_s &= b^k_s + v^{k+1}_s,
\end{align*}
$$

i.e., the position of particle $s$ is incremented by a velocity vector $v^{k+1}_s$, which is computed based on the velocity vector $v^k_s$ in the previous generation, the distance between the particle and the best self-found solution $p^k_s - b^k_s$, and the distance between the particle and the global best solution found by the swarm $p^k - b^k$. The parameter $w$ is called the inertia weight and is usually chosen as a decreasing function in the generation index, facilitating global search in early generations of the algorithm and local search in later generations. The weights $s_1$ and $s_2$ are called the cognition and social learning rates, and they are usually kept constant throughout the generations.

The uniformly distributed random numbers $r_1$ and $r_2$ on $[0,1]$ add randomness into the trajectory of each particle. In our numerical studies, the parameter setting $s_1 = s_2 = 1$, $w = 1 - 0.02k$ is used. For our optimization problem, the particle positions correspond to ADC resolution vectors $b$, hence each fractional valued particle position obtained from the update needs to be mapped to a feasible solution by rounding and fitting the values into the set $\{0, \ldots, b_{\text{max}}\}$. After the mapping and fitting, the objective function is evaluated at each new particle position, and the local best solutions as well as the global best solution need to be updated. The algorithm terminates when a maximal number of generations $G$ is reached. Depending on the dimension of the optimization, the values of $S$ and $G$ can be increased to improve the performance of the PSO. One could also repeat the algorithm for several runs and pick the best solution among all obtained global best solutions. In our simulations we take $S = G = 20$ and run the algorithm 20 times for each parameter setting.

In the above discussions, the number of antennas the receiver has is a fixed value. An interesting question we would like to investigate based on simulation results is, how does the energy-efficiency of the receiver change with increasing number of antennas. When capacity or spectrum-efficiency is the performance metric under consideration, having more receive antennas always help improve the system performance. However, as energy-efficiency depends also on the power consumption of the receiver, it does not necessarily increase when more receive antennas are employed. If the receive parameters are carefully chosen, e.g., ADC resolutions are adapted based on the methods proposed above, increment in energy-efficiency might still be achieved via adding receive antennas, i.e., by exploiting the diversity while controlling the power dissipation appropriately. Since antenna number is also an integer, the gain in energy-efficiency with respect to increased number of antennas is hard to obtain analytically, for which reason we leave the question to be studied quantitatively in the next section where simulation results are presented.

IV. SIMULATION RESULTS

We aim to compare the performance of the methods proposed in the last section, and study the variation in receiver energy-efficiency with respect to $\gamma$ and $M$, with numerical simulations. Fixed parameters such as $c_0$, $c_1$ and those used in the PSO method have been given previously. The number of transmit antennas $N$ is fixed to 1 here for a first study, which may have an influence on the optimal solutions of the problem yet the comparative results we obtain can be expected representative. For the offline methods, $10^4$ independently generated channel realizations are used to obtain the optimal ADC resolution and the optimal number of active antennas. For real simulations, each method is tested with another $10^4$ independent channel realizations, and the average performance for each parameter setting is illustrated in this section.

The first thing we find, which is in fact not shown in any of the figures, is that the offline and online versions of the uniform ADC resolution adaptation scheme without AS perform nearly the same. The online version certainly achieves better energy-efficiency than the offline version, but the difference is extremely small and in most cases, i.e., for various values of $\gamma$ and $M$, they achieve exactly the same energy-efficiency. This is due to the steadiness in the optimal uniform ADC resolution given different channel conditions. Consequently, we do not distinguish the two versions of this method in the following, and refer to it simply as NAS (no antenna selection). Accordingly, the uniform ADC resolution adaptation scheme with AS is called SAS (static antenna selection) for its offline version, and AAS (adaptive antenna selection) for its online version, respectively.

The performance of NAS, SAS, and AAS methods are compared in Fig. 2 with two fixed antenna numbers $M = 4$ and $M = 10$, for a range of average SNR values. For such small $M$, we are able to find the optimal solution to (4) with exhaustive search, which allows us to quantitatively see the suboptimality of each method in Fig. 2(a) and Fig. 2(b). As can be expected, AAS outperforms SAS, which in turn outperforms NAS. With small values of $\gamma$, the three methods all perform very well, achieving around 95% or even higher percentage of the optimal energy-efficiency. As $\gamma$ grows, the performance...
We study the performance of the three methods with increasing number of antennas in Fig. 3, where the cases that $\gamma = -30$ dB, $\gamma = 0$ dB, and $\gamma = 30$ dB are tested. From the figures in the first row, we see that the energy-efficiency achieved with the NAS method does not always increase, leading to an optimal number of antennas. The SAS and the AAS methods, on the other hand, always benefit from having more receive antennas due to the selection process, yet the improvement in energy-efficiency becomes much smaller with large $M$. This relation is shown more clearly in Fig. 3(i), where the number of antennas from which the increment in energy-efficiency falls below 1% is drawn for SAS and AAS methods. We learn from the figures that, with medium-to-large values of $\gamma$, the receiver selects a very small subset of antennas, the signals from which are to be quantized with relatively high resolution such as 4 or 5 bits per sample. In this case, equipping the receiver with less than 20 or even less than 10 antennas is enough to achieve near-optimal performance. When $\gamma$ is small, the signals from more antennas are required to be processed with lower ADC resolution, and we need much more receive antennas to achieve near-optimal performance.

V. CONCLUSION

We propose and compare several adaptation schemes of the ADC resolutions for a multi-antenna receiver with perfect CSI, aiming at the maximization of energy-efficiency of the system measured in number of information bits per consumed Joule of energy. Based on the numerical simulations, we find that the combination of antenna selection and the offline optimization of a uniform ADC resolution for the active antennas achieves good balance between performance and computational complexity. Moreover, the effect of deploying a large number of receive antennas is also discussed, which depends very much on the adaptation scheme that is employed.

A few issues need to be addressed regarding the work itself and its possible extensions. First of all, we have assumed that the receiver has perfect knowledge about the instantaneous CSI. Although in the end, we learn that the SAS method seems to be satisfactory both in performance and in complexity, which has less dependency on the CSI compared to some other schemes we proposed, the problem of channel estimation with inaccurately accessed pilot symbols can not be circumvented. The impact of quantization of receive pilots which in turn influences the quality of the channel estimation should be investigated in future work, and the ADC resolution applied for the pilot symbols should be chosen appropriately. Secondly, the energy consumed for the antenna selection process and the associated signal processing is not included in our power consumption model, meaning that the results we obtain could be rather optimistic for a real system. Thirdly, when the performance of a receiver with a large number of antennas is evaluated, the correlation between the antennas might be necessary to be taken into account, depending on the physical deployment of the antennas e.g. shape and spacing of the array. Such consideration goes beyond the scope of this work, but is certainly an interesting direction for further investigation.

REFERENCES

Figure 3. Comparison of NAS, SAS, and AAS methods with increasing number of antennas


