

# Bilateral Teleoperation over the Internet: the Time Varying Delay Problem<sup>1</sup>

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## Abstract

This paper addresses the problem of time-varying communication delay in force reflecting bilateral teleoperation. The problem is motivated by the increasing use of the Internet as a communication medium where the time delay is variable depending on factors such as congestion, bandwidth, or distance. The well-known scattering formalism introduced in [1] preserves passivity of the communication channel in general only for constant transmission delay. We demonstrate how to recover both passivity and tracking performance using a modified control architecture that incorporates time-varying gains into the scattering transformation and feedforward position control. Experimental results using a single-degree of freedom master/slave system are presented

## 1 Introduction

Bilateral Teleoperation has challenged researchers in both control theory and robotics over the last few decades. The breakthrough was achieved in [1] where

the concepts from scattering theory, passivity and network theory were used to derive a control law which guaranteed stability [2] of the teleoperator independent of the (constant) delay. These results were then extended in [4], where the notions of wave-variables, impedance matching and wavefilters were introduced. This paper is motivated by the use of the Internet as the communication medium connecting the master and slave manipulators, where transmission delays are variable. For teleoperation over the Internet the delay varies with such factors as congestion, bandwidth, or distance, and these varying delays may severely degrade performance or even result in an unstable system. There has to date been relatively little research on this problem. Some preliminary results are contained in [5, 3]. Recently, some interesting results were obtained in [6]. A simple modification to the scattering transformation of [1] was proposed, that inserts a time varying gain into the communication block which guarantees passivity for arbitrary time varying delays provided a bound on the rate of change of the time delay is known. Readily available network statistics can be used to estimate the delay variation needed to compute the gain compensation.

Passivity does not guarantee good performance. In this paper we investigate a modified control architecture that introduces time-varying gains into the scattering

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<sup>1</sup>This research was partially supported by the National Science Foundation under grants ECS-0122412, HS-0233314, and CCR-0209202, and by the Office of Naval Research under grant N-14-02-1-0011

transformation [6] and explicitly uses the position data from the master to generate a feedforward position control for the slave manipulator. We demonstrate how passivity and tracking performance can be recovered via such a configuration. The efficacy of the approach is demonstrated experimentally.

## 2 Background

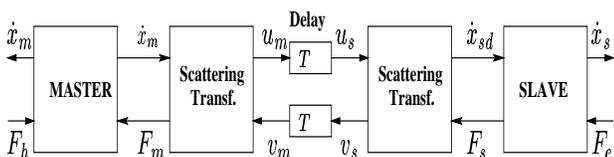
The standard bilateral teleoperation system with the scattering transformation is shown in Figure 1. The scattering transformation approach in [1] or the equivalent wave variable transformation proposed in [4] guarantee passivity of the network block for constant delay in the network. This transformation is given, using the notation of [4], as

$$\begin{aligned} u_m &= \frac{1}{\sqrt{2b}}(F_m + b\dot{x}_m) ; & v_m &= \frac{1}{\sqrt{2b}}(F_m - b\dot{x}_m) \\ u_s &= \frac{1}{\sqrt{2b}}(F_s + b\dot{x}_{sd}) ; & v_s &= \frac{1}{\sqrt{2b}}(F_s - b\dot{x}_{sd}) \end{aligned} \quad (1)$$

where  $\dot{x}_m$  and  $\dot{x}_s$  are the respective velocities for the master and slave.  $F_h$  is the operator torque and  $F_e$  is the environment torque.  $F_m$  is the force that is reflected back to the master from the slave robot. The force  $F_s$  is given as

$$F_s(t) = K_s \int_0^t (\dot{x}_{sd} - \dot{x}_s) dt + B_{s2}(\dot{x}_{sd} - \dot{x}_s) \quad (2)$$

where  $\dot{x}_{sd}$  is the velocity derived from the scattering transformation at the slave side. The power inflow



**Figure 1:** Scattering Transformation for Bilateral Teleoperation with Time Delay

into the communication block at any time is given by

$$P_{in}(t) = \dot{x}_m(t)F_m(t) - \dot{x}_{sd}(t)F_s(t) \quad (3)$$

In the case that the network delay is constant we have

$$\begin{aligned} u_s(t) &= u_m(t - T) \\ v_m(t) &= v_s(t - T) \end{aligned}$$

where  $T$  is constant. Assuming that the initial energy is zero, it is easily computed that the total energy stored

in the communications during the signal transmission between master and slave is given by

$$\begin{aligned} E &= \int_0^t P_{in}(\tau) d\tau = \int_0^t (\dot{x}_m(\tau)F_m(\tau) - \dot{x}_{sd}(\tau)F_s(\tau)) d\tau \\ &= \frac{1}{2} \left\{ \int_{t-T}^t u_m(\tau)^2 + v_s(\tau)^2 d\tau \right\} \geq 0 \end{aligned}$$

and, therefore, the system is passive independent of the magnitude of the delay  $T$ . The above result does not hold if  $T = T(t)$ , i.e., the delay is time-varying. In this case, the transmission equations become

$$\begin{aligned} u_s(t) &= u_m(t - T_1(t)) \\ v_m(t) &= v_s(t - T_2(t)) \end{aligned}$$

where,  $T_1(t)$  is the delay in the forward path and  $T_2(t)$  is the delay in the feedback path. We assume here that

$$\frac{dT_i}{d\tau} < 1 ; \quad i = 1, 2$$

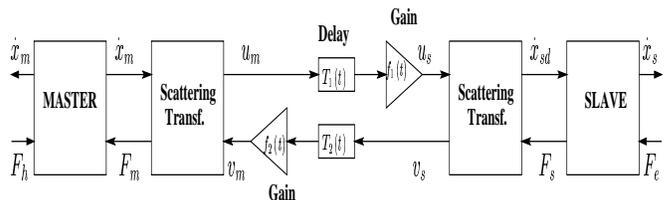
Substituting these equations into (3), the energy stored in the communications is computed as (see [6] for details)

$$\begin{aligned} \int_0^t P_{in}(\tau) d\tau &= \frac{1}{2} \left\{ \int_{t-T_1(t)}^t u_m(\tau)^2 + \int_{t-T_2(t)}^t v_s(\tau)^2 d\tau \right. \\ &\quad - \int_0^{t-T_1(t)} \frac{T'_1(\sigma)}{1-T'_1(\sigma)} u_m(\sigma)^2 d\sigma \\ &\quad \left. - \int_0^{t-T_2(t)} \frac{T'_2(\sigma)}{1-T'_2(\sigma)} v_s(\sigma)^2 d\sigma \right\} \quad (4) \end{aligned}$$

where  $\sigma = \tau - T_i(\tau) := g_i(\tau)$  and  $T'_i(\sigma) := \frac{dT_i}{d\tau} |_{\tau=g_i^{-1}(\sigma)}$ .

The presence of the last two terms in (4) show that passivity is no longer guaranteed for time-varying delays. Consider the modified architecture shown in Figure 2, where a time varying gain  $f_i$  has been inserted after the time varying delay block. The new transmission equations are given by

$$\begin{aligned} u_s(t) &= f_1(t)u_m(t - T_1(t)) \\ v_m(t) &= f_2(t)v_s(t - T_2(t)) \end{aligned}$$



**Figure 2:** Time Varying Gain  $f_i(t)$  inserted in the Communication Channel

Computing the total energy as before yields

$$\begin{aligned}
E = & \frac{1}{2} \left\{ \int_{t-T_1(t)}^t u_m(\tau)^2 + \int_{t-T_2(t)}^t v_s(\tau)^2 d\tau \right. \\
& + \int_0^{t-T_1(t)} \left( \frac{1-T'_1-f_1^2}{1-T'_1} \right) u_m(\sigma)^2 d\sigma \\
& \left. + \int_0^{t-T_2(t)} \left( \frac{1-T'_2-f_2^2}{1-T'_2} \right) v_s(\sigma)^2 d\sigma \right\} \quad (5)
\end{aligned}$$

Hence, if we choose  $f_i^2 = 1 - T'_i$  in the above expressions, the second terms are eliminated and the system is passive. In fact, one can see that passivity is preserved provided the gains,  $f_i$ , are chosen to satisfy

$$f_i^2 \leq 1 - \frac{dT_i}{dt} \quad ; \quad i = 1, 2 \quad (6)$$

### 3 Stability

In this paper we first extend the results of [6] to establish Lyapunov stability of the teleoperator with appropriately selected time-varying gains to passify the communications. It is assumed that

- The human operator and the environment can be modelled as passive systems.
- The operator and the environmental force are bounded by known functions of the master and the slave velocities respectively.
- All signals belong to the  $\mathcal{L}_{2e}$ , the extended  $\mathcal{L}_2$  space.

The master and the slave dynamics are given by

$$\begin{aligned}
M_m \ddot{x}_m + B_m \dot{x}_m &= F_h - F_m \\
M_s \ddot{x}_s + B_{s1} \dot{x}_s &= F_s - F_e \quad (7)
\end{aligned}$$

where  $M_m$  and  $M_s$  are the respective inertias and  $B_m$ ,  $B_{s1}$  represent the master and the slave damping respectively. The state vector of the system is given as

$$x \triangleq (\dot{x}_m | \dot{x}_s | \Delta x) \quad (8)$$

where  $\Delta x = x_{sd} - x_s$ . Define a positive definite Lyapunov function for the system as

$$\begin{aligned}
V = & \frac{1}{2} \{ M_m \dot{x}_m^2 + M_s \dot{x}_s^2 + K_s \Delta x^2 \} + \int_0^t F_e \dot{x}_s d\tau \\
& - \int_0^t F_h \dot{x}_m d\tau + \int_0^t (F_m \dot{x}_m - F_s \dot{x}_{sd}) d\tau \quad (9)
\end{aligned}$$

The human operator and the remote environment are passive (by assumption). Hence

$$\int_0^t F_e \dot{x}_s d\tau \geq 0 \quad ; \quad - \int_0^t F_h \dot{x}_m d\tau \geq 0$$

As the communications are passive (from (5),(6))

$$\int_0^t (F_m \dot{x}_m - F_s \dot{x}_{sd}) d\tau \geq 0$$

Thus the candidate Lyapunov function is positive-definite. The derivative of (9) along trajectories of the system is given by

$$\begin{aligned}
\dot{V} = & M_m \dot{x}_m \ddot{x}_m + M_s \dot{x}_s \ddot{x}_s + K_s \Delta x (\dot{x}_{sd} - \dot{x}_s) \\
& + F_m \dot{x}_m - F_s \dot{x}_{sd} + F_e \dot{x}_s - F_h \dot{x}_m \\
= & \dot{x}_m (-B_m \dot{x}_m + F_h - F_m) + \dot{x}_s (-B_{s1} \dot{x}_s + F_s - F_e) \\
& + K_s \Delta x (\dot{x}_{sd} - \dot{x}_s) + F_m \dot{x}_m - F_s \dot{x}_{sd} + F_e \dot{x}_s - F_h \dot{x}_m \\
= & -B_m \dot{x}_m^2 - B_{s1} \dot{x}_s^2 + (\dot{x}_{sd} - \Delta v) F_s \\
& + (F_s - B_{s2} \Delta v) (\Delta v) - F_s \dot{x}_{sd} \\
= & -B_m \dot{x}_m^2 - B_{s1} \dot{x}_s^2 - B_{s2} \Delta v^2 \leq 0 \quad (10)
\end{aligned}$$

where  $\Delta v = \dot{x}_{sd} - \dot{x}_s$ . As the derivative of the Lyapunov function is negative-semidefinite, the system (7) is stable in the sense of Lyapunov.  $V(x, t)$  is lower bounded, negative-semidefinite and its derivative (10) is uniformly continuous in time. Applying Barbalat's Lemma we see that  $\dot{V}(x, t) \rightarrow 0$  as  $t \rightarrow \infty$ . Using this fact and (10) we get that  $\dot{x}_m, \dot{x}_s$  and  $\Delta v$  asymptotically converge to zero.

### 4 Tracking Performance

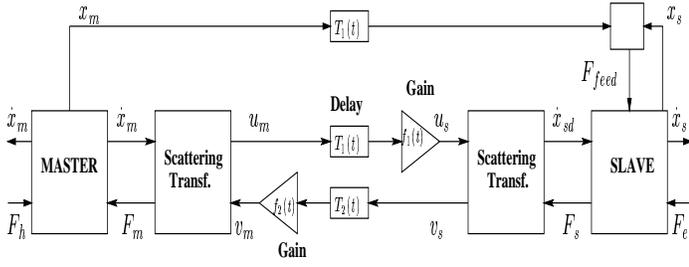
In the previous section asymptotic stability of the teleoperator, as designed in [6] to handle time-varying delays, was demonstrated. The velocity tracking error  $\Delta v$  asymptotically approaches zero but  $\Delta x$ , which is a measure of the position tracking error, is only guaranteed to be stable in the sense of Lyapunov. Furthermore with large delays, as seen in [6], the position tracking becomes quite unsatisfactory. Define the position tracking error as

$$e = x_m(t - T_1(t)) - x_s(t) \quad (11)$$

where  $x_m(t - T_1(t))$  is the delayed master position received on the slave side. To recover tracking performance, we propose a modified control configuration as shown in Figure 3.

**Proposition 1** Consider the additional feedforward control

$$F_{feed} = K_f \text{sat}_p(e) \quad (12)$$



**Figure 3:** Control Architecture with Feedforward Position Control

acting on the slave robot. Then for a range of the gain ( $0 < K_f < K^*$ ) and appropriate initial state  $x(0)$  satisfying

$$V(x(0)) \leq V_{bound}$$

the state of the system (8) remains bounded.

Here  $sat$  is the saturation function defined as

$$\begin{aligned} sat_p(e) &= e & |e| \leq p \\ &= p \frac{e}{|e|} & |e| > p \end{aligned}$$

**Proof** To implement this controller, the master position data are communicated across the channel to the slave side. The equations of motion of the system are now given as

$$\begin{aligned} M_m \ddot{x}_m + B_m \dot{x}_m &= F_h - F_m \\ M_s \ddot{x}_s + B_{s1} \dot{x}_s &= F_{feed} + F_s - F_e \end{aligned} \quad (13)$$

The stability of the system is investigated using the same candidate Lyapunov function (9). Proceeding as before, the derivative along solutions of (13) is given as

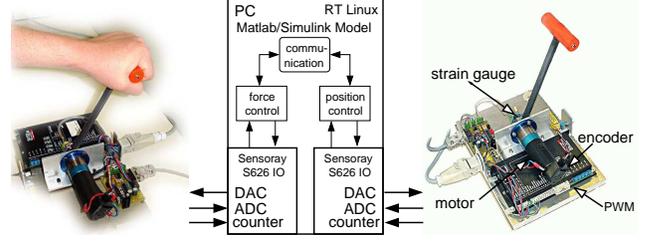
$$\begin{aligned} \dot{V} &= -B_m \dot{x}_m^2 - B_{s1} \dot{x}_s^2 - B_{s2} \Delta v^2 \\ &\quad + K_f sat(x_m(t - T_1(t)) - x_s(t)) \dot{x}_s \\ &\leq -B_m \dot{x}_m^2 - B_{s1} \dot{x}_s^2 - B_{s2} \Delta v^2 \\ &\quad + \frac{1}{2} \{ (sat(x_m(t - T_1(t)) - x_s(t))^2 + (K_f \dot{x}_s)^2 \} \\ &\leq -B_m \dot{x}_m^2 - (B_{s1} - \frac{1}{2} K_f^2) \dot{x}_s^2 - B_{s2} \Delta v^2 + \frac{1}{2} sat(e)^2 \\ &\leq -B_m \dot{x}_m^2 - (B_{s1} - \frac{1}{2} K_f^2) \dot{x}_s^2 - B_{s2} \Delta v^2 + \frac{p^2}{2} \end{aligned}$$

It is easily seen from the above inequality that to ensure stability of the system the feed-forward gain  $K_f$  has to satisfy

$$0 < K_f < \sqrt{2B_{s1}}$$

It follows that  $\dot{V} \leq 0$  if  $\dot{x}_m \geq \frac{p}{\sqrt{2B_m}}$ ,  $\dot{x}_s \geq \frac{p}{\sqrt{2B_{s1} - K_f^2}}$  or  $\Delta v \geq \frac{p}{\sqrt{2B_{s2}}}$ . If all the above inequalities are violated, then from (13) we get,

$$\begin{aligned} |\Delta x| &\leq \frac{|M_s L| + \frac{B_{s1} p}{\sqrt{2B_{s1} - K_f^2}} + \sqrt{\frac{B_{s2}}{2}} p + K_f p + F_{eb}}{K_s} \\ |\Delta x| &\leq c \end{aligned}$$



**Figure 4:** Experimental Testbed

where  $L$  is the Lipschitz constant for the compact set  $\dot{x}_s \leq \frac{p}{\sqrt{2B_{s1} - K_f^2}}$  and  $F_{eb} = \max(F_e)$  over the same set. It is to be noted that the sum of the last three integral terms in (9) is upper bounded (using assumptions and Schwartz inequality). Let this bound be given by  $I_{bound}$ . Define

$$\begin{aligned} V_{bound} &= M_m \frac{p^2}{2\sqrt{B_m}} + M_s \frac{p^2}{\sqrt{4B_{s1} - 2K_f^2}} \\ &\quad + K_s c^2 + I_{bound} \end{aligned}$$

Then all solutions to (13) with initial conditions  $x(0)$  satisfying

$$V(x(0)) \leq V_{bound}$$

remain inside the set defined by

$$V(x(t)) \leq V_{bound}$$

Thus with appropriate feed-forward gain and initial conditions, the state of the system (8) remains bounded. This completes the proof. ■

The boundedness of the tracking error (11) has not been established and is still an open problem. However it can be argued that in steady state with constant master position, slave velocity and acceleration zero, the tracking error goes to zero (from 13).

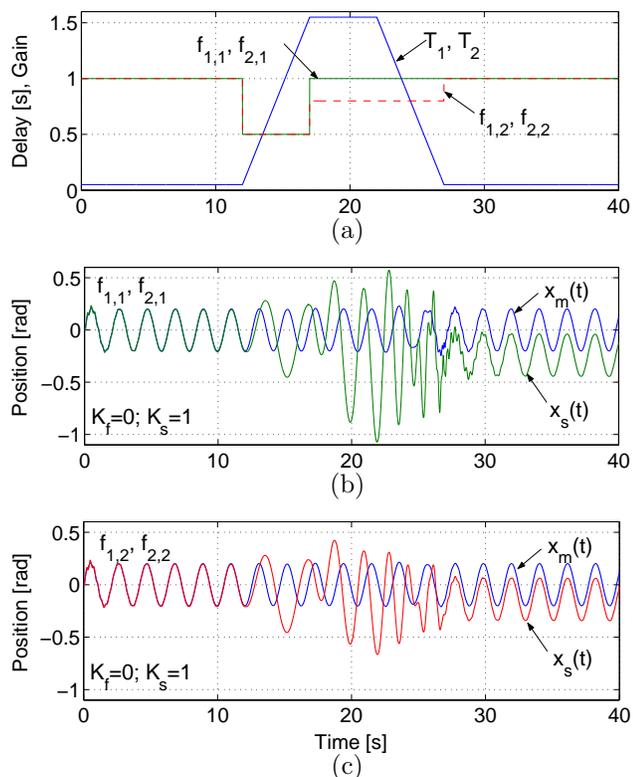
## 5 Experimental Results

In the following experiments the benefits of the additional position feedforward on position tracking are verified.

The experimental testbed consists of two identical single degree of freedom force feedback paddles connected to a PC; the original design of the paddles can be found in [7]. The basic configuration is shown in Fig. 4. The paddle DC motor torque is controlled by the PWM amplifier, which operates in current control with the reference given by a voltage from the D/A converter

output of the I/O board. The force applied to the paddle lever, attached at the motor axis, is measured through the bending of the lever by a strain gauge bridge at the bottom of the lever with the strain being amplified and converted by an A/D converter of the I/O board. The position of the lever, measured by an optic pulse incremental encoder on the motor axis is processed by a quadrature encoder on the I/O board. The control loops and the model of the communication channel are composed of MATLAB/SIMULINK blocksets; standalone realtime code for RT Linux is automatically generated from the SIMULINK model with a sample time  $T_A = 0.001$ s.

All experiments are performed in free space. A substi-

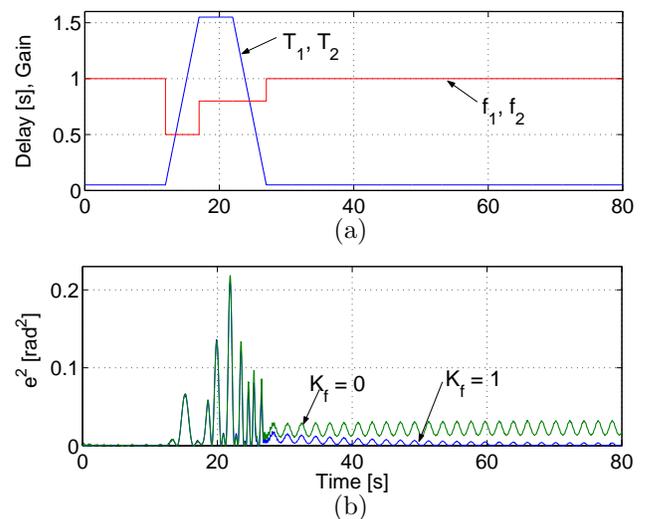


**Figure 5:** Shockwave phenomenon

tute model of the human operator exerts a force to the open loop force controlled master paddle thereby commanding the master position  $x_m(t)$ . The master velocity is transmitted via the passified transmission line with the parameter of the scattering transformation set to  $b = 1$ , in the first experiment there is no position feedforward, i.e.  $K_f = 0$ . The slave is PI velocity controlled with the output  $\dot{x}_{sd}(t)$  of the scattering transformation being the reference and the controller parameters  $B_{s2} = 22$ ,  $K_s = 1$  according to (2). The communication line is modeled by a varying time delay  $T_1(t)$  in the forward and  $T_2(t)$  in the backward path, both rising and then falling again with a constant slope of

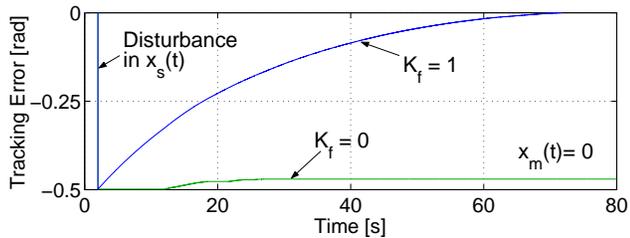
$|T'_1| = |T'_2| = 0.3$ , see Fig. 5(a) and Fig. 6(a). Theoretically, passivity with respect to the time varying delay characteristic is preserved if the gains  $f_1(t), f_2(t)$  satisfy (6), hence they need to be adjusted only when the communication time delay increases, see  $f_{1,1}(t), f_{2,1}(t)$  in Fig. 5(a). In the phases of varying and high delay tracking performance deteriorates, during decreasing delay we observe a shockwave like phenomenon resulting in oscillations of the slave position, a non-recoverable high tracking error remains as shown in Fig. 5(b). After adjusting the gains  $f_{1,2}(t), f_{2,2}(t)$  according to Fig. 5(a) the shockwaves are reduced to an acceptable level, the remaining tracking error is decreased but still non-negligible, see Fig. 5(c).

In the following experiment we examine the effect of the position feedforward with the same conditions as before, see Fig. 6(a), an additional position feedforward with varying gain, first  $K_f = 0$ , then  $K_f = 1$  is introduced. As a contribution to the competitive behavior of the I-control and the position feedforward control the I-controller parameter and the position feedforward gain are set accordingly  $K_s = 1$  if  $K_f = 0$  and  $K_s = 0$  if  $K_f = 1$ . Stability is preserved throughout the experiment. As we have observed before if the master position is not fed forward ( $K_f = 0$ ) then a non-recoverable position drift occurs caused by the varying delay, see Fig. 6(b). In case of additional position feedforward ( $K_f = 1$ ) the tracking error  $e(t)$  decreases and converges to zero.



**Figure 6:** Squared tracking error  $e^2(t)$  without and with position feedforward

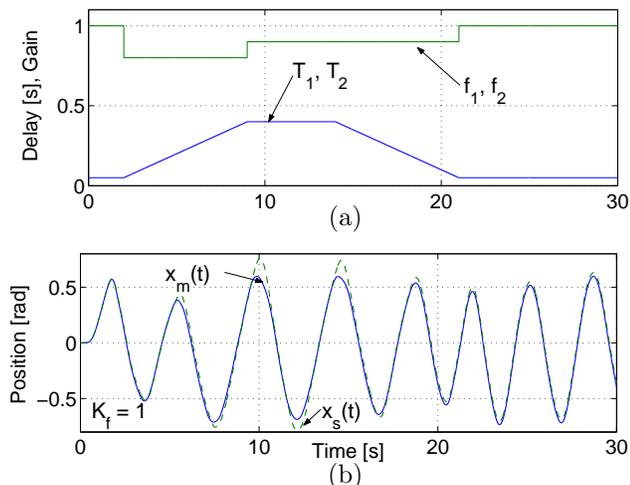
For a constant master position signal  $x_m(t) = 0$  and a disturbance in the slave position at  $t = 2$ s we obtain a similar result: there is a position drift for the standard architecture, the tracking error converges to zero as



**Figure 7:** Tracking error  $e(t)$  for constant master position and disturbance in slave position

predicted in case of additional feedforward, see Fig. 7.

In the last experiment a human operator manipulates the force controlled master paddle. The slope of the delay is set to  $|T'_1| = |T'_2| = 0.06$ , the gains  $f_1(t), f_2(t)$  according Fig. 8(a). Again stability and superior tracking performance is confirmed, see Fig. 8(b).



**Figure 8:** Master and slave position with position feedforward and human in the loop

## 6 Conclusions

In this paper we have extended the scattering formulation for teleoperation over networks with time-varying delays. A novel approach in order to obtain a stable and transparent teleoperation system using time varying gains and position feedforward has been proposed. Lyapunov stability of the passivation scheme of [6] was established. A feed-forward control scheme has been developed which improves position tracking in the system without destabilizing it. However, no bounds on the tracking error were derived. We hope to address this issue in our future research.

## References

- [1] Anderson, R.J., and Spong, M.W., “Bilateral Control of Teleoperators with Time Delay,” *IEEE Transactions on Automatic Control*, AC-34, No. 5, pp. 494–501, May, 1989.
- [2] Anderson, R.J., and Spong, M.W., “Asymptotic stability for force reflecting teleoperators with time delay,” *The International Journal of Robotics Research*, Vol. 11, pp. 135-149, April, 1992.
- [3] Kosuge, K., Murayama, H., and Takeo, K., “Bilateral Feedback Control of Telemanipulators via Computer Network,” *Proc. IROS96*, pp 1380-1385, 1996.
- [4] Niemeyer, G. and Slotine, J., “Stable Adaptive Teleoperation,” *Int. J. of Oceanic Engineering*, Vol 16, No. 1, pp 152-162, 1991.
- [5] Niemeyer, G. and Slotine, J., “Towards Force-Reflecting Teleoperation over the Internet,” *Proc. 1998 ICRA*, Leuven, Belgium, 1998, pp.1909-1915.
- [6] Lozano, R., Chopra, N., and Spong, M.W., “Passivation of Force Reflecting Bilateral Teleoperators with Time Varying Delay,” *Mechatronics’02*, Enschede, Netherlands, June 24-26, 2002
- [7] H. Baier, M. Buss, F. Freyberger, J. Hoogen, P. Kammermeier, and G. Schmidt, “Distributed PC-Based Haptic, Visual and Acoustic Telepresence System—Experiments in Virtual and Remote Environments,” in *Proceedings of the IEEE Virtual Reality Conference VR’99*, (Houston, TX), pp. 118–125, 1999.