Extended Variational Multiscale Methods for Turbulent Bubbly Channel Flow

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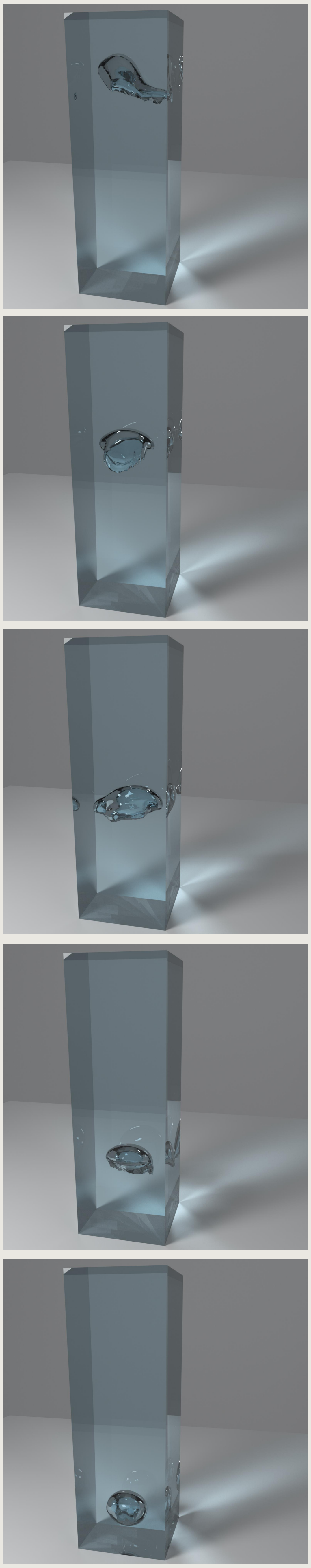

Figure 4: Rendered view of the turbulent bubbly channel flow at different time steps

Motivation

In technical applications, two-phase flows are often encountered. Since the majority of technical flows are turbulent, the simulation of turbulent two-phase flows is particular important. However, computational approaches to two-phase flows are considerably more challenging than to single-phase turbulent flows. The increase in computational power over recent years has enabled first steps towards the simulation of fully resolved two-phase flows. Nevertheless, such simulations are still very expensive and often infeasible for realistic applications. Here, a computational method for large-eddy simulation (LES) is proposed, with the particular feature, that the phase interface is captured as a sharp contour.

Numerical Method

A variational multiscale formulation of the incompressible Navier-Stokes equations with an additional surface tension term is given by

$$\left(\mathbf{w}^h, \rho\frac{\partial \mathbf{u}^h}{\partial t}\right)_\Omega + \left(\mathbf{w}^h, \rho\mathbf{u}^h \cdot \nabla\mathbf{u}^h\right)_\Omega - \left(\nabla\cdot\mathbf{w}^h, p^h\right)_\Omega + \left(\boldsymbol{\varepsilon}\left(\mathbf{w}^h\right), 2\mu\boldsymbol{\varepsilon}\left(\mathbf{u}^h\right)\right)_\Omega + \left(q^h, \nabla\cdot\mathbf{u}^h\right)_\Omega$$
$$+\left(\rho\mathbf{u}^h\cdot\nabla\mathbf{w}^h, \tau_M\mathcal{R}_M^h\right)_{\Omega'} - \left(\nabla q^h, \tau_M\mathcal{R}_M^h\right)_{\Omega'} + \left(\nabla\cdot\mathbf{w}^h, \tau_C\mathcal{R}_C^h\right)_{\Omega'} =$$
$$\left(\mathbf{w}^h, \rho\mathbf{f}\right)_\Omega + \left(\mathbf{w}^h, \mathbf{h}\right)_{\Gamma_h} + \left(\mathbf{w}^h, \gamma\kappa\mathbf{n}_{int}\right)_{\Gamma_{int}},$$

The phase interface is tracked via a level-set method. For this purpose, a scalar field Φ is introduced, governed by the same problem domain as the fluid field.

$$\frac{\partial\Phi}{\partial t} + \mathbf{u}\cdot\nabla\Phi = 0.$$

The phase interface is identified by the zero level-set $\Phi(\mathbf{x}, t) = 0$. It is located at an arbitrary position, usually not coinciding with element boundaries.

Via an extended finite element method (XFEM), as proposed in [1], which is based on enrichment shape functions, associated with additional degrees of freedom, various discontinuities within elements can be represented. This way kinks in the velocity field at the interface, due to the viscosity difference of the two phases, can be captured (see Fig. 1). Furthermore pressure jumps due to surface tension can be accurately represented.

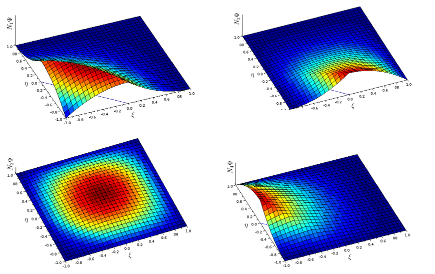

Figure 1: Kink-enriched shape functions for bi-linearly interpolated quadrilateral element.

Example

Results from an LES of a fully developed turbulent channel flow, in which a gaseous bubble rises, are shown here. They are compared to results from a detached direct numerical simulation reported in [2]. Statistics are collected over four independent ensembles, adding up to a total of 4000 sampled steps. The resulting mean velocity in the channel direction is illustrated in Fig. 2. The flow at Reynolds number $Re_\tau = 180$ is simulated on a grid of 122x50x41 tri-linearly interpolated elements.

Rising bubble and surrounding fluid have a density ratio of 0.5, and the Eötvos and Morton numbers are $Eo = 3.341$ and $Mo = 3.452\cdot 10^{-12}$ respectively. The bubble is driven by its buoyancy, which acts in the same direction as the channel flow. This ensures that the bubble does not drift towards the channel walls, but oscillates around a centered position, as can be seen by the mean void fraction in Fig. 2 and the instantaneous snapshots in Fig. 3, 4.

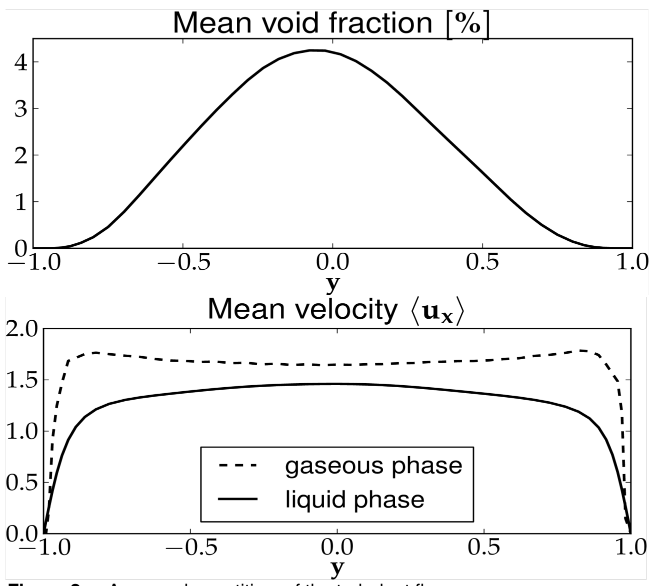

Figure 2: Averaged quantities of the turbulent flow

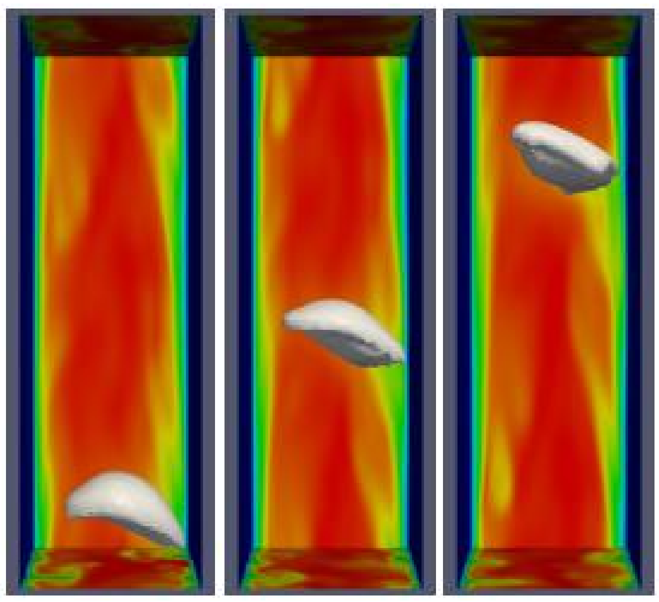

Figure 3: Instantaneous velocity field (colored velocity magnitude) of turbulent channel flow with rising bubble.

Conclusions and Outlook

It was shown that the XFEM technology in conjunction with a level-set based interface capturing can successfully be applied to LES of turbulent two-phase flows. On this basis we intend to improve the turbulence modeling in future works. A promising approach for multi-phase problems is the inclusion of a multifractal subgrid-scale model, which was recently developed and applied to turbulent incompressible single-phase flows in [3].

References

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