On Multicasting Prioritized Messages

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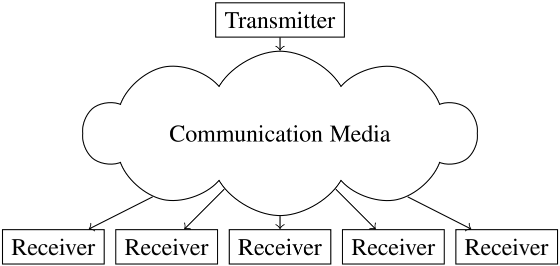
Joint work with Vinod Prabhakaran, Suhas Diggavi, Christina Fragouli

March 5, 2014

Problem setup



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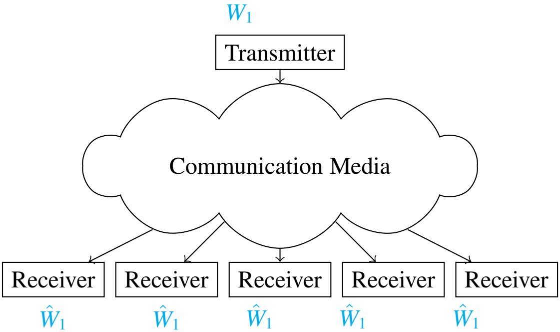
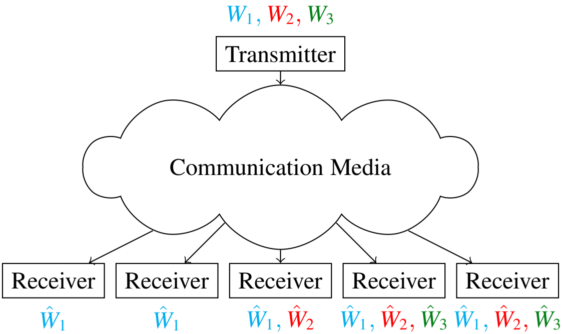


- Ahlswede, Li, Cai and Yeung (2000)
- Avestimehr, Diggavi and Tse (2007)

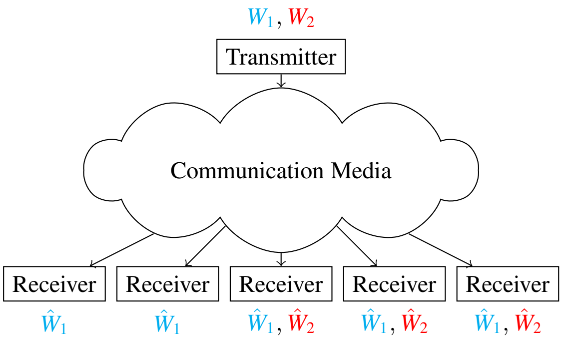
Problem setup: prioritized messages



Video Streaming over Heterogeneous Networks
Scalable Video Coding (SVC standard)

- Korner and Marton (1977); Nair and El-Gamal (2008)
- Ngai and Yeung (2004), Erez and Feder (2003), and Ramamoorthy and Wessel (2009)

Problem setup: objective



- A high priority (common) message of rate R_1 and a low priority (private) message of rate R_2
- public receivers and private receivers
- What are the ultimate communication rates?
- Optimal or Near optimal communication schemes?

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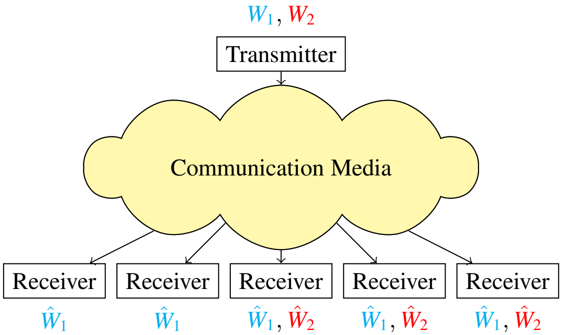

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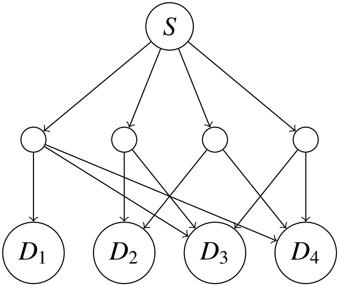
Outline

Outline

A combinatorial network model: combination networks



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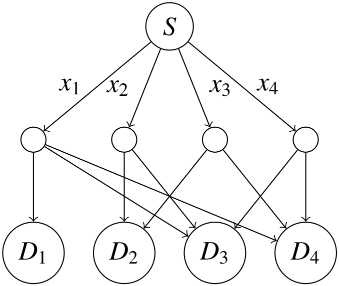

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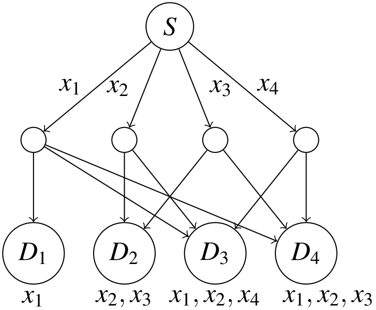

- A simple combinatorial model to capture the interaction of the signals
- Connections to linear deterministic broadcast channels

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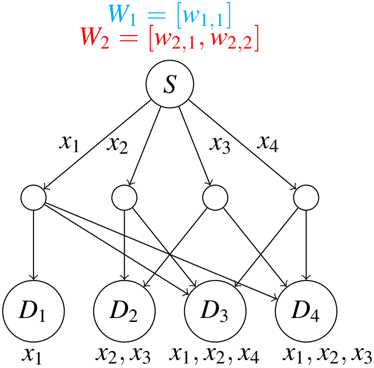

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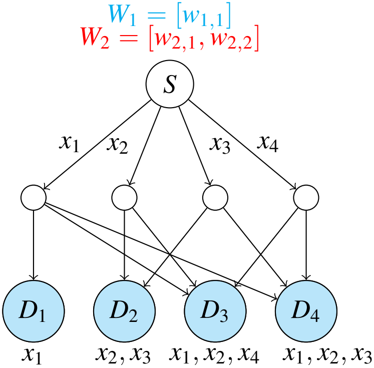

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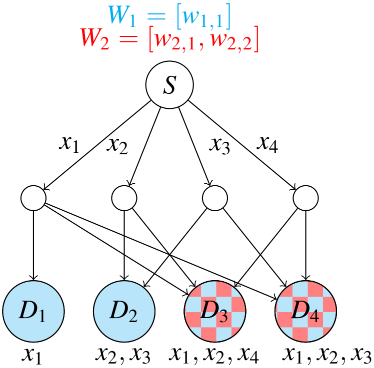

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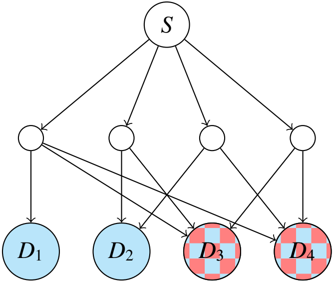
Notation



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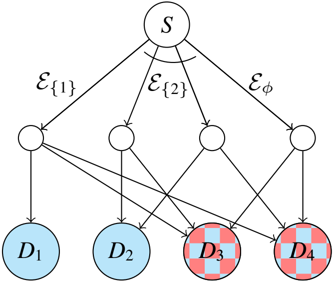

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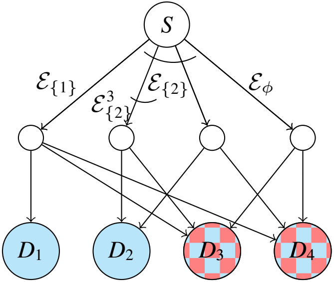

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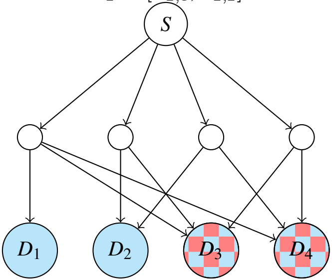
- $\mathcal{E}^p_S$, $S \subseteq \{1,2\}, p \in \{3,4\}$: in $\mathcal{E}_S$ but also connected to private receiver p

Outline

The challenge

$$W_1 = [w_{1,1}]$$

$$W_2 = [w_{2,1}, w_{2,2}]$$



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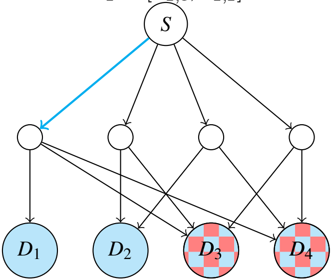

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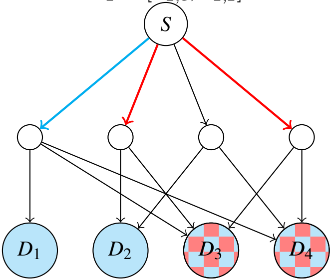

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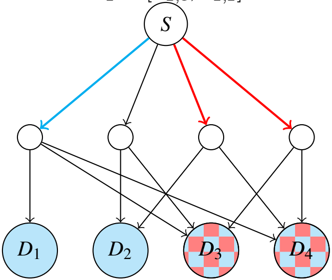

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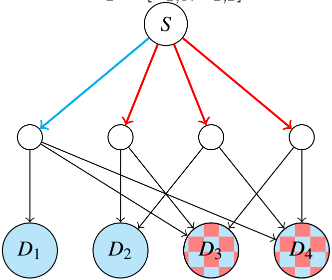

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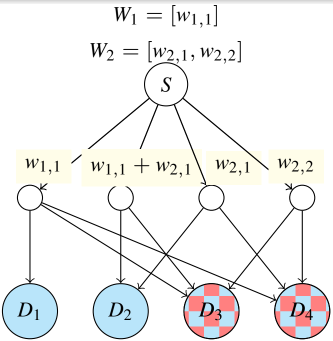

Mixing of the common and private messages is necessary; but in a controlled manner

One has to reveal (partial) information about the private message to public receivers!

Main Results

1. An achievable rate-region using a standard **linear superposition encoding** schemes.

capacity region for two public and any number of private receivers.

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capacity region for two public and any number of private receivers.

2. The rate-region is enlarged by employing a proper **pre-encoding** at the transmitter.

capacity region for three (or fewer) public and any number of private receivers.

3. A **block Markov encoding** scheme may improve both previous schemes.

capacity region for three (or fewer) public and any number of private receivers.

Outline

Rate splitting and linear superposition coding

- let $W = [w_{1,1} \dots w_{1,R_1} w_{2,1} \dots w_{2,R_2}]^T$
- let $X = \mathbf{A} \cdot W$
- reveal information about the private messages to public receivers through a zero-structured encoding matrix
- a linear superposition coding scheme

$$
\mathbf{A} = \begin{array}{c|c|c|c|c|l}
\overset{R_1}{\longleftrightarrow} & \overset{\alpha_{\{1,2\}}}{\leftrightarrow} & \overset{\alpha_{\{1\}}}{\leftrightarrow} & \overset{\alpha_{\{2\}}}{\leftrightarrow} & \overset{\alpha_{\phi}}{\leftrightarrow} & \\
\hline
 & & 0 & 0 & 0 & \updownarrow |\mathcal{E}_{\{1,2\}}| \\
\hline
 & & & 0 & 0 & \updownarrow |\mathcal{E}_{\{1\}}| \\
\hline
 & & 0 & & 0 & \updownarrow |\mathcal{E}_{\{2\}}| \\
\hline
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$$

$$R_2 = \alpha_{\{1,2\}} + \alpha_{\{2\}} + \alpha_{\{1\}} + \alpha_{\phi}$$

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$$R_2 = \alpha_{\{1,2\}} + \alpha_{\{2\}} + \alpha_{\{1\}} + \alpha_{\phi}$$

- choose appropriate parameters, and complete the matrix

Rate-region I

A rate pair (R_1, R_2) is achievable if there exist variables $\alpha_\phi, \alpha_{\{1\}}, \alpha_{\{2\}}, \alpha_{\{1,2\}}$, s.t.

Structural constraints:

$$\alpha_S \geq 0 \quad \forall S \subseteq \{1,2\}$$
$$R_2 = \sum \alpha_S$$

Decoding constraints at public receiver $i \in \{1,2\}$:

$$R_1 + \sum_{S \ni i} \alpha_S \leq \sum_{S \ni i} |\mathcal{E}_S|$$

Decoding constraints at private receiver p:

$$R_2 \leq \sum_{S \in \mathcal{T}} \alpha_S + \sum_{S \in \mathcal{T}^c} |\mathcal{E}_S^p| \quad \forall \mathcal{T} \subseteq 2^{\{1,2\}} \text{ superset saturated}$$
$$R_1 + R_2 \leq \sum_{S \subseteq \{1,2\}} |\mathcal{E}_S^p|$$

The converse holds for two public and any number of private receivers, characterizing the capacity region.

Two public and any number of private receivers

Theorem

Rate (R_1, R_2) is achievable <u>*if and only if*</u>

$$R_1 \leq \min \left(|\mathcal{E}_{\{1\}}| + |\mathcal{E}_{\{1,2\}}|, |\mathcal{E}_{\{2\}}| + |\mathcal{E}_{\{1,2\}}| \right)$$

$$R_1 + R_2 \leq \min_{p \in I_2} \left\{ |\mathcal{E}^p_\phi| + |\mathcal{E}^p_{\{1\}}| + |\mathcal{E}^p_{\{2\}}| + |\mathcal{E}^p_{\{1,2\}}| \right\}$$

$$2R_1 + R_2 \leq \min_{p \in I_2} \left\{ |\mathcal{E}_{\{1\}}| + 2|\mathcal{E}_{\{1,2\}}| + |\mathcal{E}_{\{2\}}| + |\mathcal{E}^p_\phi| \right\}$$

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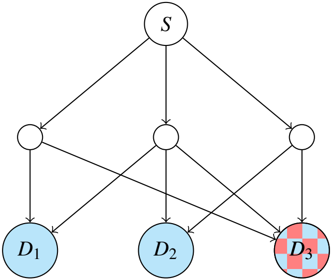
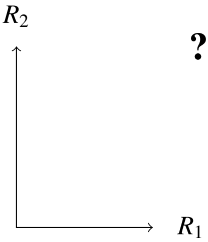


$$R_1 \le 2$$
$$R_1 + R_2 \le 3$$
$$2R_1 + R_2 \le 4$$



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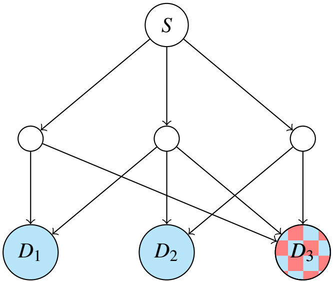

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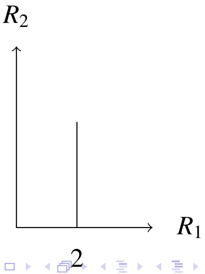

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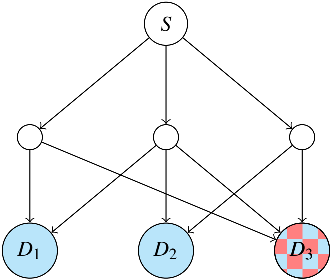

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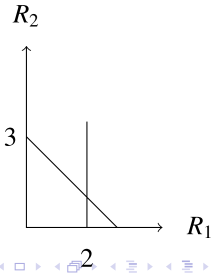

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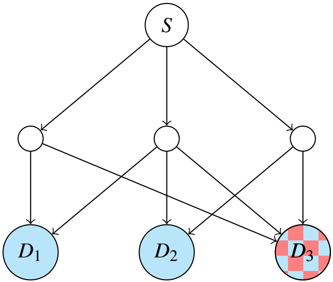

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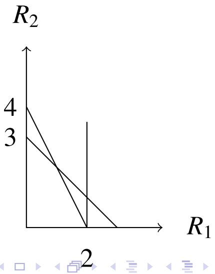


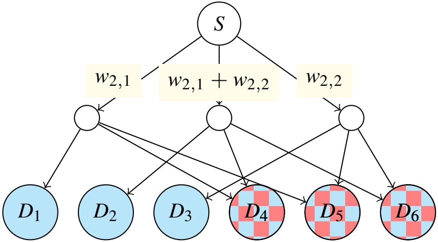
Outline

When there are more than two public receivers...

- $(0, 2)$ is not achievable using the previous scheme!

$W_1 = []$

$W_2 = [w_{2,1}, w_{2,2}]$



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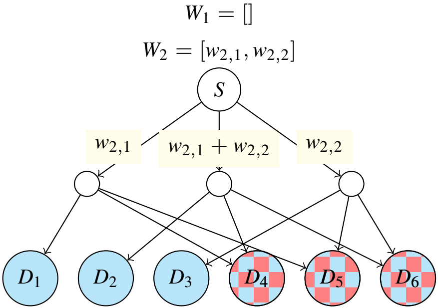
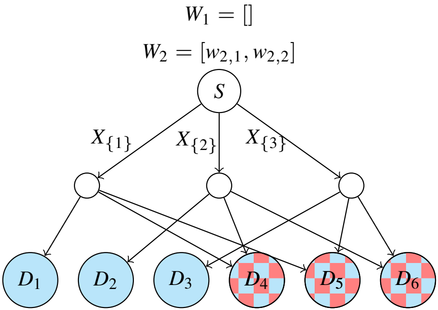


The private information revealed to different subsets of public receivers need not be independent

Appropriate pre-encoding



- pre-encode $W_2 = [w_{2,1}, w_{2,2}]^T$ into $W_2' = [w_{2,1}', w_{2,2}', w_{2,3}']$
- now use an structured encoding matrix

$$\begin{bmatrix} X_{\{1\}} \\ X_{\{2\}} \\ X_{\{3\}} \end{bmatrix} = \left[\begin{array}{c|c|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{bmatrix} w_{2,1}' \\ w_{2,2}' \\ w_{2,3}' \end{bmatrix}.$$

Rate-region II

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Structural constraints:

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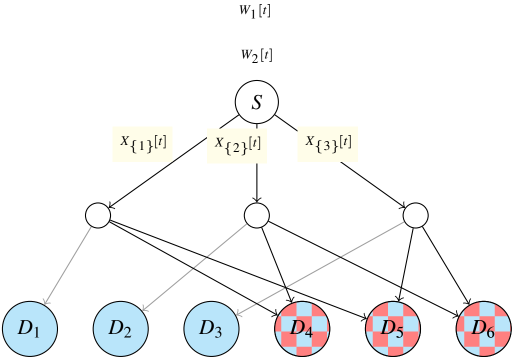
Beyond pre-encoding: dependency through time



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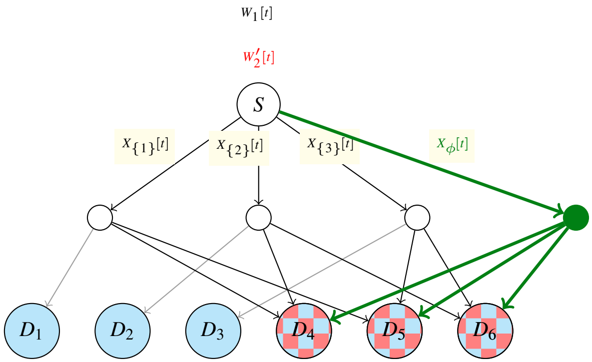

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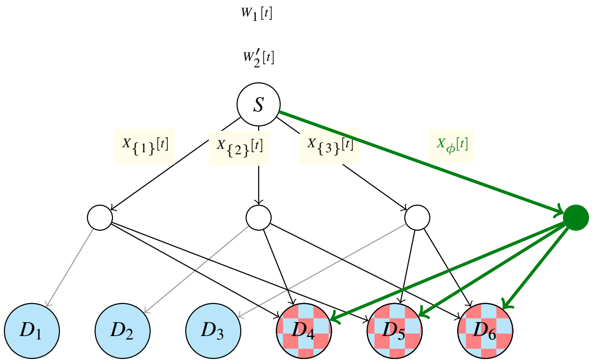

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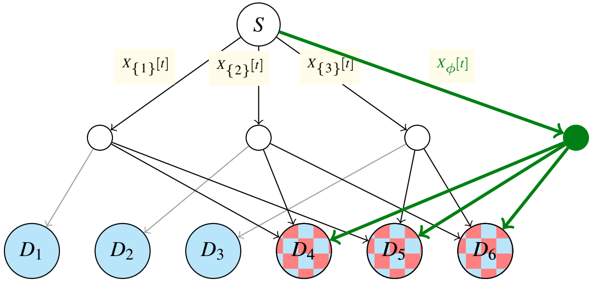
- $(R_1 = 0, R_2' = 3)$ is achievable using the linear superposition encoding scheme, over the extended channel
- use it to achieve rate pair $(0, 2)$ over the original network: block Markov encoding and backwards decoding

Beyond pre-encoding: dependency through time

$W_1[t],\ W_1[t+1] = []$

$W_2'[t],\ W_2'[t+1] = [w_{2,1}'[t+1], w_{2,2}'[t+1], w_{2,3}'[t+1]]$



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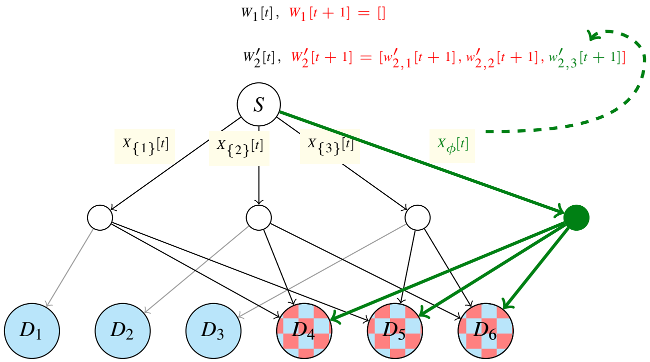

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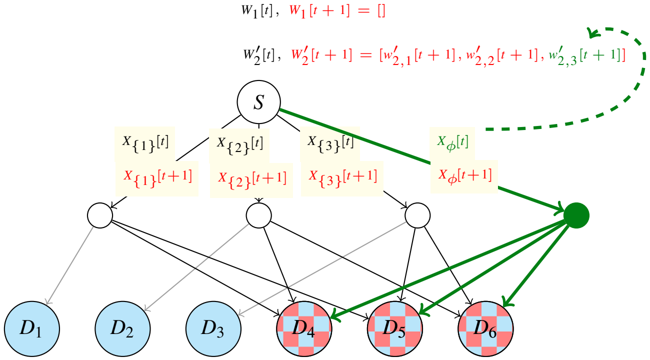
Beyond pre-encoding: dependency through time



- how to achieve rate pair $(R_1 = 0, R_2 = 2)$?
- $(R_1 = 0, R'_2 = 3)$ is achievable using the linear superposition encoding scheme, over the extended channel
- use it to achieve rate pair $(0, 2)$ over the original network: block Markov encoding and backwards decoding

Rate-region III

A rate pair (R_1, R_2) is achievable if there exist $\alpha_\phi, \alpha_{\{1\}}, \alpha_{\{2\}}, \alpha_{\{1,2\}}$, s.t.

$$\alpha_{\{1,2\}} \geq 0, \quad \alpha_{\{1\}} + \alpha_{\{1,2\}} \geq 0, \quad \alpha_{\{2\}} + \alpha_{\{1,2\}} \geq 0$$

$$\alpha_{\{1\}} + \alpha_{\{2\}} + \alpha_{\{1,2\}} \geq 0$$

$$\alpha_\phi + \alpha_{\{1\}} + \alpha_{\{2\}} + \alpha_{\{1,2\}} \geq 0$$

$$R_2 = \sum \alpha_S$$

Decoding constraints at public receiver $i \in \{1,2\}$:

$$\sum_{S \ni i} \alpha_S \leq \sum_{S \in \mathcal{T}} \alpha_S + \sum_{S \in \mathcal{T}^c,\, S \ni i} |\mathcal{E}_S| \quad \forall \mathcal{T} \subseteq \{\{i\}\star\} \text{ superset saturated}$$

$$R_1 + \sum_{S \ni i} \alpha_S \leq \sum_{S \ni i} |\mathcal{E}_S|$$

Decoding constraints at private receiver p:

$$R_2 \leq \sum_{S \in \mathcal{T}} \alpha_S + \sum_{S \in \mathcal{T}^c} |\mathcal{E}_S^p| \quad \forall \mathcal{T} \subseteq 2^{\{1,2\}} \text{superset saturated}$$

$$R_1 + R_2 \leq \sum_{S \subseteq \{1,2\}} |\mathcal{E}_S^p|$$

The converse holds for three (or fewer) public and any number of private receivers, characterizing the capacity region.

Outline

Optimality results

Discussions delegated to the end of the presentation, if of your interest!

Outline

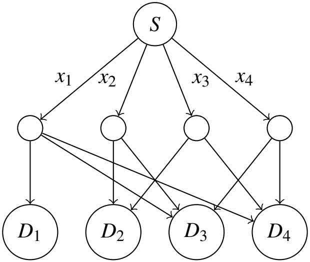
Connections with linear deterministic broadcast channels

$$Y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}$$

$$Y_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix}$$

$$Y_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix}$$



Connections with linear deterministic broadcast channels

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$$Y_2 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_3 \end{bmatrix}$$

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$$Y_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + x_2 \\ x_2 + 3x_3 + 2x_4 \end{bmatrix}$$

Connections with linear deterministic broadcast channels

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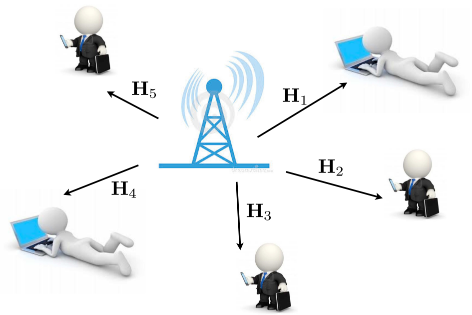

Capacity result

The capacity region of a linear deterministic broadcast channel with two public receivers and any number of private receivers is given by

$$\begin{aligned} R_1 &\leq \min_{i\in I} r_{\{i\}} \\ R_1+R_2 &\leq \min_{i\in I_2} r_{\{i\}} \\ 2R_1+R_2 &\leq \min_{i\in I_2}\{r_{\{1\}}+r_{\{2\}}+r_{\{1,2,i\}}-r_{\{1,2\}}\}, \end{aligned}$$

where the size of $\mathbb{F}$ is larger than K. The rates given above are expressed in $\log_{|\mathbb{F}|}(\cdot)$.

- $r_{\{i\}} \triangleq \text{rank}(\mathbf{H}_i)$
- $r_{\{i_1,\cdots,i_{|\mathcal{S}|}\}} \triangleq \text{rank}\begin{bmatrix}\mathbf{H}_{i_1}\\ \vdots \\ \mathbf{H}_{i_{|\mathcal{S}|}}\end{bmatrix}$

Example

$$\mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

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$$r_{12} = 3$$

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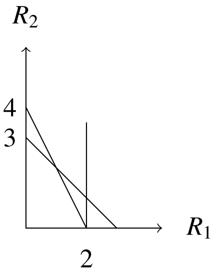

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- Combination networks turn out to be a rich class of networks and a rich class of linear deterministic broadcast channels
- Discussed three encoding schemes, and their regimes of optimality
- Generalizing these schemes to linear deterministic broadcast channels seems very promising