EM based Per-Subcarrier ML Channel Estimation for Filter Bank Multicarrier Systems

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***Abstract*—Filter bank based multicarrier (FBMC) systems present an alternative solution to cyclic prefix based orthogonal frequency division multiplexing (CP-OFDM) in wireless environments with multipath propagation. In this contribution, we propose a novel method of per-subcarrier maximum likelihood (ML) narrowband channel estimation as an extension of the scheme recently developed by the same authors. The main difference is that our new estimation method assumes that only the training sequence transmitted in the observed subcarrier is known and unknown data symbols are transmitted in the two immediately adjacent subcarriers. The method is based on the expectation maximization (EM) algorithm and allows iteratively to converge to the ML solution. The main advantage of the method is the increase in the spectral efficiency, since less subcarriers need to be filled with training symbols. Our simulation results show that if enough training and number of iterations are employed, a similar performance to the original ML algorithm, where the 3 subcarriers are filled with training, can be achieved.**

I. Introduction

In this contribution we consider FBMC systems in wireless environments with multipath propagation. In contrast to CP-OFDM, where a rectangular pulse shaping is used, here a finite impulse response (FIR) prototype filter with a longer impulse response than the symbol period, i.e. the filter length is bigger than the total number of subcarriers M. The prototype filter is modulated by complex exponentials. As a consequence, we can achieve more spectrally concentrated subcarriers that overlap only with their neighbors. Moreover, the FBMC system has no inclusion of the CP and as consequence more spectral efficiency is achieved at the cost of higher complexity in the equalization of frequency selective channels.

In order to achieve subcarrier orthogonality, i.e. inter-symbol interference (ISI) and inter-channel interference (ICI)-free received symbols the real and imaginary parts of the input symbols in each subcarrier must be staggered by $T/2$, resulting in the so called Offset-QAM (OQAM) modulation. Furthermore, the prototype filter can be designed according to different goals, like, for example, its spectral properties or its impulse response properties. In this work we restrict to the use of a finite length approximation of the root raised cosine (RRC) filter with unity roll-off, that can be calculated in closed form. This choice of the prototype will indeed introduce some ISI and ICI, but, first, if the filter is long enough the interference is negligible compared to other impairments, like noise and channel response, and secondly, we employ a linear equalizer to compensate for the propagation channel in each subcarrier that also compensates the prototype imperfection.

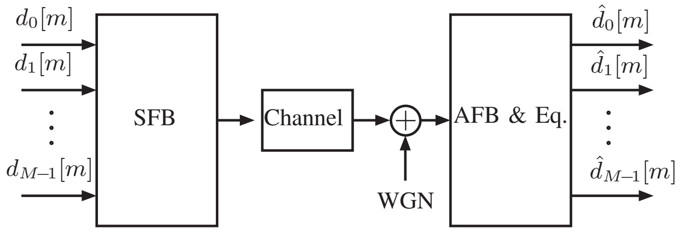

Figure 1. FBMC System Overview.

In [1]–[3], the authors have presented methods to analytically design per-subcarrier fractionally spaced equalizers that combat ISI and ICI inserted by the multipath channel. They have assumed perfect channel impulse response knowledge at the receiver side. In [4] and [5], the authors have presented methods for the estimation of the narrowband multipath channel viewed in each subcarrier. In that works the authors assume that, to estimate the narrowband channel in each subcarrier, not only the trainings sequence in the subcarrier into consideration is known, but also the adjacent overlapping subcarriers have known training sequences transmitted through them. In this contribution, we will extend the results for the ML channel estimator of [5] by considering that only the input sequence in the observed subcarrier is known and the data transmitted in the adjacent subcarrier is treated as interference. To find the ML channel estimation, we employ the iterative method of Expectation Maximization (EM).

II. FBMC System and Subcarrier Model

A high level model of the FBMC system is shown in Fig. 1. This filter bank configuration is known as transmultiplex in the signal processing literature [6]. A synthesis filter bank (SFB) performs a frequency division multiplexing of the QAM data symbols $d_k[m]$ on parallel subcarriers in a rate of $1/T$ at the transmitter. An analysis filter bank (AFB) at the receiver separates the data on each subcarrier. We assume a static frequency selective channel and AWGN between SFB and AFB. Usually some of the M subcarriers are left empty to limit the transmit signal spectrum.

Here we regard exponentially modulated SFB and AFB. This means that only one prototype low-pass filter has to be designed and the other sub-filters are obtained by modulating it as follows [7]

$$h_k[l] = h_\mathrm{P}[l] \exp\left(\mathrm{j}\,\frac{2\pi}{M} k \left(l - \frac{L_\mathrm{P}-1}{2}\right)\right),\ l = 0, \ldots, L_\mathrm{P}-1,$$

ISBN 978-3-8007-3529-7

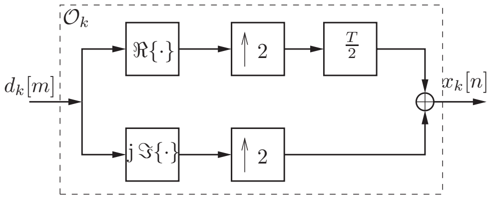

Figure 2. O-QAM staggering for odd indexed subcarrier

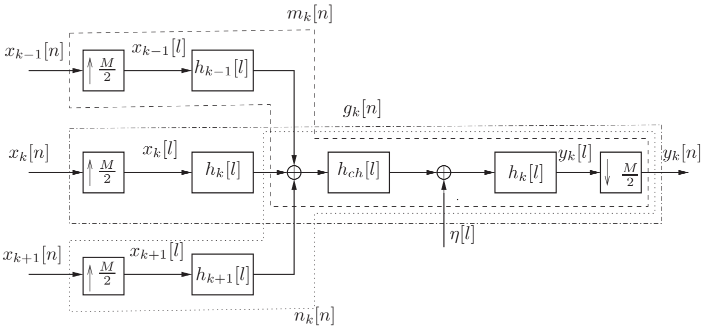

Figure 3. Subcarrier output model

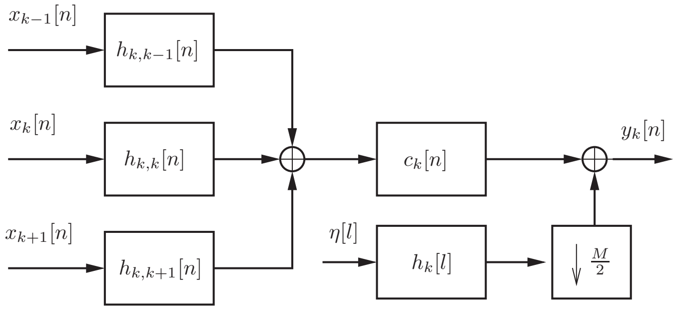

Figure 4. Subcarrier model for FBMC system

where $h_{\mathrm{P}}[l]$ is the impulse response of the prototype filter with length L_{P}. The prototype is chosen as an approximation of a Nyquist filter with roll-off factor one and as a consequence only the spectrum of contiguous subcarriers overlap and non-contiguous subcarriers are separated by a high stop-band attenuation. For example, an FIR RRC with length $L_{\mathrm{P}} = KM + 1$ can be used, where K is the overlapping factor that determines how many symbols superpose each other in time. K should be kept as small as possible to limit the complexity and to reduce the time-domain spreading of the symbols, in particular in case time-variant propagation channels are considered.

Because the prototype filter is longer than the number of subcarriers ($L_{\mathrm{P}} > M$), to maintain the orthogonality between all of them and for all time instants, the complex QAM input symbols $d_k[m]$ need to have their real and complex parts staggered by $T/2$ resulting in the so-called OQAM modulation [8]. The OQAM staggering for even indexed subcarriers is illustrated in Fig. 2. For odd indexed subcarriers the delay of $T/2$ is located at the lower branch with purely imaginary numbers. The OQAM de-staggering is performed at the receiver by the application of flow-graph reversal [9] in Fig. 2, substitution of up-samplers by down-samplers and exchange of blocks $\Re\{\cdot\}$ and $\mathrm{j}\,\Im\{\cdot\}$.

The signals of all subcarriers are up-sampled by $M/2$ after the OQAM staggering, filtered and added. A broadband signal is then generated and digital-to-analog (DA) converted into two IQ baseband signals that will be analog processed (filtered, up-converted to IF and RF, amplified, etc.) and transmitted. At the receiver side the RF signal is amplified, brought to baseband, filtered and then analog-to-digital (AD) converted. The digital received signal is then filtered by the analysis filters and down-sampled by $M/2$ to generate the OQAM staggered subcarrier signals.

The fact we have assumed only contiguous subcarriers overlap in the frequency domain, allows us to construct a model for the output of one subcarrier as shown in Fig. 3. The inputs $x_k[n]$ are OQAM symbols and the received subcarrier signals $y_k[n]$ still have to be equalized and de-staggered before further processing of the resulting QAM symbols. As a consequence, in this subcarrier model the input and output sampling rates are $2/T$. We have assumed here a multipath channel but with perfect time and frequency synchronization. In other words, no time or frequency shifts (Carrier frequency offset or Doppler shift or spread) are present. A more realistic model should involve this and other issues that are out of the scope of this contribution.

III. Per-subcarrier ML Channel Estimation

We assume that a per-subcarrier linear or DFE equalizer is employed and as a consequence a per-subcarrier estimator is sufficient for the equalizers design. In this section, we further assume that training sequences are not only employed in the subcarriers of interest, but also in their contiguous subcarriers. But later we will assume that only the observed subcarrier contains training.

To perform the per-subcarrier channel estimation, we will model the multipath channel viewed at each subcarrier as a narrowband channel with a short impulse response and represent it in a lower sampling rate, namely the double of the QAM symbol rate $2/T$.

Like in [4], [5], we also assume here that the subcarrier model depicted in Fig. 3 can be transformed in the model of Fig. 4. According to this model the received signal at subcarrier k can be arranged in an observation vector of length L_{o} and is given by

$$\begin{aligned}\mathbf{y}_k[n] = (&\mathbf{X}_k[n]\mathbf{H}_{k,k} + \mathbf{X}_{k-1}[n]\mathbf{H}_{k,k-1} \\ &+ \mathbf{X}'_{k+1}[n]\mathbf{H}_{k,k+1})\mathbf{c}_k + \boldsymbol{\Gamma}_k\boldsymbol{\eta}[l],\end{aligned} \tag{1}$$

where $\mathbf{X}_k[n], \mathbf{X}_{k-1}[n], \mathbf{X}_{k+1}[n] \in \mathbb{C}^{L_{\mathrm{o}} \times L_{\mathrm{g}}}$ are Hankel matrices obtained from the training sequences of length $L_{\mathrm{t}} = L_{\mathrm{g}} + L_{\mathrm{o}} - 1$, $\mathbf{H}_{k,k}, \mathbf{H}_{k,k-1}, \mathbf{H}_{k,k+1} \in \mathbb{C}^{L_{\mathrm{g}} \times L_{\mathrm{c}}}$ are Toeplitz matrices containing the resulting filters impulse responses, $L_{\mathrm{g}} = L_{\mathrm{h}} + L_{\mathrm{c}} - 1$, $L_{\mathrm{h}_k} = \left\lceil \frac{2(L_{\mathrm{P}}-1)}{M/2} \right\rceil$, and $\mathbf{c}_k \in \mathbb{C}^{L_{\mathrm{c}}}$ contains the channel impulse response. $\boldsymbol{\Gamma}_k \in \mathbb{C}^{L_{\mathrm{o}} \times (L_{\mathrm{P}} + \lceil \frac{M}{2} L_{\mathrm{o}} \rceil)}$ is obtained by taking each $\frac{M}{2}$-th row of the convolution matrix composed by the analysis filter impulse response $h_k[l]$. This is the reason why the vector $\boldsymbol{\eta}[l] \in \mathbb{C}^{(L_{\mathrm{P}} + \lceil \frac{M}{2} L_{\mathrm{o}} \rceil)}$, which contains white Gaussian noise (WGN) samples with zero mean and variance σ_η^2, is defined in the high sampling rate M/T.

If we call $\mathbf{S}_k[n] = \mathbf{X}_k[n]\mathbf{H}_{k,k}$ and $\mathbf{U}_k[n] = \mathbf{X}_{k-1}[n]\mathbf{H}_{k,k-1} + \mathbf{X}'_{k+1}[n]\mathbf{H}_{k,k+1}$ and drop the time and

ISBN 978-3-8007-3529-7

subcarrier index for notation simplification we get

$$\mathbf{y}=(\mathbf{S}+\mathbf{U})\mathbf{c}+\boldsymbol{\nu}. \quad (2)$$

We can see that in the linear model of (2) the noise is zero mean Gaussian distributed with covariance matrix $\mathbf{R}_{\nu\nu} = \sigma_\eta^2 \mathbf{\Gamma}\mathbf{\Gamma}^{\mathrm{H}}$ and the observation $\mathbf{y}$ given $\mathbf{c}$ is Gaussian distributed. The maximum likelihood (ML) estimate of $\mathbf{c}$ in this case is given by

$$\hat{\mathbf{c}}_{\mathrm{ML}} = \arg\max_{\mathbf{c}} p(\mathbf{y}|\mathbf{c}) = \arg\min_{\mathbf{c}} J(\mathbf{c}) \quad (3)$$

where

$$J(\mathbf{c}) = (\mathbf{y}-(\mathbf{S}+\mathbf{U})\mathbf{c})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{y}-(\mathbf{S}+\mathbf{U})\mathbf{c}). \quad (4)$$

If $\mathbf{R}_{\nu\nu}$ is independent of $\mathbf{c}$ and $((\mathbf{S}+\mathbf{U})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\mathbf{U}))$ is invertible, as we will assume here, we just need to apply the derivative and make it equal to zero

$$\frac{\partial J(\mathbf{c})}{\partial \mathbf{c}^{\mathrm{H}}} = (\mathbf{S}+\mathbf{U})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\mathbf{U})\mathbf{c} - (\mathbf{S}+\mathbf{U})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y} = \mathbf{0}, \quad (5)$$

Then the ML estimate of the narrowband multipath channel in each subcarrier is given by

$$\hat{\mathbf{c}} = ((\mathbf{S}+\mathbf{U})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\mathbf{U}))^{-1}(\mathbf{S}+\mathbf{U})^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y}. \quad (6)$$

We have shown in [4], [5] that the estimator in (6) works fine provided enough training is employed in the three consecutive subcarriers. We should note that the channel length is a design parameter of the estimator. It can be different for each subcarrier depending on how frequency selective the channel is for the corresponding portion of the spectrum in the subcarrier.

IV. Expectation Maximization ML Channel Estimation

The entries of $\mathbf{S}$ in (2) are a result of the convolution between the training in subcarrier k and the resulting subcarrier filter. Although the resulting filter has a purely real impulse response if the prototype has a purely real impulse response, the resulting convolution with the O-QAM staggered training sequence results in a complex signal. Consequently all entries of $\mathbf{S}$ are complex valued. In contrast, the entries of $\mathbf{U}$ are alternating purely real and purely imaginary, because in addition to the O-QAM data signals in subcarriers $k-1$ and $k+1$ also the impulse response between the two adjacent subcarriers has alternating real and imaginary coefficients. As a consequence, the interference term $\mathbf{u}$ has improper statics [10] and if one is supposed to estimate it, a Widely Linear (WL) processing can be employed.

Now, we can rewrite the observation vector in (2) on each subcarrier as

$$\mathbf{y} = \mathbf{S}\mathbf{c} + \mathbf{C}\mathbf{u} + \boldsymbol{\nu} = \mathbf{S}\mathbf{c} + \mathbf{C}'\mathcal{O}\mathbf{H}_{\mathrm{u}}\mathbf{x}_{\mathrm{u}} + \boldsymbol{\nu}, \quad (7)$$

where now the vector $\mathbf{u} \in \mathbb{R}^{L_{\mathrm{g}}}$, or equivalently $\mathbf{x}_{\mathrm{u}}$, in each subcarrier is assumed as unknown, $\mathbf{C} = \mathbf{C}'\mathcal{O}$ and $\mathbf{C}'$ is a convolution matrix containing the channel impulse response $\mathbf{c}$. We also use the definition

$$\mathcal{O} = \begin{cases} \mathrm{diag}[1, \mathrm{j}, 1, ...], & \text{for } k \text{ even} \\ \mathrm{diag}[\mathrm{j}, 1, \mathrm{j}, ...], & \text{for } k \text{ odd.} \end{cases} \quad (8)$$

Note that in this case $\mathbf{H}_{\mathrm{u}} \in \mathbb{R}^{L_{\mathrm{g}} \times 2L_{\mathrm{t}}}$ is a purely real matrix related to the matrices $\mathbf{H}_{k,k-1}$ and $\mathbf{H}_{k,k+1}$ in 1.

Although the exact values of $\mathbf{u}$ are unknown, we assume here that its statistics are known. We define the interference covariance matrix as

$$\mathbf{R}_{uu} = \frac{\sigma_d^2}{2}\mathbf{H}_{\mathrm{u}}\mathbf{H}_{\mathrm{u}}^{\mathrm{H}}, \quad (9)$$

where we have assumed $\mathbf{x}_{\mathrm{u}}$ to be i.i.d. and Gaussian distributed with zero mean and variance $\sigma_d^2/2$. This is usually a good approximation although we know that $\mathbf{x}_{\mathrm{u}}$ is composed by symbols taken from a finite dictionary.

For the linear model of (7) the ML estimator has no closed solution and an efficient way to reach its performance is to employ the iterative expectation maximization (EM) algorithm [11].

The EM algorithm works here as follows. Before the first iteration, a rough channel estimation is obtained that ignores the interference and only considers the training sequence in the observed subcarrier. This estimation is given by

$$\hat{\mathbf{c}}_0 = (\mathbf{S}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{S})^{-1}\mathbf{S}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y}. \quad (10)$$

Then, the iterative process starts. For each iteration i the algorithm is divided into two steps: the E-step and the M-step.

In the E-step an approximation of the ML function (here its derivative) is obtained by taking the expected value of it conditioned to the channel estimated in the iteration before and the observed sequence as follows

$$\mathrm{E}_{\mathbf{u}|\mathbf{y},\mathbf{c}_i}\left[\frac{\partial J(\mathbf{c}_i)}{\partial \mathbf{c}_i^{\mathrm{H}}}\right] = \Big(\mathbf{S}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\mathrm{E}[\mathbf{U}]) + \mathrm{E}[\mathbf{U}]^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{S} + \mathrm{E}[\mathbf{U}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{U}]\Big)\mathbf{c}_i - (\mathbf{S}+\mathrm{E}[\mathbf{U}])^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y}. \quad (11)$$

As one can see the result is a function of $\mathrm{E}[\mathbf{u}]$ and $\mathrm{E}[\mathbf{U}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{U}]$. $\mathrm{E}[\mathbf{u}] = \hat{\mathbf{u}}_i$ can be viewed as an instantaneous estimate of the interference term $\mathbf{u}$ in the i-th iteration.

To express the last expectation in (11) also in terms of the estimate $\hat{\mathbf{u}}_i$, we have first to write the matrix $\mathbf{U}$ as a function of vector $\mathbf{u}$:

$$\mathbf{U} = \sum_{j=1}^{L_{\mathrm{c}}} \mathbf{D}_j \mathcal{O}\mathbf{u}\mathbf{e}_j^{\mathrm{T}}, \quad (12)$$

where $\mathbf{D}_j = [\mathbf{0}_{j-1}\mathbf{I}_{L_{\mathrm{t}}}\mathbf{0}_{L_{\mathrm{o}}-L_{\mathrm{t}}-j}]$ is a matrix that selects rows of $\mathbf{u}$ and $\mathbf{e}_j$ is a vector with one in the j-th row and the rest of its elements are zero.

Then we plug (12) in the expected value

$$\begin{aligned}\mathrm{E}\big[\mathbf{U}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{U}\big] &= \mathrm{E}\left[\sum_{l=1}^{L_{\mathrm{c}}}\mathbf{e}_l\mathbf{u}^{\mathrm{T}}\mathcal{O}^{\mathrm{H}}\mathbf{D}_l^{\mathrm{T}}\mathbf{R}_{\nu\nu}^{-1}\sum_{j=1}^{L_{\mathrm{c}}}\mathbf{D}_j\mathcal{O}\mathbf{u}\mathbf{e}_j^{\mathrm{T}}\right] \\ &= \sum_{l=1}^{L_{\mathrm{c}}}\mathbf{e}_l\sum_{j=1}^{L_{\mathrm{c}}}\mathrm{E}\left[\mathbf{u}^{\mathrm{T}}\mathcal{O}^{\mathrm{H}}\mathbf{D}_l^{\mathrm{T}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{D}_j\mathcal{O}\mathbf{u}\right]\mathbf{e}_j^{\mathrm{T}} \\ &= \sum_{l=1}^{L_{\mathrm{c}}}\mathbf{e}_l\sum_{j=1}^{L_{\mathrm{c}}}\mathrm{tr}\left(\mathcal{O}^{\mathrm{H}}\mathbf{D}_l^{\mathrm{T}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{D}_j\mathcal{O}\mathrm{E}\big[\mathbf{u}\mathbf{u}^{\mathrm{T}}\big]\right)\mathbf{e}_j^{\mathrm{T}}.\end{aligned} \quad (13)$$

ISBN 978-3-8007-3529-7

We can further say that

$$\mathrm{E}\big[\mathbf{u}\mathbf{u}^{\mathrm{T}}\big] = \mathbf{R}_{\epsilon\epsilon,i} + \mathrm{E}[\mathbf{u}]\mathrm{E}[\mathbf{u}]^{\mathrm{T}} = \mathbf{R}_{\epsilon\epsilon,i} + \hat{\mathbf{u}}_i\hat{\mathbf{u}}_i^{\mathrm{T}}, \tag{14}$$

where $\mathbf{R}_{\epsilon\epsilon,i}$ is the covariance matrix of the estimation error of $\mathbf{u}$ in the i-th iteration. Consequently, we can write

$$\begin{aligned}\mathrm{E}\big[\mathbf{U}^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{U}\big] &= \sum_{l=1}^{L_c}\mathbf{e}_l\sum_{j=1}^{L_c}\mathrm{tr}\left(\mathcal{O}^{\mathrm{H}}\mathbf{D}_l^{\mathrm{T}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{D}_j\mathcal{O}\mathbf{R}_{\epsilon\epsilon,i}\right)\mathbf{e}_j^{\mathrm{T}}\\ &\quad+\mathrm{E}[\mathbf{U}]^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathrm{E}[\mathbf{U}],\\ &=\mathbf{\Psi}_i+\mathrm{E}[\mathbf{U}]^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathrm{E}[\mathbf{U}],\end{aligned} \tag{15}$$

where $[\mathbf{\Psi}_i]_{j,l}=\mathrm{tr}(\mathcal{O}^{\mathrm{H}}\mathbf{D}_j^{\mathrm{T}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{D}_l\mathcal{O}\mathbf{R}_{\epsilon\epsilon,i})$. We finally get

$$\begin{aligned}\mathrm{E}_{\mathbf{u}|\mathbf{y},\mathbf{c}_i}\left[\frac{\partial J(\mathbf{c}_i)}{\partial \mathbf{c}_i^{\mathrm{H}}}\right]&=\left((\mathbf{S}+\hat{\mathbf{U}}_i)^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\hat{\mathbf{U}}_i)+\mathbf{\Psi}_i\right)\mathbf{c}_i\\ &\quad-(\mathbf{S}+\hat{\mathbf{U}}_i)^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y}.\end{aligned} \tag{16}$$

Given that an estimation of the channel and the training sequence are known, we can subtract this signal from the observation signal. Then we can process the resulting vector with a minimum mean squared error (MMSE) estimator. As we mentioned before, the interference vector is composed by purely real and purely imaginary terms, and for that a WL MMSE estimation [10] can be employed and it is given by

$$\mathrm{E}[\mathbf{u}_i] = \hat{\mathbf{u}}_i = 2\Re\{\mathbf{W}^{\mathrm{H}}(\mathbf{y}-\mathbf{S}\hat{\mathbf{c}}_i)\}, \tag{17}$$

where after some calculation one obtains

$$\mathbf{W} = (\mathbf{R}_{yy} - \mathbf{P}_{yy}^{\mathrm{T}}\mathbf{R}_{yy}^{-\mathrm{T}}\mathbf{P}_{yy}^{\mathrm{H}})^{-1}(\mathbf{R}_{uy}^{\mathrm{H}} - \mathbf{P}_{yy}^{\mathrm{T}}\mathbf{R}_{yy}^{-\mathrm{T}}\mathbf{P}_{uy}^{\mathrm{H}}), \tag{18}$$

and we can show that the covariance matrices are defined by

$$\mathbf{R}_{yy} = \hat{\mathbf{C}}_i\mathbf{R}_{uu}\hat{\mathbf{C}}_i^{\mathrm{H}} + \mathbf{R}_{\nu\nu},\ \text{and}\ \mathbf{R}_{uy} = \mathbf{R}_{uu}\hat{\mathbf{C}}_i^{\mathrm{H}}, \tag{19}$$

and the pseudo-covariance matrices by

$$\mathbf{P}_{yy} = \hat{\mathbf{C}}_i\mathbf{R}_{uu}\hat{\mathbf{C}}_i^{\mathrm{T}},\ \text{and}\ \mathbf{P}_{uy} = \mathbf{R}_{uu}\hat{\mathbf{C}}_i^{\mathrm{T}}. \tag{20}$$

where $\hat{\mathbf{C}}_i = \hat{\mathbf{C}}_i'\mathcal{O}$ and $\hat{\mathbf{C}}_i'$ is a convolution matrix containing the estimated channel impulse response $\hat{\mathbf{c}}_i$. Moreover, the error covariance matrix of the WL MMSE estimation is

$$\mathbf{R}_{\epsilon\epsilon,i}=\mathbf{R}_{uu} - 2\Re\{\mathbf{W}\mathbf{R}_{uy}^{\mathrm{H}}\}. \tag{21}$$

Finally, the M-step is performed, where $J(\mathbf{c}_i)$ is minimized resulting in the new channel estimate

$$\hat{\mathbf{c}}_{i+1} =((\mathbf{S}+\hat{\mathbf{U}}_i)^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}(\mathbf{S}+\hat{\mathbf{U}}_i)+\mathbf{\Psi}_i)^{-1}(\mathbf{S}+\hat{\mathbf{U}}_i)^{\mathrm{H}}\mathbf{R}_{\nu\nu}^{-1}\mathbf{y}. \tag{22}$$

The estimation of $\mathbf{u}$ and $\mathbf{c}$ are then repeated N_{EM} times until convergence is achieved.

V. Numerical Results

To evaluate the performance of the narrowband ML per-subcarrier channel estimation employing the EM algorithm, we have performed some numerical simulations. As broadband multipath channel model, we have chosen static ITU vehicular A at a bandwidth of 10 MHz and sampling rate $M/T = 15.36$ MHz. The FBMC system was employed with $M = 256$ subcarriers from which 210 were occupied with training and symbols. The resulting subcarrier spacing is 60 kHz. We have assumed a preamble based channel estimation, where the number of observations L_{o} determines the length of the training. For the prototype an RRC filter with $K = 4$ ($L_{\mathrm{P}} = 1025$) and unitary roll-off was chosen. With this configuration and scenario a length of $L_{\mathrm{c}_k} = 5$ for the narrowband multipath channels and $L_{\mathrm{e}} = 5$ for the equalizers have shown to be sufficient for all subcarriers. We have chosen 16-QAM constellation for FBMC and 32-QAM for CP-OFDM. Moreover, the CP length was $L_{cp} = 64$. With this combination of constellation size and CP length, both systems achieve the same data throughput. For the training sequences QPSK symbols were employed.

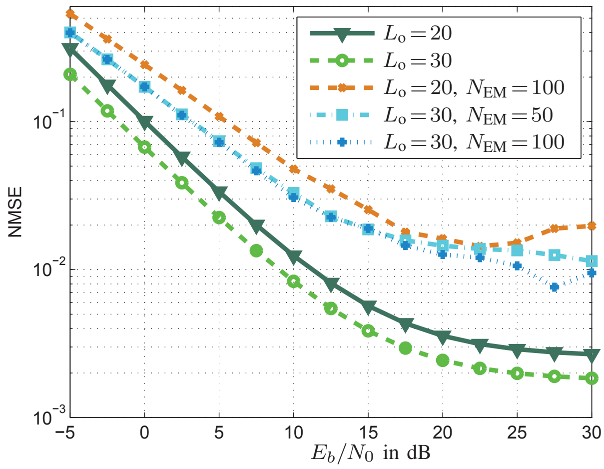

Figure 5. NMSE as a function of E_b/N_0 for different estimators parameters

We have performed Monte-Carlo simulations with 200 channel realizations for different E_b/N_0 and transmitted 100 symbols on each subcarrier after the random selected training symbols.

In Fig. 5, the normalized MSE (NMSE) of the narrowband channel estimation as a function of E_b/N_0 is depicted for different observation lengths L_{o} and for different number of iterations N_{EM}. Also the results for the ML narrowband channel estimation with known training on the adjacent subcarriers are shown as references. We can observe a considerable loss in E_b/N_0 that increases with the decrease of the noise power.

For the same simulations as in Fig. 5, we show in Fig. 6 the BER as a function of E_b/N_0. Here we have also included the results for perfect channel state information (PCSI). We can observe here that for the BER the gap between the two estimation methods is not so high as in the NMSE case. We can also note that for sufficient number of observations and iterations, quiet good estimates of the narrowband channels can be achieved.

Although the convergence of the EM algorithm is not always guaranteed in general we observe from the numerical results that the proposed algorithm provides practcally always a stable solution after a certain number of iterations.

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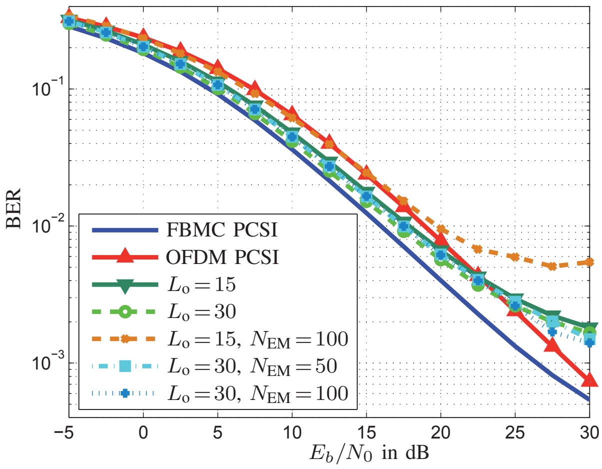

Figure 6. BER as a function of E_b/N_0 for different estimators parameters

VI. Conclusions

We have presented a novel method of per-subcarrier narrowband frequency selective channel estimation. It is based on the Expectation Maximization algorithm to iteratively approach the performance of the maximum likelihood estimator. Furthermore, the method assumes that only training symbols contained in the subcarrier of interest are known. This can be very useful in situations where training is sparsely distributed along the subcarriers. In this case an interpolation is necessary to be employed in order to obtain the channels in the data-only filled subcarriers.

Our simulations have shown that the performance in terms of BER can really approach to the performance of the less spectrally efficient estimator from our previous contributions, provided that enough training or observations and iterations are employed.

While an increase in the length of the training reduces the spectral efficiency, the number of iterations only increases the computational complexity. After a critical number of observations, and consequently training length, is reached, only the number of iterations will influence the BER performance regardles of the SNR.

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