Recent developments in mixed integer linear programming formulations for the resource-constrained project scheduling problem

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Outline

1. RCPSP
2. MILP for RCPSP
3. Standard and novel MILP formulations
   - Pseudo-polynomial time-indexed formulations
   - Extended time-indexed formulations and valid inequalities
   - Compact sequencing and natural date variable formulations
   - Compact event-based formulations
4. Synthesis of theoretical and experimental results
5. Perspectives
6. References
The Resource-Constrained Project Scheduling Problem (RCPSP)

- A central problem in many industrial applications
  - Project management, manufacturing, process industry, parallel processor architectures
- The “standard” RCPSP: An NP-hard problem posing a computational challenge since the eighties
  - 686 citations on PSPLIP (Google Scholar) 1/1/2014
  - 48 (out of 480) still open instances with 60 activities and 4 resources from PSPLIB
The RCPSP : data

- $R$ set of resources, limited constant availability $B_k \geq 0$,
- A set of activities, duration $p_i \geq 0$, resource requirement $b_{ik} \geq 0$ on each resource $k$,
- $E$ set of precedence constraints $(i, j)$, $i, j \in A$, $i < j$
- $T$ time interval (scheduling horizon)

$$|R| = 1, B = 4, T = [0, 30)$$

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The RCPSP: variables, objective and constraints

- $S_i \geq 0$ start time of activity $i$
- $C_{\text{max}}$ makespan or total project duration

**RCPSP (conceptual formulation)**

$$\min C_{\text{max}} = \max_{i \in A} S_i + p_i$$

subject to:

1. $S_j \geq S_i + p_i \quad (i, j) \in E$ \hspace{1cm} *Precedence constraints*
2. $\sum_{i \in A(t)} b_{ik} \leq B_k \quad t \in T, k \in R$ \hspace{1cm} *Resource constraints*
3. $S_j \geq 0 \quad i \in A$

where $A(t) = \{ j \in A | t \in [S_j, S_j + p_j) \}$, $\forall t \in T$
The RCPSP : solution example

\[ |R| = 1, B = 4, T = [0, 30] \]

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The RCPSP: complexity, variants and methods

- Strongly NP-hard
- Generalizes single/parallel machine, X-shop problems
- Many relevant variants
  - Other objectives: \( \min \sum_{i \in A} w_i (S_i + p_i) \)
  - Generalized precedence constraints \( S_j \geq S_i + l_{ij} \)
  - Setup times, multiple modes, non-renewable resources, ...
  - Uncertainty \( p_i \in [p_{i\text{min}}, p_{i\text{max}}], p_i \sim \mathcal{N}(\mu_i, \sigma_i^2) \)
- Exact and heuristic Methods
  - Heuristics and metaheuristics
  - Dedicated branch and bound methods
  - Specific lower bounds
  - Constraint programming (CP) or hybrid SAT/CP
  - Mixed Integer Linear Programming (MILP)
The RCPSP: pre-processing and trivial bounds

- Upper bounds $|T|$: parallel or serial list scheduling heuristics
- CPM lower bound: longest 0–$n + 1$ path (16)
- Resource lower bound $\max_{k \in R} \sum_{i \in A} b_{ik} \times p_i / B_k$ (16.5 → 17)
- Reduce time windows $[ES_i, LS_i]$ by constraint propagation:

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- Temporal constraint propagation $TW$
- Temporal + Resource constraint propagation $TW^+$
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The scheduling polyhedron

Example (release dates \( r_i \), deadlines \( \tilde{d}_i \))

\(|A| = 2, \ |R| = 1, \ b_1 = b_2 = B = 1, \ \rho_1 = 3, \ \rho_2 = 2, \ \tilde{r}_1 = 0, \ \tilde{r}_2 = 1, \ \tilde{d}_1 = 9, \ \tilde{d}_2 = 7\).

Objective function \( f(S) = S_1 + S_2 + \rho_1 + \rho_2 \).

\( (P) \) can be solved by LP on \( \text{conv}(S) \)

\[
(P) \min S_1 + S_2 + 5 \\
S_1 \geq 0 \\
S_2 \geq 1 \\
S_1 \leq 6 \\
S_2 \leq 5 \\
S_2 \geq S_1 + 3 \lor S_1 \geq S_2 + 2
\]
MILP for RCPSP: principle

Let $S$, $cS$ and $\mathcal{S}$ denote the start time vector, the linear objective and the feasible set of the RCPSP.

Let $x$ denote a vector of additional $p$ binary variables.

The MILP $\min_{S,x}\{cS|MS + Nx \leq q, S \geq 0, x \in \{0, 1\}^p\}$ is a correct formulation for the RCPSP if we have

$$S = \{S \geq 0 | \exists x \in \{0, 1\}^p, MS + Nx \leq q\}$$

$S$ can be searched by branch and bound (and cut)

- Branching: tree search on $x$
- Bounding: solve at each node the LP relaxation by considering unfixed $x_q \in [0, 1]$ (and possibly incorporating valid inequalities)

The bound is tight if the relaxed set

$$\tilde{S} = \{S \geq 0 | \exists x \in [0, 1]^p, MS + Nx \leq q\}$$

is close to $\text{conv}(S)$.
MILP for RCPSP : example and issues

- Design a MIP formulation for the scheduling problem
- Solve by branch-and-bound

\[
\begin{align*}
\min & \quad S_1 + S_2 + 5 S_1 \\
\text{s.t.} & \quad S_2 \geq 0 \\
& \quad S_1 \leq 6 \\
& \quad S_2 \leq 5 \\
& \quad S_2 - S_1 + 8 x \geq 3 \\
& \quad S_1 - S_2 + 7 (1 - x) \geq 2 \\
& \quad x \in \{0, 1\}
\end{align*}
\]
MILP for RCPSP: example and issues

- Design a MIP formulation for the scheduling problem
- Solve by branch-and-bound

\[(P) \min S_1 + S_2 + 5 \]
\[S_1 \geq 0 \]
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\[S_1 \leq 6 \]
\[S_2 \leq 5 \]
\[S_2 - S_1 + 8x \geq 3 \]
\[S_1 - S_2 + 7(1 - x) \geq 2 \]
\[x \in \{0, 1\} \]

The projection of the MILP feasible set on $S$ maps $S$.
MILP for RCPSP : example and issues

- Design a MIP formulation for the scheduling problem
- Solve by branch-and-bound

\[(P) \min S_1 + S_2 + 5\]
\[S_1 \geq 0\]
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MILP for RCPSP: example and issues

- Design a MIP formulation for the scheduling problem
- Solve by branch-and-bound

\[
\begin{align*}
(P) \min & \quad S_1 + S_2 + 5 \\
S_1 & \geq 0 \\
S_2 & \geq 1 \\
S_1 & \leq 6 \\
S_2 & \leq 5 \\
S_2 - S_1 + 8x & \geq 3 \\
S_1 - S_2 + 7(1 - x) & \geq 2 \\
x & \in \{0, 1\}
\end{align*}
\]

Root node LB = 6
issue \( x = 0.5 \) always feasible
Design a MIP formulation for the scheduling problem

Solve by branch-and-bound

\[(P) \min S_1 + S_2 + 5 \]

\[S_1 \geq 0\]
\[S_2 \geq 1\]
\[S_1 \leq 6\]
\[S_2 \leq 5\]
\[S_2 - S_1 + 8x \geq 3\]
\[S_1 - S_2 + 7(1 - x) \geq 2\]
\[x \in \{0, 1\}\]

Left node \(x = 1\), obj=9
MILP for RCPSP: example and issues

- Design a MIP formulation for the scheduling problem
- Solve by branch-and-bound

\[ (P) \min S_1 + S_2 + 5 \]
\[ S_1 \geq 0 \]
\[ S_2 \geq 1 \]
\[ S_1 \leq 6 \]
\[ S_2 \leq 5 \]
\[ S_2 - S_1 + 8x \geq 3 \]
\[ S_1 - S_2 + 7(1 - x) \geq 2 \]
\[ x \in \{0, 1\} \]

Right node $x = 0$, obj=8
MILP for RCPSP: tradeoffs

- Designing pseudo-polynomial or extended formulations
  - Pros: obtain better LP relaxations, early node pruning in the search tree
  - Cons: increase of the MILP size (number of binary variables, constraints) towards pseudo-polynomial and even exponential sizes (need of column and cut generation techniques)

- Design compact formulations (polynomial size)
  - Pros: fast node evaluation, mode nodes explored
  - Cons: need to generate cuts
[Queyranne and Schulz 1994] classify the scheduling MILP for scheduling according to the type of decision variables, each yielding different families of valid inequalities.

1. Time-indexed variables
2. Linear-ordering variables → Strict-order or sequencing variables
3. Positional dates and assignment variables → Event-based formulations
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4 Synthesis of theoretical and experimental results

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For integer data, $S$ can be restricted to its integer vectors $S^{\text{int}}$.

“Pulse” binary variable $x_{it} = 1 \iff S_i = t$, for $t \in T = \mathcal{T} \cap \mathbb{N}$

Pseudo-polynomial number of variables $|A||T|$
The aggregated time-indexed formulation

- \( S_i = \sum_{t \in T} t \cdot x_{it} \)
- \( A(t) = \{ i \in A | \exists \tau \in \{ t - p_i + 1, \ldots, t \}, x_{i\tau} = 1 \} \)

\[(DT) \quad \text{Min.} \quad \sum_{t \in T} tx_{n+1,t} \]
\[\text{s.t.} \quad \sum_{t \in T} tx_{jt} - \sum_{t \in H} tx_{it} \geq p_i \quad (i, j) \in E \]
\[\sum_{i \in V} \sum_{\tau = t - p_i + 1}^t b_{ik} x_{i\tau} \leq B_k \quad t \in T; \quad k \in \mathcal{R} \]
\[\sum_{t \in T} x_{it} = 1 \quad i \in A \]
\[x_{it} \in \{0, 1\} \quad i \in A \]

[Pritsker et al. 1969]
Back to the small example: a better relaxation...

\[(P) \min S_1 + S_2 + 5\]

\[S_1 = x_{1,1} + 2x_{1,2} + 3x_{1,3} + 4x_{1,4} + 5x_{1,5} + 6x_{1,6}\]

\[S_2 = x_{2,1} + 2x_{2,2} + 3x_{2,3} + 4x_{2,4} + 5x_{2,5}\]

\[x_{1,0} + x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} = 1\]

\[x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} = 1\]

\[x_{1,0} + x_{1,1} + x_{2,1} \leq 1\]

\[x_{2,1} + x_{2,2} + x_{1,0} + x_{1,1} + x_{1,2} \leq 1\]

\[x_{2,2} + x_{2,3} + x_{1,1} + x_{1,2} + x_{1,3} \leq 1\]

\[x_{2,3} + x_{2,4} + x_{1,2} + x_{1,3} + x_{1,4} \leq 1\]

\[x_{2,4} + x_{2,5} + x_{1,3} + x_{1,4} + x_{1,5} \leq 1\]

\[x_{2,5} + x_{1,4} + x_{1,5} + x_{1,6} \leq 1\]

\[x_{1,t} \in \{0, 1\} \quad t \in \{0, \ldots, 6\}\]

\[x_{2,t} \in \{0, 1\} \quad t \in \{1, \ldots, 5\}\]
Back to the small example: a better relaxation...

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\[x_{1,0} + x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} = 1\]

\[x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} = 1\]

\[x_{1,0} + x_{1,1} + x_{2,1} \leq 1\]

\[x_{2,1} + x_{2,2} + x_{1,0} + x_{1,1} + x_{1,2} \leq 1\]

\[x_{2,2} + x_{2,3} + x_{1,1} + x_{1,2} + x_{1,3} \leq 1\]

\[x_{2,3} + x_{2,4} + x_{1,2} + x_{1,3} + x_{1,4} \leq 1\]

\[x_{2,4} + x_{2,5} + x_{1,3} + x_{1,4} + x_{1,5} \leq 1\]

\[x_{2,5} + x_{1,4} + x_{1,5} + x_{1,6} \leq 1\]

\[x_{1,t} \in \{0, 1\} \quad t \in \{0, \ldots, 6\}\]

\[x_{2,t} \in \{0, 1\} \quad t \in \{1, \ldots, 5\}\]

In this example \(\tilde{S} = \text{conv}(S)\) and the relaxation is tight...
Back to the small example: a better relaxation...

\[(P) \min S_1 + S_2 + 5\]

\[S_1 = x_{1,1} + 2x_{1,2} + 3x_{1,3} + 4x_{1,4} + 5x_{1,5} + 6x_{1,6}\]

\[S_2 = x_{2,1} + 2x_{2,2} + 3x_{2,3} + 4x_{2,4} + 5x_{2,5}\]

\[x_{1,0} + x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} = 1\]

\[x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} = 1\]

\[x_{1,0} + x_{1,1} + x_{2,1} \leq 1\]

\[x_{2,1} + x_{2,2} + x_{1,0} + x_{1,1} + x_{1,2} \leq 1\]

\[x_{2,2} + x_{2,3} + x_{1,1} + x_{1,2} + x_{1,3} \leq 1\]

\[x_{2,3} + x_{2,4} + x_{1,2} + x_{1,3} + x_{1,4} \leq 1\]

\[x_{2,4} + x_{2,5} + x_{1,3} + x_{1,4} + x_{1,5} \leq 1\]

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\[x_{1,t} \in \{0, 1\} \quad t \in \{0, \ldots, 6\}\]

\[x_{2,t} \in \{0, 1\} \quad t \in \{1, \ldots, 5\}\]

In this example \(\tilde{S} = \text{conv}(S)\) and the relaxation is tight...

... but we need 11 binary variables for a 2-task example.
Standard and novel MILP formulations... but not so good in general

\[ |R| = 1, B = 4, \mathcal{T} = [0, 30) \]

\begin{tabular}{ccc}
  \( i \) & \( p_i \) & \( b_i \) \\
  1 & 3 & 2 \\
  2 & 5 & 3 \\
  3 & 1 & 3 \\
  4 & 3 & 1 \\
  5 & 2 & 1 \\
  6 & 4 & 2 \\
  7 & 5 & 3 \\
  8 & 6 & 1 \\
  9 & 4 & 1 \\
  10 & 4 & 1 \\
\end{tabular}

\[ \text{Bound} = 16.46 \ (17) \ (\text{not better than trivial Res. Bount}) \]
The disaggregated time-indexed formulation (DDT)

The model can be reinforced by disaggregation of the precedence constraints, i.e. replacing precedence constraints by

\[
\sum_{\tau=0}^{t-p_i} x_{i\tau} - \sum_{\tau=0}^{t} x_{j\tau} \geq 0 \quad (i, j) \in E; \quad t \in T
\]

[Christofides et al. 1997]

- Modeling the logical relation: \( S_j \leq t \Rightarrow S_i \leq t - p_i \)
- The constraint matrix without resource constraints is totally unimodular.
- Total unimodularity preserved by lagrangean relaxation of the resource constraints Also efficiently computable by a max flow algorithm [Möhring et al. 2003]
DDT : relaxation quality

\[ |R| = 1, B = 4, T = [0, 30) \]

\[ \begin{array}{c|cc}
   i & p_i & b_i \\
   \hline
   1 & 3 & 2 \\
   2 & 5 & 3 \\
   3 & 1 & 3 \\
   4 & 3 & 1 \\
   5 & 2 & 1 \\
   6 & 4 & 2 \\
   7 & 5 & 3 \\
   8 & 6 & 1 \\
   9 & 4 & 1 \\
   10 & 4 & 1 \\
\end{array} \]

Bound = 17.14 (18) Strictly better than trivial bounds
Time-indexed step variables

- “Step” binary variable $\xi_{it} = 1 \iff S_i \leq t$, for $t \in T$

- Introduced by [Pritsker and Watters 1968] rediscovered several times... [citations removed]
Time-indexed formulations with step variables

- The time-indexed formulation with step variable (SDDT) can be obtained by (DDT) by the following transformation:

\[ \xi_{it} = \sum_{\tau=0}^{t} x_{it} \]

- Conversely, \( x_{it} = \xi_{it} - \xi_{it-1} \)

- This is a non-singular transformation (NST)

- Formulations that can be obtained from each other by a NST are strictly equivalent. They have the same \( \tilde{S} \) and the same relaxation value.

- [Bianco and Caramia 2013] present a variant of the step formulation based on variables \( \xi'_{it} = 1 \Leftrightarrow S_i + p_i \leq t \). We can shown that it is equivalent to (SDDT) by NST [A. 2013].
On/off time-indexed step variables

- "On/off" binary variable

\[ \mu_{it} = 1 \iff t \in [S_i, S_i + p_i] \]

Consider the following non singular transformation:

- \( \mu_{it} = \sum_{\tau = t - p_i + 1}^{t} x_{i\tau} \)
- \( x_{it} = \sum_{k=0}^{\lfloor t/p_i \rfloor} \mu_{i,t-kp_i} - \sum_{k=0}^{\lfloor (t-1)/p_i \rfloor} \mu_{i,t-kp_i-1} \)

[A. 2013] Applying the transformation yields a time-indexed formulations with on/off variables OODDT equivalent to DDT and tighter than that of [Klein 2000].

- Many "new" formulations presented in the literature are in fact weaker than or equivalent to DDT.

- Need to be distinguished from actual cutting planes or extended formulations.
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Extended formulations

- Formulation having better relaxations...
- ... with an exponential number of constraints and/or variables
- Need to use cut and/or column generation techniques

Small example again. $S^E$ dominant set of earliest schedules Let $x_s = 1$ iff schedule $S^s = S^E$ is selected. $S_i = \sum_{s \in S^E} S_i^s x_s$

\[
\begin{align*}
S_1 &= \sum_{s} C_i = 8 \\
S_2 &= \sum_{s} C_i = 9
\end{align*}
\]

\[
\begin{align*}
\min S_1 + S_2 + 5 \\
S_1 &= 3x_2 \\
S_2 &= 3x_1 + x_2 \\
x_1 + x_2 &= 1 \\
x_1, x_2 &\in \{0, 1\}
\end{align*}
\]
Minimal forbidden set (MFS) $F$: a minimal set of activities that cannot be scheduled in parallel:

$\sum_{i \in F} b_{ik} > B_k$ and $\forall j \in C, \sum_{i \in F \setminus \{j\}} b_{ik} \leq B_k$

$F = \\{\{1, 2\}, \{1, 3\}, \{2, 3\}, \ldots, \{7, 8, 9\}, \ldots\}$

There is in general an exponential number of MFS.

Can be reduced by excluding MFS having two activities with a precedence relation or non intersecting time windows.
Valid inequalities

- Forbidden set-based valid inequalities [Hardin et al 2008]
  - Basic inequality: \( \sum_{i \in A} \sum_{s=t-p_i+1}^{t} x_{is} \leq |F| - 1, \quad \forall F \in \mathcal{F} \)
  - The resource constraints can be replaced by this set of inequalities \( \rightarrow \) extended formulation
  - A more general family of inequalities: extension to an interval of length \( v \)
    \[
    \sum_{i \in F \setminus \{j\}} \sum_{s=t-p_i+1+v}^t x_{is} + \sum_{s=t-p_j+1}^{t+v} x_{js} \leq |F| - 1 \quad \forall F \in \mathcal{F}
    \]

- Lifting procedure and separation heuristic

Feasible subsets

- Feasible subset $P$: a set of activities that can be scheduled in parallel:
  \[ \sum_{i \in P} b_{ik} \leq B_k \text{ and } (i, j) \notin TA \text{ and } [ES_i, LS_i + p_i] \cap [ES_j, LS_j + p_j] \neq \emptyset \]

\[ P = \{\{1\}, \{2\}, \ldots, \{10\}, \{1, 5\}, \{2, 4\}, \ldots, \} \]

- There is in general an exponential number of FS.

- A schedule: an assignment of feasible subset to each time period
  1–2: \{1\}; 3–5: \{2, 4\}; 6,7: \{2\}; 8: \{3\}; 9,10: \{5, 6\}; ...
The feasible subset-based formulation (FS)

- obtained from (DDT) by replacing the resource constraints by

\[
\begin{align*}
\text{s. t.} \quad & \sum_{P \in P_i} \sum_{t \in T} y_{Pt} = p_i \quad i \in A, \ p_i \geq 1 \\
& \sum_{P \in \overline{P}} y_{Pt} \leq 1 \quad t \in T \\
& x_i^t - \sum_{P \in P_i} y_{Pt} - \sum_{P \in P_i} y_{P,t-1} \geq 0 \quad i \in A; \ t \in T \\
& y_{At} \in \{0, 1\} \quad P \in \mathcal{P}; \ t \in \cap_{i \in P} \{ES_i, \ldots, LS_i\}
\end{align*}
\]

where $P_i \subseteq \mathcal{P}$ is the set of all feasible subsets that contain activity $i$.

[Mingozzi et al 1998]
Lower bounds based on the feasible subset-based formulation

- Weighted Node packing combinatorial bound issued from the dual of the preemptive relaxation [Mingozzi et al. 1998]
- Destructive preemptive relaxation solved by constraint propagation and column generation or lagrangian relaxation [Brucker and Knust 2000, Demassey et al 2004, Baptiste and Demassey 2004]
- Preemptive FS solved by branch and price. [Moukrim et al. 2013]
Limits of time-indexed formulations

1. Equivalent relaxations does not mean equivalent behaviour of the MILP solver for obtaining solutions
   - [Bianco and Caramia 2013] show that the $\xi_{it}$ formulation outperforms others in terms of integer solving

2. Even weaker relaxations may yield better integer solutions
   - Well-known that (DT) formulation may also perform better than (DDT) formulation for integer solving.

3. Time-indexed formulation cannot be used for problems where large horizons are needed
   - Some examples with 15 activities are out of reach of time-indexed formulation [Kone et al. 2011]

Need of compact and/or hybrid formulations
Outline

1. RCPSP
2. MILP for RCPSP
3. Standard and novel MILP formulations
   - Pseudo-polynomial time-indexed formulations
   - Extended time-indexed formulations and valid inequalities
   - Compact sequencing and natural date variable formulations
   - Compact event-based formulations
4. Synthesis of theoretical and experimental results
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Sequencing or strict ordering variable

- Principle: adding precedence constraints such that all resource conflicts are resolved
- Any schedule satisfying these new precedence constraints is feasible
- Sequencing variable $z_{ij} = 1 \iff S_j \geq S_i + p_i$
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A first formulation based on forbidden sets

The set of additional precedence constraints has to “destroy” all forbidden sets.

\[
\begin{align*}
\text{Min. } S_{n+1} \\
\text{s.t. } & z_{ij} + z_{ji} \leq 1 \quad i, j \in V, \ i < j \\
& z_{ij} + z_{jh} - z_{ih} \leq 1 \quad i, j, h \in V, \ i \neq j \neq h) \\
& z_{ij} = 1 \quad (i, j) \in E \\
& S_j - S_i + (1 - M_{ij})z_{ij} \geq p_i \quad i, j \in V, \ i \neq j \\
& \sum_{i,j \in F, i \neq j} z_{ij} \geq 1 \quad F \in \mathcal{F} \\
& z_{ij} \in \{0, 1\} \quad i, j \in V, \ i \neq j
\end{align*}
\]

[Alvarez-Valdés and Tamarit 1993]
Extension of the disjunctive formulation for the job-shop problem [Balas 1985] with an exponential number of constraints
Resource flow variables

\[ \phi_{ij}^k \geq 0 : \text{numbers of units of resource } k \text{ transferred from } i \text{ to } j \]
Resource flow variables

$\phi_{ij}^k \geq 0$ : numbers of units of resource $k$ transferred from $i$ to $j$
Resource flow variables

\[ \phi^k_{ij} \geq 0 : \text{numbers of units of resource } k \text{ transferred from } i \text{ to } j \]

Enforcing sequencing variables to be compatible with the flow

\[ \phi^k_{ij} > 0 \Rightarrow z_{ij} = 1 \]
A formulation based on resource flows

- Replace the forbidden set constraints by the following flow constraints

\[
\phi_{ij}^k - \min(\tilde{r}_{ik}, \tilde{r}_{jk})z_{ij} \leq 0 \quad (i, j \in V, \ i \neq j, \ \forall k \in \mathcal{R})
\]

\[
\sum_{j \in V \setminus \{i\}} \phi_{ij}^k = \tilde{r}_{ik} \quad (i \in V \setminus \{n + 1\})
\]

\[
\sum_{i \in V \setminus \{j\}} \phi_{ij}^k = \tilde{r}_{jk} \quad (j \in V \setminus \{0\})
\]

\[
0 \leq \phi_{ij}^k \leq \min(\tilde{r}_{ik}, \tilde{r}_{jk}) \quad (i, j \in V, \ i \neq n+1, \ j \neq 0, \ i \neq j; \ k \in \mathcal{R})
\]

- \(O(|A|^2R)\) additional continuous variables

Valid inequalities for sequencing formulations

- Relaxation of poor quality, need to generate valid inequalities


Example 2: constraint propagation-based cutting planes [Demassey et al 2005]

- Compute conditional distances \( d_{ij}^k \preceq l \), \( d_{ij}^l \preceq k \) and \( d_{ij}^k \parallel l \) by CP
- Lifted distance inequalities

\[
S_j - S_i \geq d_{ij}^h \parallel l + (d_{ij}^h \preceq l - d_{ij}^h \parallel l) z_{hl} + (d_{ij}^l \preceq h - d_{ij}^h \parallel l) z_{lh}
\]
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Start and End Event variables

- $\mathcal{E}$: set of remarkable events.
- $t_e \geq 0$: event date: representing the start and end of at least one activity.
- Start binary assignment variables $a_{ie}^- = 1 \iff S_i = t_e$
- End binary assignment variables $a_{ie}^+ = 1 \iff S_i + p_i = t_e$
- Maximum $n + 1$ events $\implies 2(n + 1)|\mathcal{E}|$ binary variables.

Extension of models proposed for machine scheduling [Lasserre and Queyranne 1994, Dauzère-Pérès and Lasserre 1995], widely used also in the process scheduling industry [Pinto and Grossmann 1995, Zapata et al 2008].
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Extension of models proposed for machine scheduling [Lasserre and Queyranne 1994, Dauzère-Pérès and Lasserre 1995], widely used also in the process scheduling industry [Pinto and Grossmann 1995, Zapata et al 2008].
On/Off Event variables

- $\mathcal{E}$: set of remarkable events.
- $t_e \geq 0$: event date: representing the start of at least one activity.
- On/off binary variable $a_{ie} = 1 \iff [S_i, S_i + p_i] \cap [t_e, t_e + 1] \neq \emptyset$
- Each activity such that $a_{ie} = 1$ can be assumed of length $[t_e, t_e + 1]$
- $n|\mathcal{E}|$ binary variables
(OE) Min. \( C_{\text{max}} \)

s.t. \( C_{\text{max}} \geq t_e + (\bar{a}_{ie} - \bar{a}_{i(e-1)})p_i \) \( (e \in E; \ i \in A) \)

\( t_0 = 0 \)
\( t_{e+1} \geq t_e \) \( (e \neq n - 1 \in E) \)
\( t_f \geq t_e + (\bar{a}_{ie} - \bar{a}_{i,e-1} - \bar{a}_{if} + \bar{a}_{i,f-1} - 1)p_i \) \( ((e, f, i) \in E^2 \times A, \ f > e \neq 0) \)

\( \sum_{e'=0}^{e-1} \bar{a}_{ie'} \geq e(1 - \bar{a}_{ie} + \bar{a}_{i,e-1}) \) \( (i \in A; \ e \neq 0 \in E) \)

\( \sum_{e'=e}^{n-1} \bar{a}_{ie'} \geq e(1 + \bar{a}_{ie} - \bar{a}_{i,e-1}) \) \( (i \in A; \ e \neq 0 \in E) \)

\( \sum_{e' \in E} \bar{a}_{ie} \geq 1 \) \( (i \in A) \)

\( \bar{a}_{ie} + \sum_{e'=0}^{e} \bar{a}_{je'} \leq 1 + (1 - \bar{a}_{ie})e \) \( (e \in E; \ (i, j) \in E) \)

\( \sum_{i=0}^{n-1} r_{ik} \bar{a}_{ie} \leq R_k \) \( (e \in E; \ k \in R) \)

\( t_e \geq 0 \) \( (e \in E) \)

\( \bar{a}_{ie} \in \{0, 1\} \) \( (i \in A; \ e \in E) \) [Koné et al. 2011]
Valid inequalities for event-based formulations

- Wanted!!

Done for the one machine problem in [Della croce et al 2014]
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## Synthesis of theoretical and experimental results

### Comparison of formulations: LB

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**LCG12**: [Schutt et al 2013] (hybrid CP/SAT method: Lazy clause generation)

**PFS13**: [Moukrim et al 2013] Preemptive feasible subset formulation solved by B&P
Comparison of formulations: exact solving

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- **MCS** [Laborie 2005] (MFS-based CP)
- **LCG** [Schutt et al 2013]

- KSD30 “highly disjunctive” instances
- PACK, BL “highly cumulative” instances
- KSD15_d: first 15 activities of KSD30 with modified durations
- PACK_d: PACK instances with modified durations
Synthesis of theoretical and experimental results

- Time indexed formulations have the best LP relaxations with $FS \succ DDT \succ DT$
- Compact formulations have poor relaxation but can be the only alternative for large scheduling horizons
  - Highly disjunctive instances: flow-based models
  - Highly cumulative instances: event-based models
  - Valid inequalities strictly necessary
- MILP vs Lazy Clause Generation
  - MILP outperformed by LCG for exact solving disjunctive instances
  - Competitive with LCG for lower bounds based on preemptive exact solving of FS through B&P.
  - Competitive with LCG for exact highly cumulative instances
Outline

1. RCPSP
2. MILP for RCPSP
3. Standard and novel MILP formulations
   - Pseudo-polynomial time-indexed formulations
   - Extended time-indexed formulations and valid inequalities
   - Compact sequencing and natural date variable formulations
   - Compact event-based formulations
4. Synthesis of theoretical and experimental results
5. Perspectives
6. References
- Time aggregation / energetic reasoning / dual feasible functions [Carlier and Néron 2000, Kooli 2012]

- Mixed continuous/discrete models [Haït and A. 2012]

- Preprocessing [Baptiste et al. 2010]

- B&P for the non-preemptive feasible set formulations


- Matheuristics [Palpant et al. 2004, Della croce et al. 2014]

- Hybrid SAT/CP/MILP

\[
\begin{align*}
\text{Find } x \\
\text{s.t.} \\
\sum_{l \in I_j} x_{jl} &= p_j, \quad \forall j \in A \\
x_{jl} &\leq \Delta_l, \quad \forall j \in A, \forall l \in I_j \\
\sum_{j \in A_l} b_{lk} x_{jl} &\leq B_k \Delta_l, \quad \forall k \in K, \forall l \in L \\
\sum_{s \in S / s \leq l} x_{is} &\geq \sum_{s = r / r = r_j} x_{js}, \quad \forall (i, j) \in A, \forall l \in I_i^j \\
x_{jl} &\geq 0, \quad \forall j \in A, \forall l \in I_j
\end{align*}
\]
Outline

1. RCPSP
2. MILP for RCPSP
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   - Pseudo-polynomial time-indexed formulations
   - Extended time-indexed formulations and valid inequalities
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In order of appearance 3/6


In order of appearance 5/6


