A mortar method for incompressible fluid flow discretized by non-matching grids in a stabilized finite element framework

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Motivation

For many applications in different fields of engineering and applied sciences, fluid systems in relatively large domains have to be investigated. Computational methods are increasingly used for such investigations. While most of the domain can often be discretized with a rather coarse discretization length without jeopardizing the overall solution quality, a rather small characteristic discretization length is required locally, for instance due to boundary layers, which need to be resolved. An adequate resolution of such boundary layers as well as the large number of elements needed to bridge from the fine boundary layer mesh to the coarse domain grid are usually linked with high computational costs. Therefore, it is desirable to develop efficient methods enabling the use of different discretizations for boundary-layer regions and the bulk of the flow domain.

Non-conforming meshes

→ Domain decomposition into sub-domains with an internal boundary (see Fig. 1 and 2)

→ Mortar method is used for weakly enforcing coupling constraints by dual Lagrange Multipliers (mortar matrices **M** and **D**)

→ The initial saddle-point system is transformed to a non-saddle-point system by trivial condensation operations (see Fig. 3)

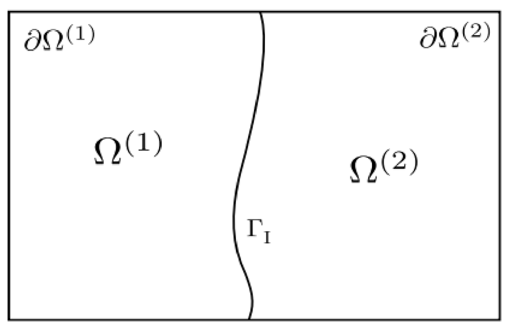

Fig. 1

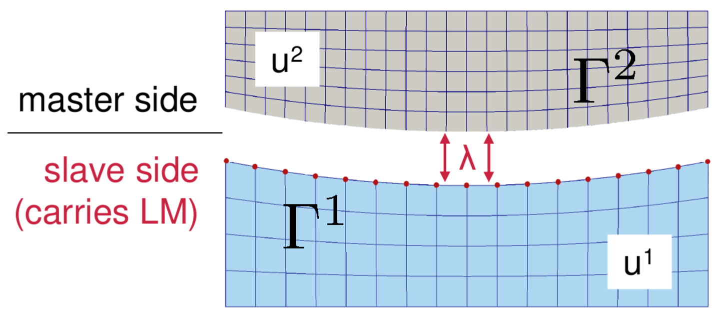

Fig. 2

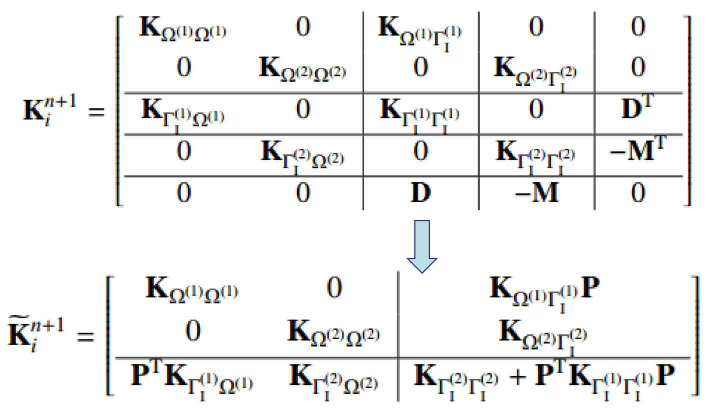

Fig. 3

Numerical examples

Beltrami flow

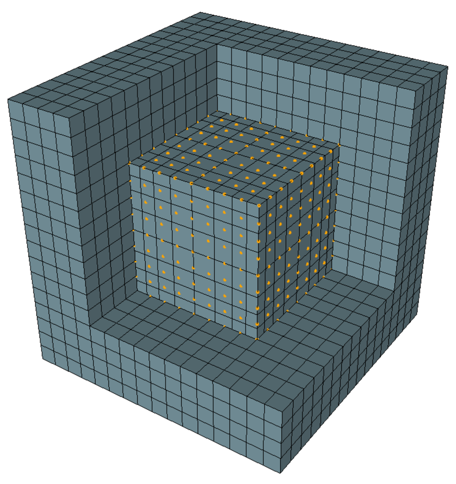

Fig. 4: Computational domain with a piecewise planar internal interface of a cubic subdomain.

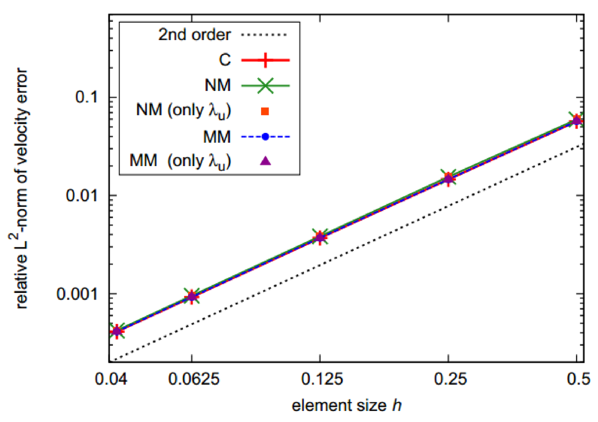

Fig. 6: Velocity error for diffusion-dominated flow.

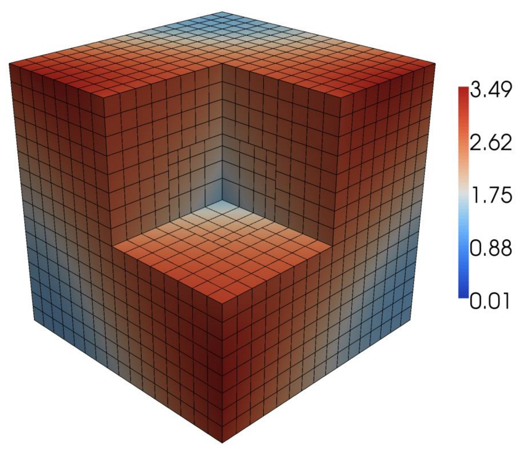

Fig. 5: Euclidean norm of velocity vector |u| for a convection dominated flow field.

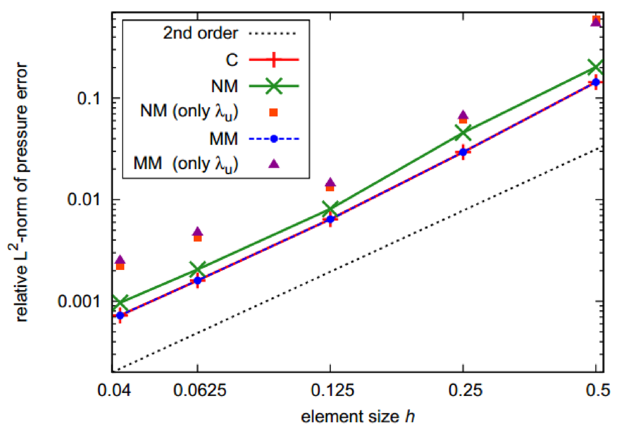

Fig. 7: Pressure error for diffusion-dominated flow.

Lid-driven cavity flow

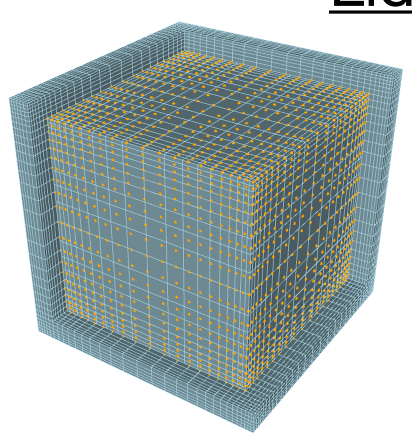

Fig. 8: Non-matching mortar-based discretization for the lid-driven cavity discretized with 32x32x32 elements.

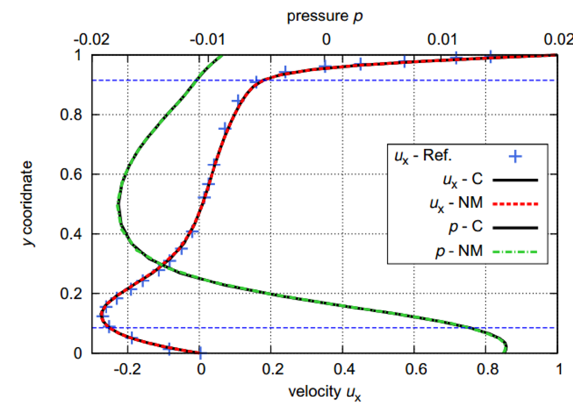

Fig. 10: Velocity component u_x and pressure p along the line P1-P2 in y-direction for 32x32x32 elements at t=100.

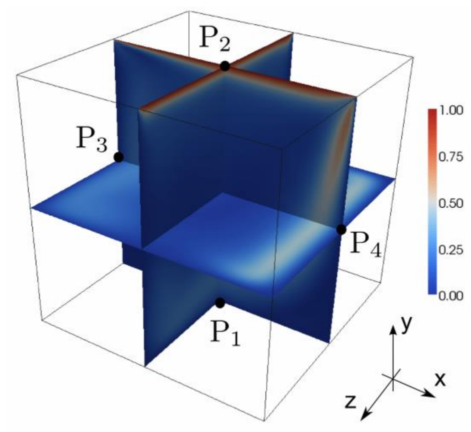

Fig 9.: Euclidean norm of velocity vector ||u|| for the 32x32x32 nonmatching mortar-based discretization in three different planes at t = 100

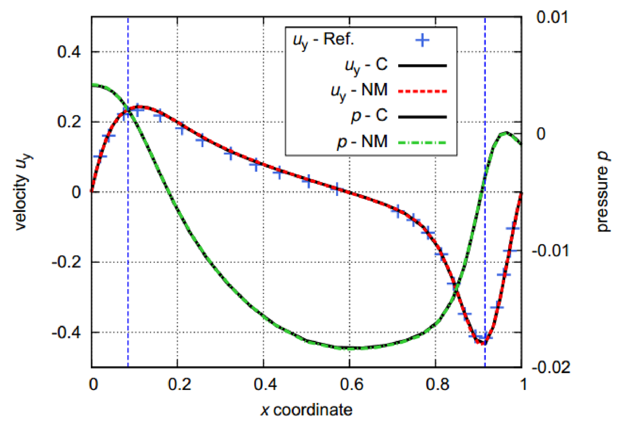

Fig. 11: Velocity component u_y and pressure p along the line P3-P4 in y-direction for 32x32x32 elements at t = 100

Blood flow through an artery with an aneurysm

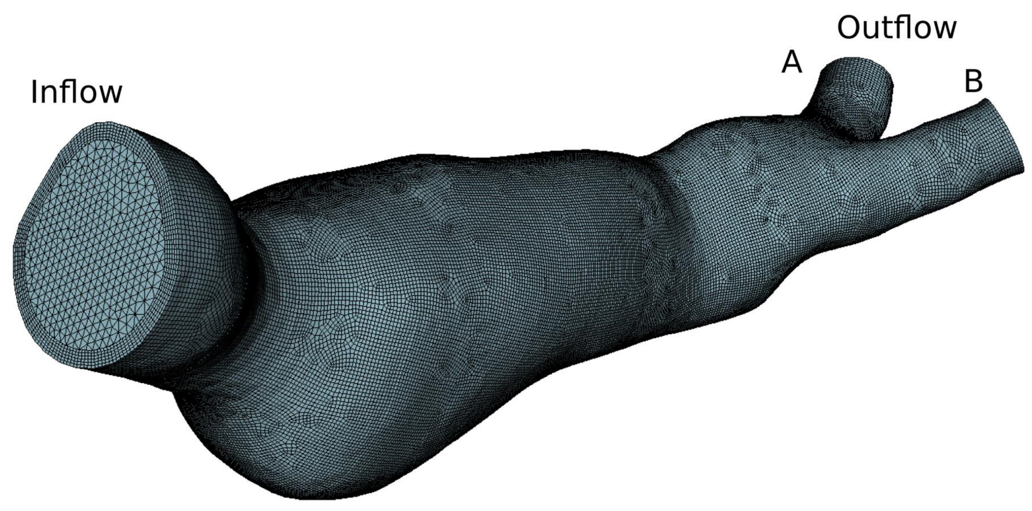

Fig. 12: Patient-specific, aortic aneurysm with structured surface mesh.

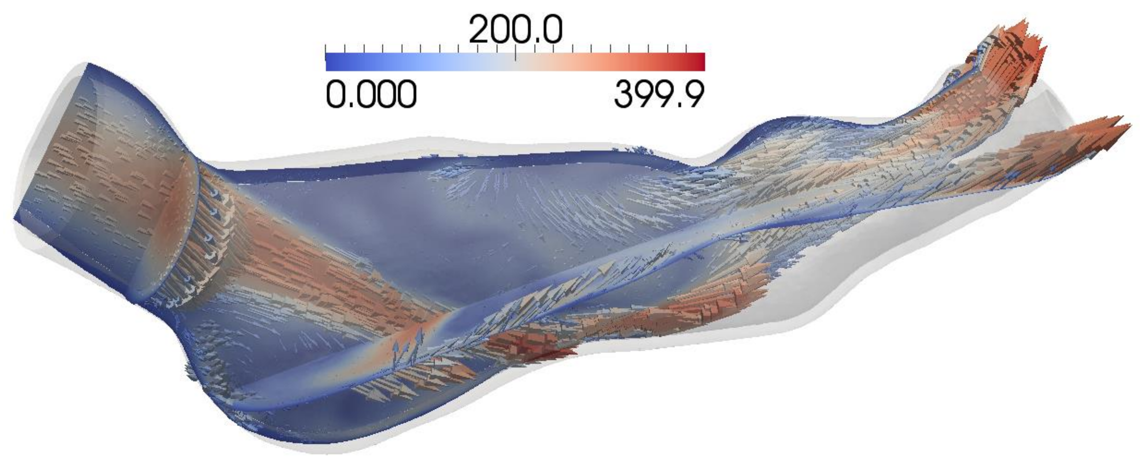

Fig. 14: Velocity field [mm/s] at t=1.5 s for the mesh shown in Fig. 12.

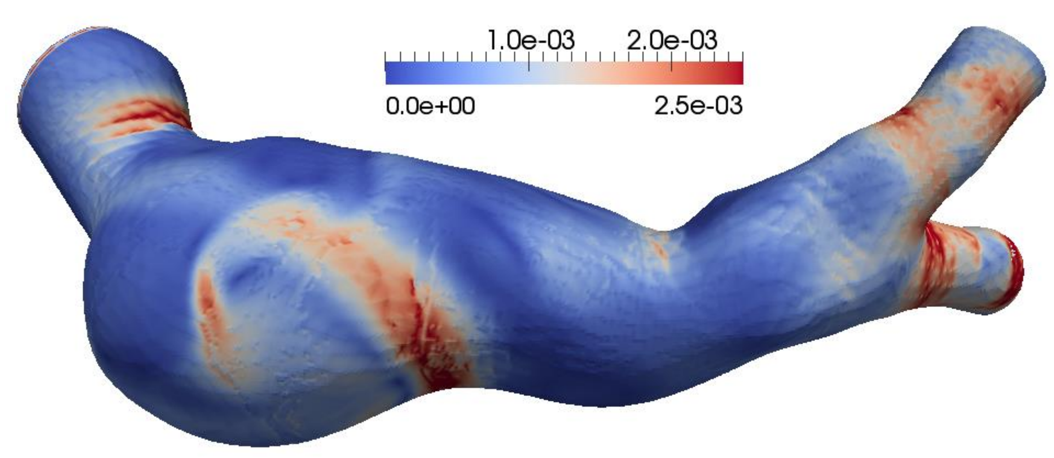

Fig. 16: Wall shear stress [kPa] for the mesh shown in Fig. 12.

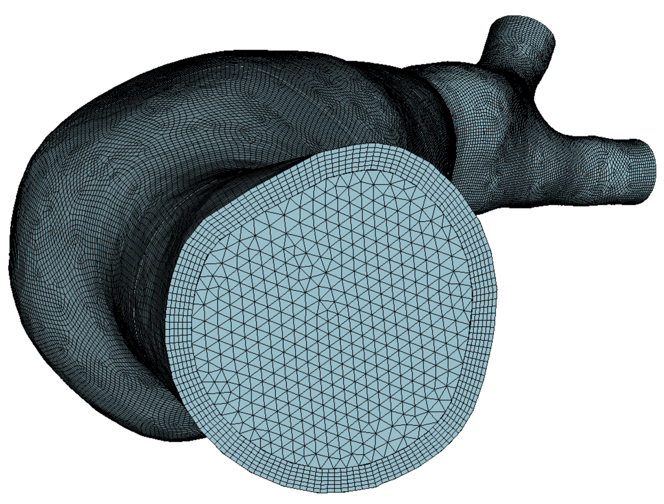

Fig.13: Inflow region of aneurysm with hexahedral boundary layer mesh and tetrahedral mesh in the bulk region.

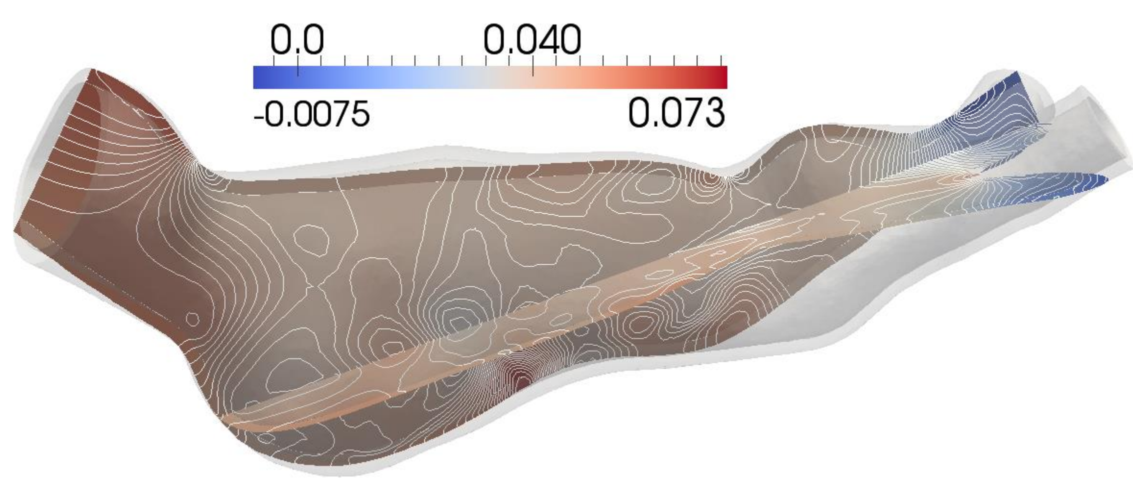

Fig. 15: Pressure field [kPa] including isolines at t=1.5 s for the mesh shown in Fig. 12.

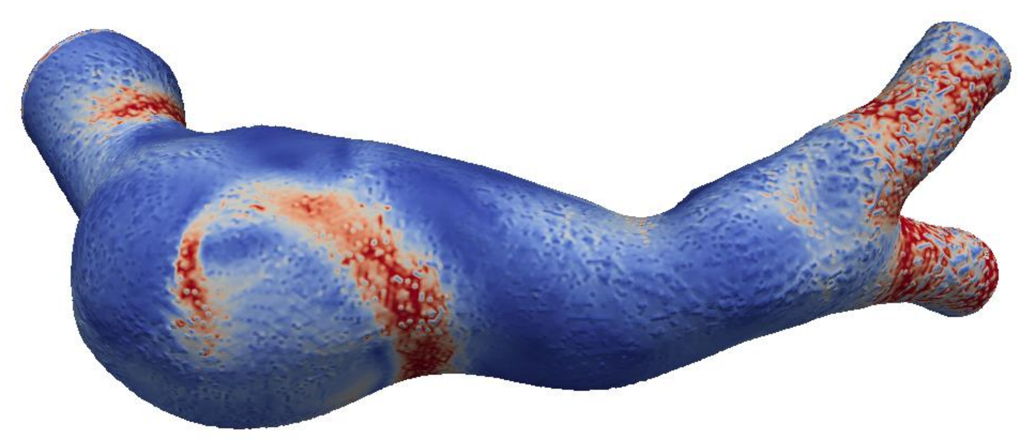

Fig. 17: Wall shear stress [kPa] compared with a mesh without internal interface consisting of a comparable number of tetrahedral elements.

References

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Acknowledgement

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