Distance based Dynamical System Modulation for Reactive Collision Avoidance

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Abstract—An algorithm which allows the robot to avoid obstacles and to reach the assigned goal is proposed. For this purpose, a dynamical system (DS) modulation matrix is calculated using the distance from the obstacles, without the need of analytical representation of the obstacles. This matrix modulates a generic first order DS, used to generate the desired path. In this way we guarantee the obstacles avoidance and the reaching of the goal. The effectiveness of the proposed approach is validated with an experiment on a 7 DOF KUKA light weight arm.

I. INTRODUCTION

In human-robot interaction scenarios the robot is required to adapt quickly to various situations and to eventual external disturbances ensuring the operator safety. When unknown obstacles and humans enter in the scene, the robot has to modify its motion quickly to avoid collisions. After the avoidance, it is desirable that the robot fulfills the assigned task as long as possible.

A widely used approach to generate collision-free paths in real-time is based on an artificial potential field [1]. The idea is to assign an attractive force to the goal and to shape the obstacles as repulsive forces, so as to reach the target avoiding obstacles. One drawback of the potential field approach is that the motion can stop in a local minimum even if a collision-free path to the goal exists.

Some authors propose to skip the local minima by modifying the dynamics of a particular DS. In [2] a repulsive force is added to a Dynamic Movement Primitive to avoid point-mass obstacles, saving the global stability in static scenarios. In [3] a combination of potential fields and circular fields is applied to a second order system to generate a smooth collision-free path. The mentioned approaches work only with a specific DS, reducing the possibility of encoding many different tasks.

A technique to modulate a generic first order DS is proposed in [4]. Given the analytical representation of the obstacles surface, a modulation matrix, which locally deforms the original system, is computed. This approach guarantees the obstacles impenetrability without modifying the equilibria of the modulated system.

Our proposed approach is based on the DS modulation for obstacle avoidance in [4]. In contrast to the original algorithm, our contributions are twofold. First, an analytical representation of the obstacles is no longer prerequisite. Instead, we use the Euclidean distance of a point from the point clouds of the objects. This gives us the possibility to work in real-time. Second, impenetrability of concave obstacles as well as convex obstacles are achieved in the proposed approach. The proposed modulation guarantees the impenetrability of the obstacles without changing the modulated DS equilibrium points. The effectiveness of our approach is proved with an experiment on a KUKA LWR IV+.

II. DISTANCE BASED MODULATION

A. Modulation Matrix

The modulation algorithm is based on the assumption that the path to follow is generated by a first order DS. By modulating the DS with a suitable matrix \( M(p) \) by

\[ \dot{p}(t) = M(p) f(p, t) \]  (1)

one can avoid obstacles and keep the stability properties of the DS\(^1\).

Assume that only one \( d \)-dimensional obstacle is present on the scene, and that the normal vector to the obstacle surface \( \tilde{n}(\bar{p}) = [\tilde{n}_1(\bar{p}) \ldots \tilde{n}_d(\bar{p})]^T \) is defined \( \forall \bar{p} \). Under these assumptions, a tangential hyperplane can be defined at each point on the surface. The matrix \( V(\bar{p}) = [\tilde{n}(\bar{p}) \tilde{v}_1(\bar{p}) \ldots \tilde{v}_{d-1}(\bar{p})] \), where \( [\tilde{v}_1(\bar{p}) \ldots \tilde{v}_{d-1}(\bar{p})] \) is a base of the tangential hyperplane, is an orthonormal basis of the \( d \)-dimensional space.

Now, let us call \( \Phi(\bar{p}) = \alpha \) the distance between the robot and the surface of the obstacle, where the positive scalar \( \alpha \in \mathbb{R} \) is a safety margin, and \( \bar{p} \) the point of minimum distance. We can define the diagonal matrix \( E(\bar{p}) = [\lambda_1(\bar{p}), \ldots, \lambda_n(\bar{p})] \), where

\[ \left\{ \begin{array}{l} \lambda_1 = 1 - \frac{1}{(\Phi(\bar{p}) + 1)^2} \quad \bar{p}^T \bar{p} < 0 \text{ or } m = 1 \\ 1 \quad \bar{p}^T \bar{p} \geq 0 \text{ and } m = 0 \end{array} \right. \]  (2)

In (2), the positive scalar \( \rho \) is the reactivity parameter, used to change the magnitude of the modulation, and the boolean variable \( m = 0, 1 \) is used to interrupt the modulation (\( m = 1 \)) after passing the obstacle (\( \bar{p}^T \bar{p} < 0 \)).

The modulation matrix can be calculated as

\[ M(\bar{p}) = V(\bar{p}_m) E(\bar{p}) V(\bar{p}_m)^{-1} \]  (3)

\(^1\)In the following pages, \( p \) denotes the generic point, \( \bar{p} \) a point on the object surface, and \( \bar{p} = p - \bar{p} \) a generic point with respect to \( \bar{p} \).
By modulating (1) with the matrix (3), it is possible to prove that a trajectory $p(t)$ can never penetrate the convex obstacle. Moreover, the modulation does not affect the equilibrium points of the modulated DS [4].

The described distance-based modulation can be directly applied in a same way, no matter how many obstacles exist in the work space. We simply calculate the distance from the closest object and the normal at the point of minimum distance. So, the number of objects does not affect the performance of our algorithm.

**B. Impenetrability of concave obstacles**

The tangential hyperplane of a concave object can intersect the surface, as shown in Fig. 1. Therefore, the point $p = p + M(p)\delta t$, calculated by integrating (1), may be located within the object. To prove the impenetrability, consider the set $I = \{p_t|\Phi(p_t) \geq 0, \forall p_t \in R^n\}$ that is the set of all the points external or belonging to the surface. A subset of $I$ is defined by

$$J_r(p) = \{p_{J_r}|r \geq max(p - \bar{p}), \forall p_{J_r} \in I \subset R^n\} \quad (4)$$

where $p - \bar{p} = M(p)\delta t$. By construction, $J_r(p)$ is an intersection-free neighbourhood of $p$.

The impenetrability of a concave object is ensured, if a neighbourhood like (4) exists for each point on the boundary of the obstacle.

**III. EXPERIMENTAL RESULTS**

In this experiment the robot has to reach two goal positions, one of which is located at the center of a box of size $40\text{cm} \times 35\text{cm} \times 20\text{cm}$. One side of the box is open (Fig. 2(a)).

Starting from a point outside the box, the robot is guided to the first goal $g_1$ inside it by the system $\dot{p}(t) = k(g_1 - p(t))$, $k = 2$. Then, starting from $g_1$ the robot comes out of the box and reaches the second goal $g_2$ of $\dot{p}(t) = k(g_2 - p(t))$, $k = 2$. A collision-free path is found by modulating this switching linear DS, as represented in Fig. 2(b). These results are obtained with $\beta = 10$, $\alpha = 0.08$, $\rho = 0.3$ and $m = 1$.

Note that, by using the algorithm in [4], the task cannot be accomplished. In this case, the box should be approximated using a bounding box, thereby preventing the robot to enter. Even if the box is approximated considering each face as a separate obstacle, there may be division by zero and the algorithm crashes.

**IV. CONCLUSIONS**

In this paper we presented a real-time collision avoidance technique that guarantees the convergence to the goal. The novelty of our approach lies in the calculation of the modulation matrix directly from a point cloud. This makes the resulting algorithm faster than other approaches which requires an analytical representation of the obstacle surface. It is shown that this algorithm works with convex and concave objects without being stuck into local minima. We implemented and evaluated the distance based DS modulation approach on a 7 DOF KUKA LWR with comparison of the original DS modulation approach.

**REFERENCES**


