Analysis of Competitive Electricity Markets under a New Model of Real-Time Retail Pricing with Ex-Post Adjustment

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***Abstract*—In this paper, we propose a new real-time retail pricing model characterized by ex-post adjustments to exanté price, and investigate the stability and efficiency properties of the ensuing closed loop system. Under this pricing mechanism, electricity is priced at the exanté price (calculated based on predicted demand) up to the amount consumed at the previous time period. Any deviation of the demand from the previous time period is penalized or reimbursed at the ex-post price (calculated based on actual demand, after consumption). It is assumed that the exanté and ex-post prices are calculated based on the aggregate consumption of the population. Therefore, although an individual consumer is a price-taker, he might adjust his behavior strategically based on the mean consumption of the population. Within this class of pricing mechanisms we investigate the social welfare and price stability properties. Simulation is used to show that the approximate dynamics with individual-mass interaction has better stability and robustness properties than pure exanté pricing.**

***Index Terms*—Real-Time Pricing, Market Stability, Economic Efficiency.**

I. Introduction

A paramount attribute of *smart grids* in comparison to today's electric power systems is an advanced communication and IT infrastructure, which enables multi-directional flow of information between consumers, producers, and system operators, and allows for a refined granularity of monitoring and control. This, in turn, would enable active participation of consumers in real-time balancing of demand and stochastically fluctuating supply. However, presence of a complex demand structure with uncertain preferences and dependence on price and other incentives poses significant challenges to realizing these potentials. Pricing mechanisms and associated markets that guarantee stability and economic efficiency are difficult to design under endogenous and exogenous uncertainties arising from uncertain consumer behavior and stochastic uncertainties that are inherent to renewable energy supply [1] [13] [5]. This paper is a step toward the design of dynamic pricing mechanisms with suitable stability and economic efficiency properties.

The existing body of literature on dynamic pricing in transportation or communication networks is extensive. See for instance [9], [4], [11] and the references therein. However, application of the underlying concepts and tools available in these fields to electric power grids poses new formidable challenges due to the asymmetry of information, the safety-critical nature of the system, and the firm coupling between physics and economics. There has been some recent work on the interaction of economics and physics. For instance, in [7] and [8], a framework is proposed for translating global objectives and constraints into real-time prices based on first-order optimality conditions, and applications to optimal power flow and transmission network congestion control problems are considered. However, the key challenges, i.e., analysis and design of controllers with provable stability and performance properties, are yet to be addressed. With the targeted scale of granularity of real-time monitoring and control up to the individual consumer level, stability and economic analysis of real-time pricing mechanisms is a priority.

In the context of stability, recent work on price volatility and stability in power systems has shown that real-time retail pricing of electricity, characterized by relaying the wholesale electricity prices to the end consumers, creates a closed-loop feedback system, which could be unstable or non-robust [12] [13]. The framework of [13] considers the consumer as an autonomous agent who myopically adjusts her usage in response to the price signal in order to maximize a concave quasi-linear utility function. The adjusted demand is, in turn, a feedback signal to the wholesale market and affects the prices for the next time step. Under exanté pricing, the price per unit of consumption is announced at the beginning of the pricing interval and, therefore, must be calculated based on predicted demand. It is binding and cannot change until the next pricing interval. The results indicate that price volatility is a function of the system's *relative price elasticity,* defined as the maximal ratio of the price-elasticity of consumers to the price elasticity of producers. As this ratio increases, the system becomes more volatile, eventually becoming unstable as the relative price elasticity approaches to one. In practice, instability—in its dynamic sense—could manifest itself as extreme volatility in the spot market prices along with wild fluctuations of demand [13]. The same notion of relative price elasticity can be used to characterize the system's robustness to external disturbances. The higher the relative price elasticity is, the higher is the incremental L2-gain from the external disturbances to the market prices. The system could be stabilized and volatility could be reduced in many different ways, e.g., via static or dynamic controllers regulating the interaction of wholesale markets and retail consumers. However, different pricing mechanisms pose different consequences on competing factors of interest.

In this paper we introduce a new pricing mechanism characterized by ex-post adjustments to exanté prices. The exanté price is calculated based on predicted demand, which is assumed to be equal to the demand during the previous pricing interval. The ex-post price is calculated at the end of each dispatch interval based on the actual accumulated consumption, which could exceed or recede the predicted demand. In our proposed pricing mechanism, electricity is priced at the exanté price up to the amount consumed at the previous time period. Only the deviation of the demand from the previous time period is penalized or reimbursed at the ex-post price. Therefore, although an individual consumer is a price-taker, he might adjust his behavior strategically based on the mean consumption of the population. Our model invokes ideas from mean-field stochastic control and results in a Nash-Cournot type dispatch model. Representation of a large number of interacting consumers by a mean-field is an adequate and plausible approximation and can be used to overcome computational intractability that generally constrains the applicability of algorithms seeking for best response dynamics to obtain Markov perfect equilibria [3]. Moreover, individual control strategies only based on individual-mass interaction can be designed such, that the population's mean state reaches an optimum in the game theoretic sense of Nash [6].

The organization of the paper is as follows. In Section II, we describe the market structure followed by the dynamics of market operation in Section III. In Section IV, we present and discuss results obtained from simulation in terms of price stability, robustness and market efficiency. Finally, in Section V we present our conclusions and an outlook for future work.

II. Preliminaries

A. Notation

The set of positive real numbers (integers) is denoted by $\mathbb{R}_+$ ($\mathbb{Z}_+$), and nonnegative real numbers (integers) by $\overline{\mathbb{R}}_+$ ($\overline{\mathbb{Z}}_+$). For a differentiable function $f : \mathbb{R}^n \to \mathbb{R}^m$, we use $\dot{f}$ to denote the Jacobian matrix of f. When f is a scalar function of a single variable, $\dot{f}$ simply denotes the derivative of f with respect to its argument: $\dot{f}(x) = df(x)/dx$.

B. Market Structure

The electricity market model adopted in this paper has three participants: (a) The producers, (b) The consumers, and (c) An independent system operator (ISO).

1) The Consumers and the Producers: Let $D = \{1, ..., N\}$ and $S = \{1, ..., M\}$ denote the index sets of consumers and producers respectively. Each consumer $j \in D$ is associated with a strictly increasing, strictly concave value function $v_j : \overline{\mathbb{R}}_+ \to \mathbb{R}$, where $v_j(x)$ can be thought of as the monetary value that consumer j derives from consuming x units of electricity. Similarly, to each producer $i \in S$, a cost function is assigned $c_i : \overline{\mathbb{R}}_+ \to \overline{\mathbb{R}}_+$, which is strictly increasing, and strictly convex, and represents the monetary cost of production per unit.

Let $d_j : \mathbb{R}_+ \to \overline{\mathbb{R}}_+$, $j \in D$, and $s_i : \mathbb{R}_+ \to \overline{\mathbb{R}}_+$, $i \in S$ be demand and supply functions mapping price to consumption and production, respectively. In the framework adopted in this paper, the producers and consumers are price-taking, utility-maximizing agents. Therefore, letting λ be the price per unit of electricity, we define

$$\begin{aligned} d_j(\lambda) &= \arg\max_{x \in \overline{\mathbb{R}}_+} \; v_j(x) - \lambda x, \qquad j \in D, \qquad (1) \\ &= \max\{0, \{x \mid \dot{v}_j(x) = \lambda\}\} \end{aligned}$$

and

$$\begin{aligned} s_i(\lambda) &= \arg\max_{x \in \overline{\mathbb{R}}_+} \; \lambda x - c_i(x), \qquad i \in S. \qquad (2) \\ &= \max\{0, \{x \mid \dot{c}_i(x) = \lambda\}\} \end{aligned}$$

In the interest of simplicity in notation and technical development, we will assume in the remainder of this paper that $d_j(\lambda) = \dot{v}^{-1}(\lambda)$ and $s_i(\lambda) = \dot{c}^{-1}(\lambda)$. This can be mathematically justified by assuming that $\dot{v}(0) = \infty$, and $\dot{c}(0) = 0$, or that $\lambda \in [\dot{c}(0), \dot{v}(0)]$.

Definition 1: The social welfare $\mathcal{S}$ is the aggregate benefit of the producers and the consumers:

$$\mathcal{S} = \sum_{j \in D} (v_j(d_j) - \lambda_j d_j) - \sum_{i \in S} (\lambda_i s_i - c_i(s_i))$$

If $\lambda_i = \lambda_j = \lambda$, $\forall i, j$, we say that λ is a uniform market clearing price, and in this case, we have:

$$\mathcal{S} = \sum_{j \in D} v_j(d_j) - \sum_{i \in S} c_i(s_i)$$

2) The Independent System Operator (ISO): The ISO is an independent, profit-neutral player in charge of optimally matching supply and demand, that is, maximizing the social welfare subject to reliability and physical constraints. The constraints include power flow constraints, transmission line constraints, generator capacity constraints, and local and system-wide reserve capacity requirements. A set of Locational Marginal Prices (LMP) emerge as the dual variables corresponding to the nodal power balance constraints of the underlying optimization problem. The interested readers are referred to [14], [16], [12], [15] for more details.

a) Real-Time Market Operation: We consider the case of real-time market operation and assume that price-sensitive consumers do not bid in the real-time market, though they may adjust their consumption in response to real-time wholesale market prices. In this case, the demand is taken as fixed over each pricing interval, and supply is matched to demand. Furthermore, in order to obtain simplified models that highlight the effects of the behavior of producers and consumers on system stability, price volatility, and system efficiency, we will make the following simplifying assumptions: (a) Resistive losses are negligible, (b) The line capacities are high enough, (c) The generator capacities are high enough, and (d) There are no reserve capacity requirements. The ISO's real-time optimization problem reduces to meeting the fixed demand at minimum cost:

$$\begin{aligned} \min \quad & \sum_{i\in S} c_i(s_i) \\ \text{s.t.} \quad & \sum_{i\in S} s_i = \sum_{j\in D} \hat{d}_j \end{aligned} \tag{3}$$

where $\hat{d}_j$ is the predicted demand of consumer j for the next time period. We assume that the system operator solves (3) and sets the price to the marginal cost of production at the minimum cost solution, i.e., the Lagrangian multiplier corresponding to the balance constraint[1]. This constitutes the basics of the real-time market clearing model adopted in this paper. More specific details regarding dynamic extensions of this model are presented in the next Section.

III. Dynamic Supply-Demand Model

In this section, we present the details of the real-time retail pricing models under examination. These models are consistent with the practice of marginal cost pricing in wholesale electricity markets, with the additional feature that the consumers adjust their usage based on the real-time wholesale market prices. The practice of defining the clearing price corresponding to each pricing interval based on the predicted demand at the beginning of the interval is called exanté pricing. As opposed to this, ex-post pricing refers to the practice of defining the clearing price for each pricing interval based on the materialized consumption at the end of the interval.

A. A New Pricing Model with Ex-Post Adjustment

Under exanté pricing, the consumers are guaranteed a price at the beginning of the interval, though the ISO needs to reimburse the generators based on the actual marginal cost of production, i.e., the ex-post price [13]. In what follows, we present a new pricing model, where the price uncertainty and the associated risk is shared. That producers guarantee the supply to balance the predicted demand at a fixed price, and excess or shortfall in consumption are charged at the ex-post price.

[1]By setting the market price to be the Lagrangian multiplier corresponding to the balance constraint, the system operator creates a competitive environment in which, the collective selfish behavior of the participants results in a system-wide optimal condition. In other words, the aggregate surplus is maximized while each agent maximizes her own net benefit. [9], [13]

Let $\hat{d}_t$ and d_t represent, respectively, the predicted and observed demands for the interval $[t, t+1]$, $t \in \mathbb{Z}_+$. Let $\hat{d}_t = d_{t-1}$, $\forall t$, that is, the predicted demand for each period is equal to the demand for the previous period. Therefore, when there is only one producer, $\lambda_t = \dot{c}(\hat{d}_t) = \dot{c}(d_{t-1})$ represents the exanté price. Any deviation δ_t of the consumer from previous consumption d_{t-1} will be charged at the ex-post price $\lambda_{t+1} = \dot{c}(d_{t-1} + \delta_t)$. Hence, at time t, the total cost to the consumer is $\lambda_t d_{t-1} + \lambda_{t+1}\delta_t$. Now, assume that the consumer optimizes energy usage according to:

$$d_t = \arg\max_{\delta_t} v(d_{t-1} + \delta_t) - \delta_t \hat{\lambda}_{t+1} - \lambda_t d_{t-1} \tag{4}$$

$$= \arg\max_{\delta_t} v(d_{t-1} + \delta_t) - \delta_t \hat{\lambda}_{t+1} \tag{5}$$

where $\hat{\lambda}_{t+1}$ denotes the ex-post price anticipated by the consumer. For the remainder of this paper, for simplicity of notation, we assume that there is only one producer with cost function c.

1) Single Stage Game-Theoretic Formulation: In the case of N agents playing a single-stage game, $d = \left(d^1, \cdots, d^N\right)$ and $\delta = \left(\delta^1, \cdots, \delta^N\right)$ are vectors for the demands and deviations for the agents. The utility function for agent i is then

$$U_i(\lambda, d) = -\lambda d^i + \max_{\delta^i} \left\{ v_i\left(d^i + \delta^i\right) - \delta^i \dot{c}\left(\sum_{k=1}^N d^k + \delta^k\right)\right\}$$

Define

$$W_i(d, \delta) = v_i\left(d^i + \delta^i\right) - \delta^i \dot{c}\left(\sum_{k=1}^N d^k + \delta^k\right)$$

If there exists an optimizing $\delta^* = \left(\delta^{1*}, \cdots, \delta^{N*}\right)$, then it is assumed that δ^* satisfies

$$\left.\frac{\partial W_i}{\partial \delta^i}\right|_{\delta=\delta^*} = 0 \quad \forall i$$

We then have the following necessary and sufficient condition for δ^* to be a Nash equilibrium:

$$\dot{v}_i\left(d^i + \delta^{i*}\right) - \ddot{c}\left(\sum_{k=1}^N d^k + \delta^{k*}\right)\delta^{i*} = \dot{c}\left(\sum_{k=1}^N d^k + \delta^{k*}\right) \quad \forall i \tag{6}$$

Equivalently, since $d^i_{t+1} = d^i_t + \delta^i_t$,

$$\dot{v}_i\left(d^i_{t+1}\right) - \ddot{c}\left(\sum_{k=1}^N d^k_{t+1}\right)\left(d^i_{t+1} - d^i_t\right) = \dot{c}\left(\sum_{k=1}^N d^k_{t+1}\right) \quad \forall i \tag{7}$$

Assuming that the consumers know the cost function c and the demand d^i_t for all other consumers at time t, the solution to the system of N simultaneous implicit equations (7) determines each agent's strategic demand for the next time period. When the cost function is quadratic, $c(x) = \rho x^2$, the discrete-time dynamical system (7), simplifies to:

$$-\dot{v}_i\left(d_{t+1}^i\right) + 2\rho d_{t+1}^i + 2\rho\sum_{k=1}^{N} d_{t+1}^k = 2\rho d_t^i \tag{8}$$

2) Mass Approximations: Dynamical system Equations (7) and (8) are models for ideal but unlikely consumer behavior. As a result, they are more realistic for modeling situations where there are only a few large consumers, or a few consumer groups consisting of a large number of small consumers that can be presented via a representative agent model. When these conditions do not hold, proper approximations may results in a model which is both more realistic and more tractable. We present two such models. The first is obtained when each agent simply ignores its own impact on the ex-post price, that is:

$$\frac{\partial}{\partial \delta^i}\dot{c}\left(\sum_{k=1}^{N} d^k + \delta^k\right) \simeq 0, \qquad \forall i$$

We then obtain the following simplified dynamics

$$\dot{v}_i\left(d^i + \delta^{i*}\right) = \dot{c}\left(\sum_{k=1}^{N} d^k + \delta^{k*}\right)$$

Equivalently,

$$\dot{v}_i\left(d_{t+1}^i\right) = \dot{c}\left(\sum_{k=1}^{N} d_{t+1}^k\right), \qquad \forall i \tag{9}$$

In the full-information case, when the vector of value functions $v = (v_1, \cdots v_N)$ is available to all agents, (9) reduces to a static system of equations and the equilibrium is reached in one step. If there is asymmetry of information, the consumers may need to either run a prediction on the aggregate demand $\sum_{k=1}^{N} d_{t+1}^k$, the ex-post price $\dot{c}\left(\sum_{k=1}^{N} d_{t+1}^k\right)$, or both.

In a more interesting scenario, we consider each agent playing against a mass of agents, while ignoring the effects of its own actions on the mass. Here, we let $\bar{v}$ and $\bar{\delta}$ be the assumed value function and demand-deviation (respectively) of the mass from the perspective of the agent, and $d = \sum_{k=1}^{N} d^k$ will be the actual demand of the mass. Then,

$$v_i\left(d^i + \delta^i\right) - \dot{c}\left(d + \bar{\delta}\right)\delta^i \to \max_{\delta^i}$$
$$\bar{v}_i\left(d + \bar{\delta}\right) - \dot{c}\left(d + \bar{\delta}\right)\bar{\delta} \to \max_{\bar{\delta}}$$

Letting $\bar{d}_{t+1} = d_t + \bar{\delta}_t$, we have the following dynamical system:

$$\dot{v}_i\left(d_{t+1}^i\right) - \dot{c}\left(\bar{d}_{t+1}\right) = 0$$
$$\dot{\bar{v}}\left(\bar{d}_{t+1}\right) - \ddot{c}\left(\bar{d}_{t+1}\right)\left(\bar{d}_{t+1} - d_t\right) = \dot{c}\left(\bar{d}_{t+1}\right)$$
$$d_t = \sum_{k=1}^{N} d_t^k$$

We consider a special case where all consumers are identical: $v_i = v_j = v$, $\forall i, j$. In this case, a plausible choice for the value function of the *mass* is given by [13]:

$$\bar{v}(x) = Nv\left(\frac{x}{N}\right)$$

We then have

$$\dot{v}\left(d_{t+1}^i\right) - \dot{c}\left(\bar{d}_{t+1}\right) = 0 \tag{10}$$

$$\dot{v}\left(\frac{\bar{d}_{t+1}}{N}\right) - \ddot{c}\left(\bar{d}_{t+1}\right)\left(\bar{d}_{t+1} - \sum_{k=1}^{N} d_t^k\right) = \dot{c}\left(\bar{d}_{t+1}\right) \tag{11}$$

Remark 1: The equations (10)–(11) were derived for the full information scenario. That is, when N is a deterministic variables and is known to all agents. More realistic and more interesting dynamics arise under asymmetry of information, e.g., when the agents have different beliefs about, or different estimates of N, which could possibly be a random process.

3) Auto-Regressive Demand Predictive: First, we consider agents who individually predict that the overall demand for energy changes according to the auto-regressive system

$$x_{t+1} = x_t + \gamma(x_t - x_{t-1})$$

where $x_t = \sum_{k=1}^{N} d_t^k$. By following the same steps as in the derivation of (7), the dynamics of the Auto-Regressive Demand Predictive model are given by

$$\dot{v}_i\left(d_{t+1}^i\right) - \dot{c}\left((1+\gamma)\,d_t - \gamma d_{t-1}\right) = 0 \tag{12}$$
$$d_t = \sum_{k=1}^{N} d_t^k$$

4) Auto-Regressive Cost Predictive: Next, we consider the case where the agents decide their own consumption based on a simple prediction of energy costs. The dynamics of the Auto-Regressive Demand Predictive model are given by

$$\dot{v}_i\left(d_{t+1}^i\right) - (1+\gamma)\,\dot{c}\left(d_t\right) + \gamma\dot{c}\left(d_{t-1}\right) = 0 \tag{13}$$
$$d_t = \sum_{k=1}^{N} d_t^k$$

5) Demand Averaging: Now, we consider the case where the agents decide on their consumption based on tracking (not predicting) the average demand. Following the same procedure as before, the dynamics of the Demand Averaging model are given by:

$$\dot{v}_i\left(d_{t+1}^i\right) = \dot{c}\left(\tilde{d}_{t+1}\right) \tag{14}$$

$$\tilde{d}_{t+1} = \tilde{d}_t + \gamma\left(\sum_{k=1}^{N} d_t^k - \tilde{d}_t\right) \tag{15}$$

6) Cost Averaging: Finally, we consider the case where the agents decide on their consumption based on tracking (not predicting) the average price. The dynamics of the Cost Averaging model are as follows:

$$\dot{v}_i\left(d_{t+1}^i\right) = \tilde{c}_{t+1} \tag{16}$$

$$\tilde{c}_{t+1} = \tilde{c}_t + \gamma\left(\dot{c}\left(\sum_{k=1}^{N} d_t^k\right) - \tilde{c}_t\right) \tag{17}$$

7) Mass Approximation with Stochastic Price Uncertainty: Assume that in the mean of the population, a single consumer's deviation from the predicted demand is negligible, and that the variation of the anticipated ex-post price is only due to unknown exogenous effects, i.e. $\hat{\lambda}_{t+1} = \lambda_t + \epsilon$. Then, a risk-averse consumer would seek to maximize the expected value of his utility

$$d_t \longmapsto \max_{\delta_t}(1-\mu)\mathbf{E}\left[v(d_{t-1}+\delta_t) - \delta_t\lambda_t - \delta_t\epsilon\right] - \mu\delta_t^2\sigma^2 \tag{18}$$

Here, $\delta_t^2\sigma^2$ is the variance of the consumer's utility, and $\mu \in [0,1]$ is a measure of risk-sensitivity. The value $\mu = 1$ corresponds to the case where a consumer does not take any risk at all. In contrast, a risk-neutral consumer with $\mu = 0$ would consume according to

$$\max_{\delta_t}\mathbf{E}\left[v(d_{t-1}+\delta_t) - \delta_t\lambda_t - \delta_t\epsilon\right] \tag{19}$$

$$\dot{v}^{-1}(\lambda_t) = d_{t-1} + \delta_t \tag{20}$$

as to him $\delta_t = 0$ in the statistical limit, i.e., $\mathbf{E}\left[\delta_t\right] = 0$. Thus, he is a pure price-taker.

Applying the first-order optimality condition and taking $\delta_t = d_t - d_{t-1}$ the price dynamics under a consumer utility as in (18) is

$$\lambda_{t+1} = \dot{c}(d_t) \tag{21}$$

$$(1-\mu)\dot{v}(d_t) - 2\mu d_t\sigma^2 = (1-u)\lambda_t - 2\mu d_{t-1}\sigma^2 \tag{22}$$

Rewriting these equations in terms of only the dynamics of the demand yields

$$(1-\mu)\dot{v}(d_t) - 2\mu d_t\sigma^2 = (1-\mu)\dot{c}(d_{t-1}) - 2\mu d_{t-1}\sigma^2 \tag{23}$$

We define the parameter $\kappa = 2\mu\sigma^2/(1-\mu)$ so that (23) becomes

$$\dot{v}(d_t) - \kappa d_t = \dot{c}(d_{t-1}) - \kappa d_{t-1} \tag{24}$$

These dynamics depend on the unknown consumption deviation δ_t. Each consumer maximizes its own expected utility based on the local parameter μ and the announced locational marginal price λ_t as global parameter. When $\kappa = 0$, we obtain the dynamics

$$\dot{v}(d_t) = \dot{c}(d_{t-1})$$

which is the same dynamic obtained in [13] under pure exanté or ex-post pricing. However, it can be verified, either by simulation or by applying the stability criterion of [13], that $\kappa > 0$ has a stabilizing effect, and for a given v and c it is more likely that (24) would be stable.

B. Exanté Pricing without Ex-Post Adjustment

The real-time retail pricing dynamics without ex-post adjustment are akin to the pricing scheme developed in [13]. Let $\hat{s}_{t+1}$ denote the supply balancing the demand predicted for the time interval $[t, t+1]$. The equations describing the market dynamics are

$$\lambda_{t+1} = \dot{c}(\hat{s}_{t+1})$$
$$\hat{s}_{t+1} = d_t$$
$$d_t = \arg\max_{x\in\mathbb{R}_+} v(x) - \lambda_t x$$

Then the price dynamics are

$$\lambda_{t+1} = \dot{c}(\dot{v}^{-1}(\lambda_t)) \tag{25}$$

Alternatively, the demand dynamics are

$$d_{t+1} = \dot{v}^{-1}\left(\dot{c}\left(d_t\right)\right)$$

Remark 2: We could consider an exanté pricing model in which the agents are not myopic and compute or predict the effects of their actions on future prices, particularly the price for the next round. However, in this paper, we do not discuss such models.

IV. Simulation Results

In this section we present simulation results for three of the proposed models: (a) Mass Approximation, (b) Cost Averaging, and (d) Demand Averaging. In all of these simulations we assume that all consumers are identical, i.e., the have the same value function. The only differentiating factor among individual consumers is their initial consumption. Furthermore, we assume that the production cost function is quadratic and the demand value function is logarithmic:

$$c\left(x\right) = \beta x^2 \tag{26}$$

$$v_i\left(x\right) = v\left(x\right) = \alpha\log\left(x\right), \qquad \forall i = 1,..,N \tag{27}$$

The economically efficient solution is the maximizer of the social welfare function $\mathcal{S}$:

$$\mathcal{S} = N\alpha\log\left(x\right) - \beta\left(Nx\right)^2 \rightarrow \max_x$$

It can be verified that the optimal solution is

$$x^* = \sqrt{\frac{\alpha}{2\beta N}} \tag{28}$$

We will use (28) to examine optimality of the equilibrium in the simulations. For each one of the models of interest, we ran 50 simulations for $N = 10$ consumers with initial conditions uniformly distributed in $[0.5, 5.5]$.

A. Mass Approximation

The simulation results corresponding to the mass approximation dynamics (10)–(11) with the functions defined in (26)-(27) are shown in Figure 1. The system always converged to the efficient equilibrium. The following statistics were reported: Average deviation of final demand to efficient equilibrium: $1.5791e-06$, Standard deviation of final demand to efficient equilibrium: $3.7153e-08$, Average number of iterations to convergence: 4.

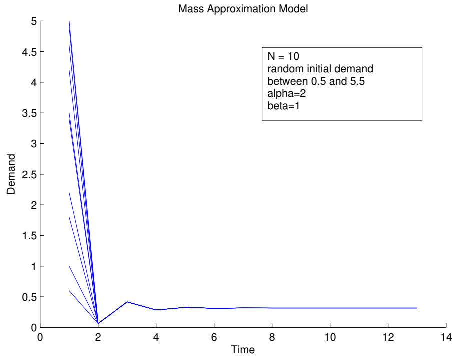

Fig. 1. Simulation of the Mass Approximation Model.

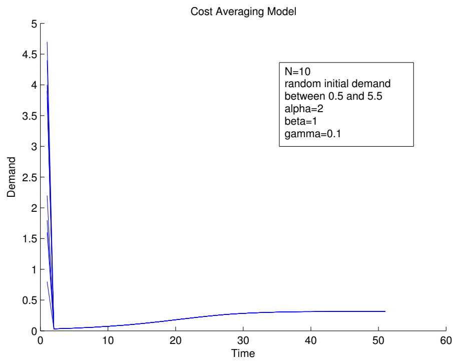

Fig. 2. Simulation of the Cost Averaging Model.

B. Cost Averaging

The simulation results corresponding to the cost averaging model (17) are shown in Figure 2. In this case too, the system always converged to the efficient equilibrium, though at a much slower rate for the chose value of $\gamma = 0.1$. The following statistics were reported: Average deviation of final demand to efficient equilibrium: $2.8507e-04$, Standard deviation of final demand to efficient equilibrium: $6.1415e-06$, Average number of iterations to convergence: 2.

C. Demand Averaging

The simulation results for the demand averaging model were qualitatively and quantitatively very similar to the cost averaging model. In both cases, convergence to the efficient equilibrium can be made faster by choosing a larger value of γ.

D. Exanté Pricing

For comparison, we examine the dynamics under pure exanté pricing. We have

$$d^i_{t+1} = \dot{v}^{-1}\left(\dot{c}\left(d_t\right)\right), \qquad \forall i = 1, .., N$$

Hence,

$$d_{t+1} = N\dot{v}^{-1}\left(\dot{c}\left(d_t\right)\right) = \frac{N\alpha}{\beta d_t} \tag{29}$$

It can be seen that the system is marginally unstable and fragile against external disturbances.

V. Conclusions and Future Work

In comparison to pure exanté or ex-post pricing, the new real-time pricing model introduced in this paper offers better stability and robustness properties under several models of consumer behavior. Furthermore, the equilibrium of the dynamical system resulting from these models was shown to be efficient. Future work includes theoretical proofs of stability and extensions of the models to situations with more asymmetry of information and stochastic uncertainty.

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