SIMULATIVE THROUGHPUT CALCULATION FOR STORAGE PLANNING

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1. ABSTRACT

Throughput calculation of an automated storage and retrieval system is an important part of the storage planning. It depends on storage configuration, storage strategies and dimensioning of the system. A unified approach for the calculation is developed by breaking down command cycles in its separate cycle time components and synthesizing them to the various specific command cycles. The time components represent typical movements and load handling steps of a storage and retrieval machine. The whole range of different types is modeled and calculated with Monte Carlo simulation, in this way the components can easily be described and to customized. The unified model for throughput calculation is merged with other models for the calculation of storage capacity, building geometry and investment in a database-aided software tool. It allows a mathematical optimization for the dimensioning of design variants and therefore the easy comparison of the latter.

Keywords: AS/RS throughput calculation, cycle time components, travel time, Monte Carlo simulation

2. INTRODUCTION

Automated storage and retrieval systems (AS/RS) are used for automatically storing and retrieving loads from specific storage shelves. The systems usually consist of several storage aisles, which are composed of two storage racks, input/output points (I/O points) and a storage and retrieval machine (S/R machine). The load handling device of a S/R machine is able to travel simultaneously in the horizontal and vertical direction and pickup and deposit loads (see Figure 1).

The objective in planning an AS/RS is to identify the most economical design. Throughput calculation is an important part of it. It describes the number of inputs and/or removals in a given time. Basis for determining the expected throughput is the calculation of the cycle times for the S/R machines. This time is significantly dependent of the storage configuration and strategy as well as the dimensioning:

- The configuration describes the physical structure of the storage (single-deep or double-deep rack, number of load handling devices on a single S/R machine, etc.)
- The strategies describe the operational sequence of a command cycle (storage assignment rule, storage retrieval rule, etc.)
- The dimensioning determines the size of the storage system and the aisles

A literature review on throughput calculations in AS/RS give Sarker and Babu (1995), Johnson and Brandeau (1996) and Roodbergen (2009).

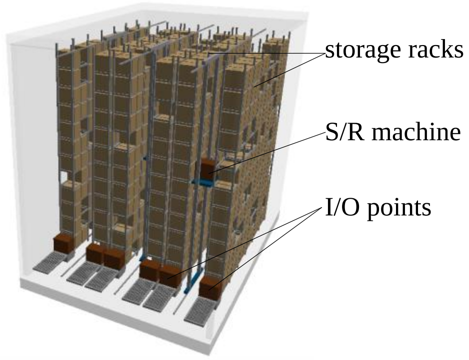

Figure 1: Automated Storage and Retrieval System (AS/RS)

The planning of storage systems is still largely manual today. Planning steps are performed sequentially and in an iterative manner. Since this method is very time-consuming, only a few design variants are considered and compared. There is no exact mathematical model for throughput calculation which is valid for a variety of design options. Hence, there is a lack of a comprehensive computer aided planning tools. The objectives of this work are:

1. Description of the developed unified approach for cycle time and throughput calculation respectively
2. Analysis and modeling of the necessary cycle time components by simulation
3. Validation and application of the approach
4. Outlook on the integration of this calculation model in a developed planning tool named LSP (Atz 2012) and the resulting opportunities

3. STORAGE AND RETRIEVAL SYSTEMS

The basis for determining the throughput of an AS/RS is the calculation of the mean cycle time of an S/R machine. A single command cycle depends on the configuration, strategies and dimensioning of the storage system.

3.1. Storage configuration

In the past decades, a variety of storage configurations have been developed to meet the different needs of modern logistics. The storage configuration describes the physical structure of the storage. The specifications of common configurations are shown in Table 1.

Table 1: Overview of important Configuration Parameters and Specifications

<table>
<tr><th>Configuration parameter</th><th colspan="6">Specifications</th></tr>
<tr><td>Depth of the rack</td><td colspan="3">Single-deep (sd)</td><td colspan="3">Double-deep (dd)</td></tr>
<tr><td>Number of load handling devices on a S/R machine</td><td colspan="2">1</td><td colspan="2">2</td><td colspan="2">3</td></tr>
<tr><td>Width of the load handling device</td><td colspan="3">Single-width</td><td colspan="3">Double-width</td></tr>
<tr><td>S/R machine type</td><td colspan="3">Fixed-aisle</td><td colspan="3">Multi-aisles</td></tr>
<tr><td>Position of the I/O point</td><td colspan="3">In the corner of the rack</td><td colspan="3">Staggered in x- or y-direction</td></tr>
</table>

A simple and often realized storage configuration is composed of aisles with single-deep racks, served by fixed-aisle S/R machines with one load handling device. Normally the load handling device is single-width and so the S/R machine is able to handle only one load at a time. Depending on the capacity and throughput requirements, double-deep storage forms with more storage shelves related to the storage volume, multiple load handling devices for a higher throughput or multi-aisles S/R machines at lower throughput requirements, are used.

3.2. Storage strategies

Each storage configuration can be operated with multiple storage strategies. The strategies determine the operational sequence of a command cycle. These strategies determine, for example, the selection of a storage shelf and the actual behavior of an S/R machine accessing load units and are usually matched to the storage configuration. The strategies can be structured as follows (Table 2):

Table 2: Overview of important Storage Strategies and Specifications

<table>
<tr><th>Storage strategy</th><th colspan="6">Specifications</th></tr>
<tr><td>Type of command cycle</td><td colspan="2">Single storage cycle</td><td colspan="2">Single retrieval cycle</td><td colspan="2">Combined cycle</td></tr>
<tr><td>Selection of storage shelf</td><td colspan="2">Random selection</td><td colspan="2">Near the retrieval shelf</td><td colspan="2">Zoning</td></tr>
<tr><td>Selection of relocation shelf</td><td colspan="2">Random selection</td><td colspan="2">Near the retrieval shelf</td><td colspan="2">Zoning</td></tr>
<tr><td>Selection of retrieval shelf</td><td colspan="3">Strict FIFO</td><td colspan="3">Alleviated FIFO</td></tr>
<tr><td>I/O point strategy</td><td colspan="3">Separate pick-up and deposit</td><td colspan="3">Parallel pick-up and deposit</td></tr>
<tr><td>Sequence strategy</td><td colspan="3">No / Random sequence</td><td colspan="3">Travel path optimization</td></tr>
</table>

Practice-relevant storage assignment rules affect strategies for the selection of a shelf for the storage or the relocation of a load. Relocation of loads becomes necessary in a double-deep storage rack when access to a covered load unit is not possible. In this case, the front load unit has to be relocated to another storage shelf first. The strategies for the selection of the storage or relocation shelf are similar. The random selection of a free shelf or the selection of a shelf near the (subsequently visited) retrieval shelf to save routes is very common.

In practice, the selection of the retrieval shelf usually depends on a strict FIFO rule. This prevents aging of the stock. In some industries, this principle is alleviated and articles of a common batch are treated equally in case of retrieval. In this case and also in case of multiple handling devices on an S/R machine, sequence strategies become important and travel path optimization can be applied.

3.3. Dimensioning

When the storage configuration and the storage strategies are fixed, the geometrical extension of the storage system can be specified. The parameters thereof are the number of rows and columns of the rack, the number of aisles, the number of S/R machines and the technical specifications of the applicable storage components (e.g. S/R machine, I/O points, etc.). An important indicator for the dimensioning is the parameter *b*, called shape factor. It describes the relationship between the kinematic characteristics of the S/R machine and the dimensions of the rack (length l_{rack} and height h_{rack}). When v_x and v_y are the maximum speeds of the S/R machine in a horizontal or a vertical direction, factor b is defined as:

$$b = \frac{h_{rack}}{l_{rack}} \cdot \frac{v_x}{v_y} \qquad (1)$$

4. THROUGHPUT CALCULATION BY SIMULATION

4.1. Unified approach for the throughput calculation

Atz (2012) describes a unified approach to calculate the throughput of several design variants with different storage configurations and storage strategies. A command cycle can be broken down into its separate cycle time components which represent typical movements and load handling steps of an S/R machine.

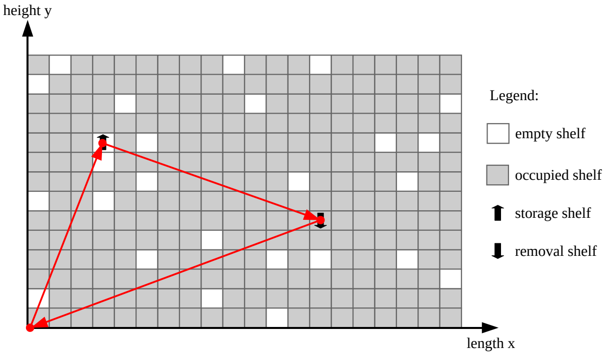

Figure 2: Travel Time Components of a Combined Command Cycle

Figure 2 illustrates a combined command cycle. In this special case it is also called a dual command cycle because the S/R machine first stores one load on an empty storage shelf and consecutively retrieves one load from another storage shelf. The pickup and deposit times (P/D times) are constant and can be deterministically calculated. On the other hand, the travel times of the S/R machine are distributed stochastically. The time for a single travel depends on the distances between the start and end position. Depending on the configuration and the strategies, the positions are distributed in different ways across the surface of the rack. For the throughput calculation it is necessary to calculate the mean time. In the example shown there are two different kinds of cycle time components:

- I/O travel time $t_{I/O,P}$: This is the average travel time from an I/O point (which is usually at the edge of the rack) to any storage shelf in the rack or vice versa.
- travel between time $t_{P,P}$: This is the average travel time from any storage shelf in the rack to another storage shelf in the rack

Different strategies and configurations require a variety of different time components to describe a whole command cycle. In AS/RS with double-deep racks certain travels (e.g. relocation of loads) are not executed every command cycle but only with certain probabilities. These probabilities depend on the percentage of filling and the load allocation in the racks. After calculating the time components they are weighted by their probabilities and synthesized to the command cycles. The command cycles can start from different I/O points and be single command cycles or combined command cycles. The decisive factor for the command cycles considered are the relevant planning requirements.

4.2. Calculation of cycle time components

There are analytical (Gudehus 1972, Bozer and White 1984) and numerical models available (Jodin 2010a) for the calculation of each cycle time component. To calculate the expected value in analytical models, each possible travel time t_T is multiplied with its probability (Formula 2). The challenge of this method is to derive the density function $f(t_T)$ analytically:

$$\mu = \overline{t_{ttc}} = \int_0^\infty t_T \cdot f\left(t_T\right) \cdot dt_T \qquad (2)$$

The Monte Carlo simulation (MC simulation) is a numerical stochastic method. It is based on random experiments which are repeated many times. The results are used to calculate the arithmetical mean of the experiment.

The MC simulation method is suitable for the calculation of a large amount of different cycle time components. The modeling is simple and the cycle time components are able to represent various storage strategies. In comparison, the analytical modeling is often too complex or fails due to too restrictive assumptions. Also the results are less accurate compared to the simulation method. An advantage is the reduced computation time. However, the two methods are interchangeable in the described unified calculation approach.

The first step in the simulation is the initialization of the two racks representing an aisle. They are divided into rows and columns. With a random based algorithm the storage racks are virtually filled with loads taking into account assignment and retrieval rules.

After this step, the necessary parameters for the subsequent synthesis of the time components can be determined from the resulting load allocation:

- ratio of empty shelves P_e
- ratio of single-occupied shelves P_s
- ratio of double-occupied shelves (in case of double-deep racks) P_d

In a second step, the starting point $P_S = (x_S, y_S)$ and the end point $P_E = (x_E, y_E)$ are determined. These are the coordinates of storage shelves or the I/O point. If there are several equiprobable options (storage shelves) to choose from based on the restrictions and requirements of storage configuration and storage strategies, then selection will again be random.

The travel time between the two points is then calculated. It can be interpreted as a single travel time measurement. Subsequently, additional measurements with new randomly selected start and/or end points are performed. The resulting measured travel time samples are t_{T1}, ..., t_{Tn}, a random sample from t_T. The arithmetic mean approximates the expected value of the cycle time component t_{ctc}:

$$\mu = \overline{t_{ctc}} \approx \frac{1}{n}\sum_{i=1}^{n} t_{T\,i} \tag{3}$$

According to Jodin (2010b), the experiment design stipulates two termination criteria, both of which must be satisfied in order to have a sufficiently accurate estimate and therefore to finish the simulation run. The first criterion is the convergence of the arithmetic mean value. The criterion is fulfilled as soon as the mean value of the travel times varies by just a small enough amount $x_{Convergence}$ (e.g. 0.01 sec). The convergence termination criterion is calculated with the following formula:

$$\left|\frac{1}{n-1000}\sum_{i=1}^{n-1000} t_{T\,i} - \frac{1}{n}\sum_{i=1}^{n} t_{T\,i}\right| \le x_{Convergence} \tag{4}$$

The second criterion is the confidence interval which indicates the reliability of the estimated expected mean value of the travel time component. With confidence level α (e.g. 99%) it indicates the probability that the true population expected value parameter is captured inside the confidence interval calculated in base of the travel time samples. With an unknown distribution, unknown expected value and an unknown variance, the confidence interval is defined by the following lower and upper end point:

$$\left[\overline{t_{ctc}} - z_{\left(1-\frac{\alpha}{2}\right)} \cdot \frac{S}{\sqrt{n}} ; \overline{t_{ctc}} + z_{\left(1-\frac{\alpha}{2}\right)} \cdot \frac{S}{\sqrt{n}}\right] \tag{5}$$

The standard deviation S is estimated from the calculated travel time samples. The confidence termination criteria $x_{Confidence}$ (e.g. 0.02 sec) is therefore given by the following formula:

$$2 \cdot z_{\left(1-\frac{\alpha}{2}\right)} \cdot \sqrt{\frac{\frac{1}{n-1} \cdot \sum_{i=1}^{n}\left(T_i - \frac{1}{n}\sum_{i=1}^{n} t_{T\,i}\right)^2}{n}} \le x_{Confidence} \tag{6}$$

The sample generation and calculation of the travel times is compared to the calculation of the termination criteria very quickly. Therefore, in order to save computing time, the termination criteria are not calculated and checked after each new sample, but only every 1.000 samples.

4.3. I/O travel time

An example of the application of the simulation approach is the modeling of the I/O travel time $t_{I/O,P}$. It is part of all storage configurations and strategies at the beginning of the resulting command cycle. Figure 3 illustrates the entrance travel of the S/R machine to two of the free storage shelves in which storage subsequently takes place.

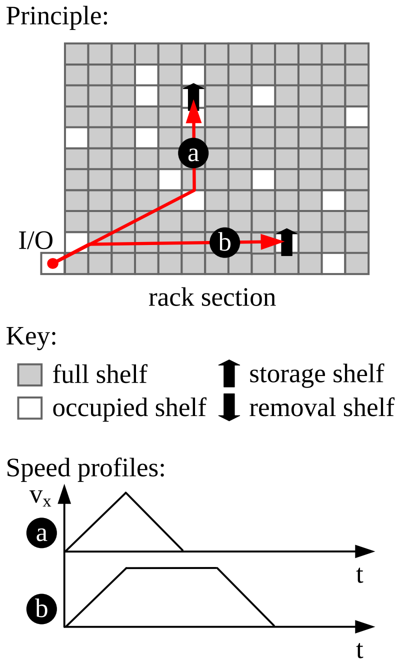

Figure 3: Principle and Speed Profiles in Horizontal Direction of the I/O Travel Time

Depending on the travel distance, the S/R machine assumes two different speed profiles in both x and y directions (in Figure 3 a and b, respectively). The speed profile is characterized through the maximum velocity v and the acceleration/deceleration a. The following equation for the travel time t_S depending on the travel distance s is derived from the laws of motion for constant velocity and constant acceleration:

$$t_S = \begin{cases} 2 \cdot \sqrt{\frac{s}{a}} & when\ s \le \frac{v^2}{a} \\ \frac{s}{v} + \frac{v}{a} & else \end{cases} \tag{7}$$

The travel times in the horizontal direction t_{SX} and the vertical direction t_{SY} are calculated independently. The travel time t_T is given by the longer duration of the simultaneous travels:

$$t_T = \max\left(t_{SX}, t_{SY}\right) \tag{8}$$

The average I/O travel time $t_{I/O,P}$ is then calculated from the arithmetic mean of n measurements of the travel time t_T from the I/O point $P_S(x_{I/O}, y_{I/O})$ to a randomly chosen free storage shelf $P_E(x_P, y_P)$:

$$t_{I/O,P} = \frac{1}{n}\sum_{i=1}^{n} t_T\left(|x_{I/O} - x_P|, |y_{I/O} - y_P|\right) \tag{9}$$

The I/O point P_S is normally located in the corner of the rack, but can also be staggered in the x or y direction depending on the storage configuration. The end point P_E is either a storage or a retrieval shelf and randomly distributed across the surface of the rack.

4.4. Travel between times

The calculation of the travel between time is similar to that of the I/O travel time. It describes the travel time between:

- a storage and a retrieval shelf
- a relocation and a retrieval shelf
- two storage shelves or two retrieval shelves (when the S/R machine is able to handle more than one load contemporary)

The calculation is therefore:

$$t_{P,P} = \frac{1}{n}\sum_{i=1}^{n} t_T\left(|\,x_{PS}-x_{PE}\,|,|\,y_{PS}-y_{PE}\,|\right) \qquad (10)$$

The retrieval shelves are uniformly distributed in a strict FIFO strategy. With random allocation strategies, the storage and relocation shelves are also uniformly distributed. In these cases, both the starting shelf $P_S = (x_S, y_S)$ and the end shelf $P_E(x_{PA}, y_{PA})$ are randomly selected either from the occupied or the empty storage shelves in the modeling.

In order to reduce the travel between time the storage shelf can be selected near the subsequently approached retrieval shelf. With a double-deep rack configuration the necessary relocation of loads can also be done as close as possible to the retrieval shelf. Since this shelf is uniformly distributed on the surface, the generally uniform allocation of loads remains. In modeling the retrieval shelf P_E is randomly selected first. Then a search for the first available storage shelf (empty or single-occupied) starts around the shelf P_E. The first choice is the shelf directly opposite in the other rack. If this is not available, the retrieval shelf directly surrounding 16 storage shelves (in both racks) are checked. If there is still no free storage shelf available the next 32 surrounding shelves are tested, etc. If there are several free storage shelves at the same distance from the shelf P_E, shelf P_S is selected randomly from amongst the free shelves. In this method, if the retrieval shelf P_E is at the edge of the rack, this may reduce the number of potential storage shelves.

If more than two uniformly distributed shelves are approached in a single command cycle, travel path optimization can be applied. The sequence of the travel route is chosen in such a way that the accumulated travel time is as short as possible. A simple heuristic for travel path optimization according to Meller (1997) is to move from the current shelf to the nearest of all the remaining shelves. The model therefore generates m remaining shelves P_E and calculates the travel time to them. The shelf requiring the shortest travel time is then selected as P_E. Of course, more complex heuristics can also be modeled.

4.5. I/O travel time in multi-aisle systems

In multi-aisle systems one S/R machine serves several aisles. There is therefore a transition area at the front of the racks so that the S/R machine is able to switch the aisle served. The I/O point is often also located in this area. The travel distance for the S/R machine is therefore divided into the transition section and the rack section. In the transition section the horizontal velocity has to be reduced whilst the lifting unit is not affected (see Figure 5).

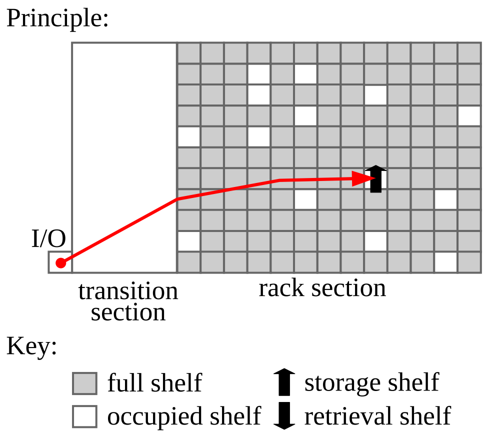

Figure 4: Principle of the I/O travel time with I/O point in Transition Section

In this situation, the velocity profiles in the transition section and in the rack section are interdependent. For example in Figure 5, case e, it is necessary to initiate deceleration in the transition section. Otherwise the remaining braking distance is too short and the S/R machine would not be able to stop in time.

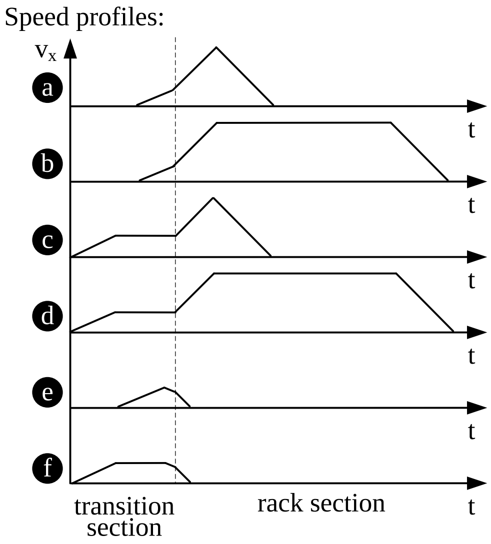

Figure 5: Speed Profiles of the I/O Travel Time in a Horizontal Direction of the with the I/O point in the Transition Section

To calculate the technically fastest possible travel time, the speed limit of the horizontal travel drive at the switchover point from the transition section to the rack section first has to be determined. The switchover speed can be limited on the one hand by the speed profile in the transition section (still in the acceleration phase or maximum velocity reached) and on the other hand by the speed profile in the aisle (reduced speed to be able to brake in time and to come to a stop at the desired point). In Figure 5, the speed limit in cases a-d are determined by the travel in the transition section and cases e-f of the travel in the aisle. The maximum achievable and allowable speed depending on the distance is:

$$v_{\max} = \min\left(v, \sqrt{2 \cdot a \cdot s}\right) \tag{11}$$

The speed has to be calculated once for the horizontal travel distance *s* in the transition section ($v_{max\ TS}$) and once for the horizontal travel distance *s* in the aisle ($v_{max\ AS}$). Acceleration and velocity are determined according to technical specifications for the traveled areas. The maximum switchover speed $v_{max\ SO}$ is the smallest allowable:

$$v_{\max SO} = \min\left(v_{\max TS}, v_{\max AS}\right) \tag{12}$$

Both the start and the end velocity are determined in this way for journeys in the transition and aisle section. In the following, a function is derived to calculate the required travel time in the two sections. Equations for the traveled distance and the travel time with superposed acceleration and constant speed are required for this:

- the instantaneous velocity depending on time *t*, with acceleration *a* and a starting velocity v_s (Formula 13)
- the traveled distance s depending on time *t*, with acceleration *a* and a starting velocity v_s (Formula 14)
- the instantaneous velocity depending on distance *s*, with acceleration *a* and a starting velocity v_s (Formula 15)

$$v = a \cdot t + v_s \tag{13}$$

$$s = \frac{a}{2} \cdot t^2 + v_a \cdot t \tag{14}$$

$$v = \sqrt{2 \cdot a \cdot s + v_a{}^2} \tag{15}$$

The maximum speed on the distance *s* for a given initial and final velocity can be calculated with the given formulae. Formula 16 expresses the value without considering the technical speed limitation for the section; Formula 17 takes this limit into account:

$$v_{m \max theoretical} = \sqrt{\frac{2 \cdot a \cdot s + v_s{}^2 + v_e{}^2}{2}} \tag{16}$$

$$v_{m \max m} = \min\left(v, \sqrt{\frac{2 \cdot a \cdot s + v_s{}^2 + v_e{}^2}{2}}\right) \tag{17}$$

A function to determine the travel time t_s for a given initial and final velocity and the distance *s* is thus determined. Formula 18 covers three cases:

- Case 1: deceleration or acceleration is not possible within distance *s*
- Case 2: travel consisting of an acceleration phase (first term) and a deceleration phase (second term)
- Case 3: travel consisting of the acceleration from the initial velocity to the technical maximum velocity (first term), deceleration from the maximum velocity to the final velocity (second term), constant velocity phase (third term)

$$t_s = \begin{cases} \text{not defined} & \text{for } s < -\dfrac{v_e{}^2 - v_s{}^2}{2 \cdot a} \\ \dfrac{v_{m \max} - v_s}{a} + \dfrac{v_{m \max} - v_e}{a} & \text{else and for } v_{m \max theoretical} < v \\ \dfrac{v - v_s}{a} + \dfrac{v - v_e}{a} + \dfrac{s}{v} - \dfrac{\left[\left(\dfrac{v - v_s}{a}\right)^3 \cdot \left(\dfrac{a}{2} + v_s\right)\right]}{v} - \dfrac{\left[\left(\dfrac{v - v_e}{a}\right)^3 \cdot \left(\dfrac{a}{2} + v_e\right)\right]}{v} & \text{else} \end{cases} \tag{18}$$

With Formula 18, travel times can be calculated for the two sections taking into account the predetermined switchover speed $v_{max\ SO}$ at the switchover point.

In the modeling of the Monte Carlo simulation, the first step is the generation of the start and end point. The start point P_S is given by the position of the I/O point in the transition section, P_E is determined analogously with fixed-aisle S/R machines and is uniformly distributed over the surface of the rack. The length of the transition section depends on the relative position of the aisle to the I/O point. This may also be different for different aisles. In a second step, the travel time is calculated according to the predetermined scheme.

4.6. Travel between time in multi aisles Systems

When a multi-aisle S/R machine performs a travel between two shelves in two different aisles, there are two switchover points from the aisles to the transition area. The number of possible speed profiles doubles to 12.

The maximum switchover speed has to be calculated for both switchover points. The maximum attainable speed at the end of the first aisle and the maximum possible speed at the beginning of the second aisle are therefore calculated with regard to the particular horizontal travel distances (Formula 11). The lower of the two speeds is decisive and may still be limited by the maximum possible velocity in the transition section. The second limiting speed in the other switchover point can be calculated starting from this first switchover speed (Formula 17). The first fixed switchover speed is used for this purpose as the initial velocity of the travel in the transition section. When both limiting switchover speeds are known, the travel time of the three segments are calculated with Formula 18.

In the modeling the start point P_S and end point P_E are again uniformly distributed over the entire surface of the rack, however not in the same aisle. The length of the transition section is affected by the number of skipped aisles.

5. APPLICATION

5.1. Comparison of cycle time components with analytical models

The described simulation models can be verified and validated using parameter studies. The MC simulation models and equivalent analytical methods where therefore implemented in a computer program. A study of the I/O travel time will be presented in the following. Because of its fundamental importance, the calculation of this cycle time component is developed in numerous analytical models. Three well-suited methods are compared with the simulation model:

- FEM: FEM 9.851 (2003) is the most important practice-related directive. The travel time calculation is based on a stochastic analytical travel distance model which was developed in the early 70s. To simplify the model, it assumes that each S/R machine travel reaches the maximum speed. For the special case $b = 1$, the resulting travel distances are converted into representative points. The expected values of the cycle time and also of the cycle time components are calculated by traveling to these points. The best accuracy is achieved when $b = 1$. The model is valid within the limits $2 \leq b \leq 0.5$.
- BW&G: Bozer and White (1984) developed a stochastic analytical travel time model. It considers only the travel at maximum speed without acceleration. The additional time which is required for acceleration and deceleration, is added by an estimation (Gudehus 1972). This model has the advantage of being valid independently of the shape factor *b*.
- CWL: The model according to Chang et al. (1995) is a further development of that of Bozer and White. In this model the speed profile is approximated by a new function which covers all speed profiles. Thus, the amount of time for acceleration and deceleration is already included in the calculation.

In the parameter study, the I/O travel time in an AS/RS with a height of *20 m* and varied length is calculated. For a better understanding of the behavior and characteristics of the different calculation models, an idealized S/R machine with infinite acceleration/deceleration and a practice-oriented S/R machine with normal acceleration/deceleration are used. The maximum velocities in the x-y direction and the accelerations/decelerations of the two S/R machines are shown in Table 3.

Table 3: Kinematic Characteristics of tested S/R machines

	Ideal S/R machine	Real S/R machine
v_x	4 m/s	4 m/s
a_x	∞ m/s^2	1 m/s^2
v_y	2 m/s	2 m/s
a_y	∞ m/s^2	1 m/s^2

The calculated values of the I/O travel time of the different methods in the parameter study are illustrated in the following diagram. It shows the results in the range of a *5-80 m* long rack. Simultaneously the shape factor *b* varies from *8* to *0.5*.

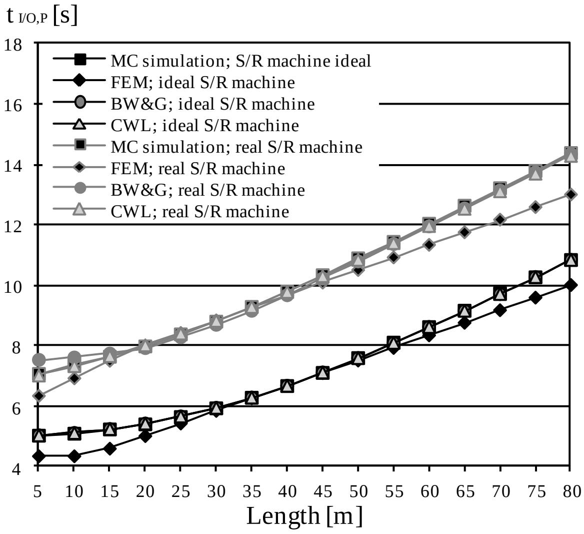

Figure 6: I/O Travel Time in an 20 m high AS/RS with an ideal and a real S/R machine

In the following Table 4, the I/O travel times of the four different calculation models for the real S/R machine are tabulated. The value of the MC simulation is the reference value to which the percentage differences of the other models were calculated.

Table 4: Comparison of different Calculation Models for the I/O Travel Time in a 20 m high AS/RS with the real S/R machine

Length [m]	Shape factor b [-]	MC simulation [s]	FEM [s]	BW& G [s]	CWL [s]
5	8.00	7.06	6.33* –10.3%	7.53 +6.6%	7.01 –0.8%
10	4.00	7.35	6.92* –5.9%	7.60 +3.5%	7.32 –0.4%
20	2.00	7.97	7.98 +0.2%	7.92 –0.7%	8.01 +0.5%
30	1.33	8.79	8.83 +0.5%	8.69 –1.2%	8.82 +0.3%
40	1.00	9.78	9.67 –1.2%	9.67 –1.2%	9.77 –0.1%
50	0.80	10.89	10.50 –3.5%	10.78 –0.9%	10.84 –0.4%
60	0.67	12.03	11.33 –5.8%	11.94 –0.7%	11.96 –0.6%
70	0.57	13.21	12.17 –7.9%	13.13 –0.6%	13.11 –0.8%
80	0.50	14.37	13.00 –9.5%	14.33 –0.3%	14.28 –0.7%

*Values outside the recommended range of the directive

The performance evaluation according to FEM 9.851 is often used in practice. The best accuracy is reached by definition when *b = 1*. At that point the results for both S/R machines agree closely with the MC simulation. With increasing deviance up to this point, the model behavior becomes worse and the differences in the calculated cycle time components became larger. The FEM model also fails to include the additional time for acceleration and deceleration for the real S/R machine correctly. The behavior of the Bozer and White model with added acceleration and deceleration time according to Gudehus corresponds exactly to the MC simulation. The match is very good over a wide range of *b*. If the acceleration influence becomes important, deviations occur at small shelf dimensions. In this case, the method of estimation of the acceleration and deceleration time is inaccurate. The model according to Chang et al. agrees closely with the simulation for both the ideal and the real S/R machine. The study allows the conclusion that the MC simulation is congruent to the expected behavior of analytical models and reproduces reality very well. Advantages over analytical models consist in the discrete modeling of the distances to and between shelves and the I/O point and the good reproduction of the different speed profiles of the S/R machine. With further parameter studies, a logical test of the cycle time component to each other allows the evaluation of consistency.

5.2. Calculation of cycle times

The expected value of cycle times can be calculated from the calculated average cycle time components. For example, the dual command cycle initially described includes the pick-up and deposit time t_{PD} four times, the average I/O travel time $t_{I/O,P}$ twice and the average travel between time $t_{P,P}$ once (see Figure 2). The mean duration of one cycle time t_{CT} is therefore composed of the sum of the separate cycle time components:

$$t_{CT} = t_{PD} + t_{I/O,P} + t_{PD} + t_{P,P} + t_{PD} + t_{I/O,P} + t_{PD} \tag{19}$$

With the real S/R machine from the previous chapter and a rack *40 m* in length and *20 m* in height, the I/O travel time is $t_{I/O,P} = 9.79\,s$ and the travel between time is $t_{P,P} = 7.75\,s$. If the pickup and deposit time t_{PD} is *9 s*, the total cycle time is *63.33 s*.

One possibility to increase throughput performance is to change the strategy for selecting the storage shelf. Rather than choosing the storage shelf randomly, it can be placed as close as possible to the retrieval shelf. The previously used term for the travel between time $t_{P,P}$ is therefore replaced by $t_{P,P\,nrs}$, the expected travel time from the nearest available storage shelf to a retrieval shelf. With *0.5* x *0.5* m large storage shelves and a percentage of filling of 90 % the MC simulation determines the value of $t_{P,P\,nrs}$ on *2,28 s*. The resulting cycle time is then shortened from *63.27* to *57.86* seconds, which means an increase of the throughput of 9.*5%* because of the new strategy.

Another way to increase the performance of S/R machines is to increase the number of load handling devices. If three load handling devices are installed on the S/R machine, up to six load units may be handled in one single command cycle. Thereby the S/R machine has to travel to a total of three storage and three retrieval shelves in alternating sequence. When the storage shelves are again sought near the retrieval shelves, $t_{P,P\,nrs}$ retains the value *2.28 s*. The retrieval shelves can be traveled to using a strategy of travel path optimization: after traveling to one of the retrieval shelves, the next selected retrieval shelf is the nearest of the remaining shelves. With two remaining retrieval shelves the travel between time $t_{P,P\;n=2}$ is *6.40 s* compared to the single travel between time $t_{P,P} = t_{P,P\,n=1} = 7.75\,s$. The cycle time is then calculated by the following formula and has the value of *112.57 s* (Formula 20). The increase in throughput compared to the first example is 68.8%.

$$\begin{aligned} t_{CT} = {} & t_{PD} + t_{I/O,P} + t_{PD} + t_{P,P\,nrs} + t_{PD} + t_{P,P\,n=2} \\ & + t_{PD} + t_{P,P\,nrs} + t_{PD} + t_{P,P\,n=1} \\ & + t_{PD} + t_{P,P\,nrs} + t_{PD} + t_{I/O,P} + t_{PD} \end{aligned} \tag{20}$$

5.3. Integration of the model

The design of AS/RS depends on a variety of parameters. The most important parameters are the storage capacity, building geometry, throughput and investment. In addition to the presented model for throughput calculations, own models for other characteristics were developed and merged into the database-aided software tool LSP. This software allows mathematical optimization for the dimensioning of design variants and therefore the easy comparison of the latter.

The objective function of the optimization is the amount of investment costs. The constraints are throughput, storage capacity, dimensions of the building, etc. A valid design variant has to meet any constraints arising from the design specifications. For each valid solution, the width and the height of the racks, the lowest number of needed aisles and the most appropriate type of S/R machine for the database are calculated and visualized (see Figure 7, screenshot of the software). The solutions are colored in accordance with the investment outlay. Similarly, good solutions can thus be easily found by planners.

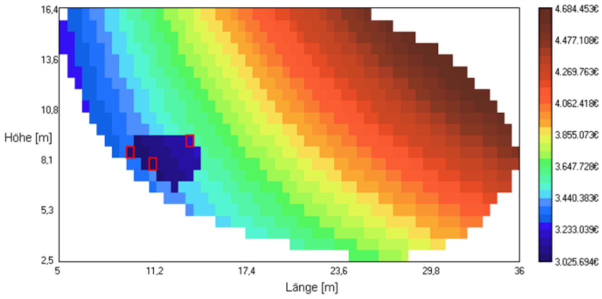

Figure 7: Solution Space of the Optimization

The optimum points found show the best dimensioned AS/RS for a specific configuration combined with strategies and allows comparison with other concepts. Automatic sensitivity analyzes are performed with the help of parameter studies to examine the effects of varying planning data on an optimally dimensioned AS/RS (see Figure 8).

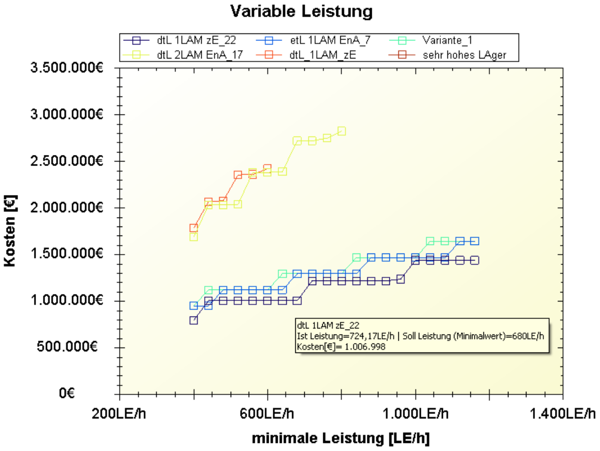

Figure 8: Sensitivity Analysis with varied Throughput Requirements

It is possible to study and visualize the effects of different size limits and increasing or decreasing throughput or capacity requirements. Usually a step function describes the cost of the optimally dimensioned storage. The optimal dimensioned storage only varies if there is a sufficiently large change in a planning requirement. The robustness with respect to changing conditions can easily be found for a solution found in this way. For alternative concepts which differ in storage configurations and/or strategies, suitable application areas can be identified and included in the planning quickly and easily.

6. CONCLUSIONS

The throughput calculation of many different practice-relevant storage configurations and storage strategies is covered with the presented unified approach. Specific command cycles are therefore synthesized from the different cycle time components which represent typical movements and load handling steps of a storage and retrieval machine. With double-deep racks some travel time components are weighted with their probability of occurrence, which depends on the percentage of filling and the load allocation in the racks.

The different cycle time components can be modeled and calculated with analytical models and the Monte Carlo simulation. The presented simulation approach is numerical and stochastic and used to calculate the arithmetical mean. It is suitable for the calculation of a large number of different cycle time components. The modeling contains the initialization of the racks with a representative allocation of loads. The starting point P_S and the end point P_E of a single travel are then determined according to the storage configuration and the strategies (assignment, retrieval rules, etc.). The travel time between the two points is then calculated. These steps are repeated and the arithmetic mean of the single measurements is calculated. Termination criteria are the convergence of the arithmetic mean value and the confidence interval.

The results are congruent to the expected behavior of analytical models and reproduce reality very closely. Advantages over analytical models consist in the discrete modeling of the distances to and between shelves and the I/O point and the good reproduction of the different speed profiles of the S/R machine. These advantages result in more general validity and a better accuracy.

Because of its formality, the unified approach for the throughput calculation has been implemented in a computer routine and merged with models for the calculation of storage capacity, building geometry and investment in a database-aided software tool. This tool allows mathematical optimization for the dimensioning of design variants and therefore the easy comparison of the latter.

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