Personalization of cardiac fiber orientations from image data using the Unscented Kalman Filter

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Motivation

We propose to estimate rule-based myocardial fiber model (RBM) parameters from DT-MRI, with the goal of personalizing the fiber architecture for cardiac simulations. The RBM is based on a space-dependent angle distribution on the heart surface and then extended to the whole domain through an harmonic lifting of the fiber vectors. For the angles estimation we use a static Unscented Kalman Filter (UKF). We also show the effect of different fiber distributions on cardiac contraction simulations.

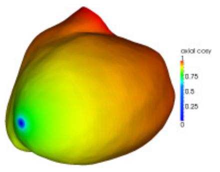
Rule-based fiber model

The construction of the spatially variant fiber model includes the following parts:

- Lifting operation (L) of surface parameter g(**x**):

$$\begin{aligned} \triangle f(\mathrm{x}) &= 0, \quad for\ \mathrm{x}\ \in \Omega \\ f(\mathrm{x}) &= g(\mathrm{x}), \quad for\ \mathrm{x}\ \in \Gamma \subset \partial\Omega \\ \partial_{\mathrm{n}_\Omega} f &= 0, \quad for\ \mathrm{x}\ \in \partial\Omega \backslash \Gamma \end{aligned}$$

- Construction of axial and circumferential coordinate system



Local axial coordinate system, created by solving (L), given Dirichlet boundary conditions (heart apex: 0, heart base: 1).

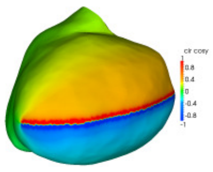
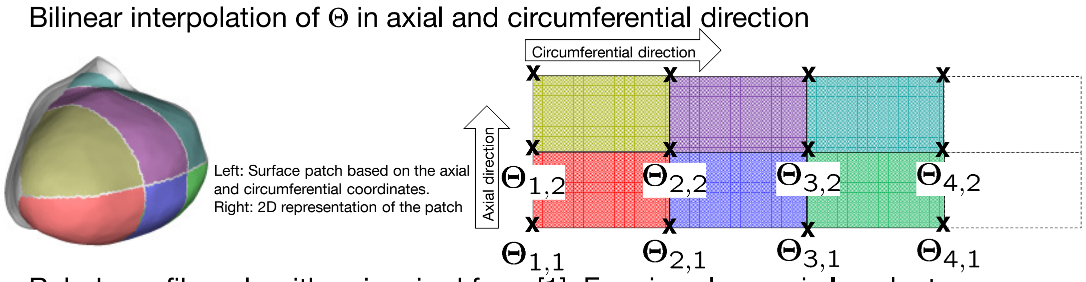


Figure shows local circumferential coordinate system, created by solving (L), given -1/1 Dirichlet condition at a given long-axis line at the left ventricular free wall.

- Bilinear interpolation of Θ in axial and circumferential direction



Left: Surface patch based on the axial and circumferential coordinates. Right: 2D representation of the patch

- Rule-base fiber algorithm, inspired from [1]: For given long-axis **l** evaluate:
  - I. For surface S_i do :
    - For every node $\mathbf{x}_j \in S_i$ do:
      1. Evaluate the local pseudonormal $\mathbf{n}(\mathbf{x}_j)$, circumferential direction $\mathbf{c}(\mathbf{x}_j)$ = cross(λ_j $\mathbf{n}(\mathbf{x}_j)$, **l**) with $\lambda(\mathrm{x}_j) = \begin{cases} +1, & for\ \mathrm{x_j} \in\ epicardium \\ -1, & for\ \mathrm{x_j} \in\ endocardium \end{cases}$
      2. Compute the interpolation of angle $\Theta_{i,j}$: $\delta(\mathbf{x}_j) = \Theta(cir(\mathbf{x}_j), axi(\mathbf{x}_j))$
      3. Generate the surface fibers:

$$\tilde{\mathrm{g}}(\mathrm{x_j}) = cos(\delta(\mathrm{x_j}))\ \mathrm{c}(\mathrm{x_j}) + sin(\delta(\mathrm{x_j}))\ \frac{cross(\mathrm{n}(\mathrm{x_j}),\mathrm{c}(\mathrm{x_j}))}{||cross(\mathrm{n}(\mathrm{x_j}),\mathrm{c}(\mathrm{x_j}))||}$$

$$\mathrm{g} = \frac{\tilde{\mathrm{g}}}{||\tilde{\mathrm{g}}||}$$

  - II. Solve the harmonic lifting (L) with g(**x**)

Unscented Kalman Filter

We aim to minimize the following functional :

$$J(\Theta) = ||\mathrm{f}_m - \mathrm{f}(\Theta)||^2_{W^{-1}} + ||\Theta - \Theta\_||^2_{P^{-1}}$$

with

- noisy fiber measurements f_m from DTMRI measurements
- rule-base algorithm **f** with variables Θ
- a priori estimate Θ_, **W** = γ **Id**, **P** = $(std_0)^2$ **Id,** γ, std_0 positive scalars
- norms $\| \|_{W-1}$ and $\| \|_{P-1}$ for weighting the terms

For solving this least squared problem for Θ we apply the reduced order unscented Kalman filter (ROUKF)[2] in a static manner:

- simplex sigma points $\mathrm{I}_k^{(i)},\ 1 \le i \le k+1$, recursively calculated via

$$[I_1^{(*)}] = \left[-\frac{1}{\sqrt{2\alpha}}, \frac{1}{\sqrt{2\alpha}}\right],\ \alpha = \frac{1}{k+1}$$

$$\left[I_d^{(*)}\right] = \begin{bmatrix} & [I_{d-1}^{(*)}] & & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \frac{1}{\sqrt{\alpha d(d+1)}} & \cdots & \frac{1}{\sqrt{\alpha d(d+1)}} & \frac{-d}{\sqrt{\alpha d(d+1)}} \end{bmatrix},\ 2 \le d \le k$$

- Innovation for each particle:

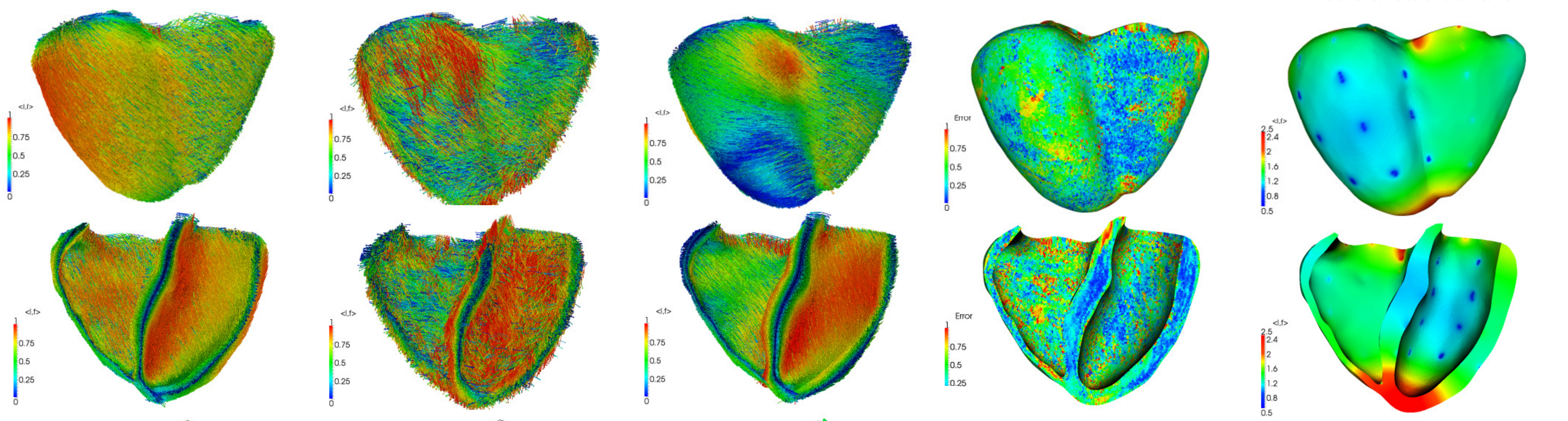
$$\begin{aligned} \mathbf{C} &= \sqrt(P),\ meaning\ Cholesky\ decomposition \\ \Theta^{(i)} &= \Theta\_ + \mathbf{C}I_k^{(i)},\ 1 \le i \le k+1,\ \left[\Theta^{(*)}\right] = \left[\Theta^{(1)} \dots \Theta^{(k+1)}\right] \\ \Gamma^{(i)} &= \mathrm{f}_m - f(\Theta^{(i)}),\ 1 \le i \le k+1,\ \left[\Gamma^{(*)}\right] = \left[\Gamma^{(1)} \dots \Gamma^{(k+1)}\right] \end{aligned}$$

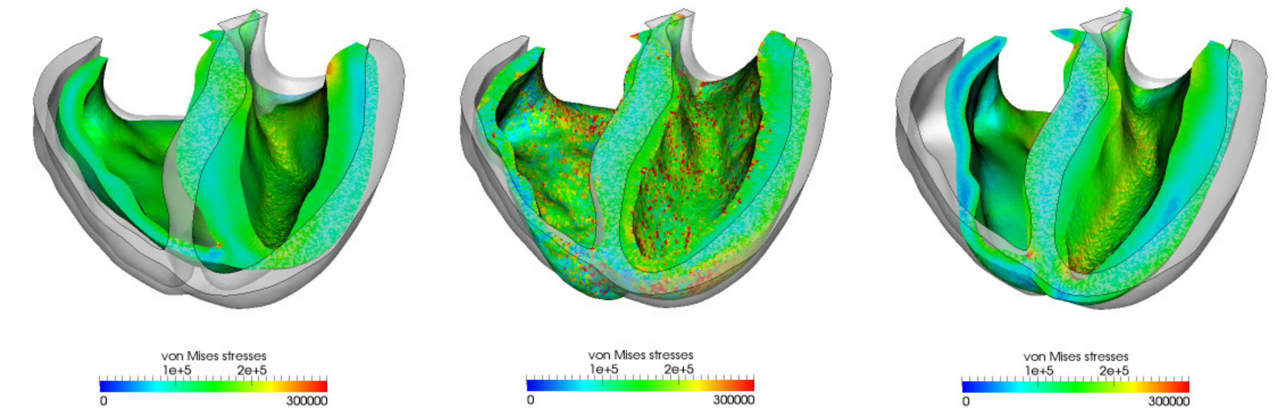
- Estimation

$$\begin{aligned} \mathbf{L}^\Theta &= \alpha[\Theta^{(*)}][\mathbf{I}_k^{(*)}]^T \\ \mathbf{L}^\Gamma &= \alpha[\Gamma^{(*)}][\mathbf{I}_k^{(*)}]^T \\ \mathbf{U} &= \alpha[\mathbf{I}_k^{(*)}][\mathbf{I}_k^{(*)}]^T + (\mathbf{L}^\Gamma)^T\ \mathbf{W}^{-1}\ \mathbf{L}^\Gamma \\ \Theta_{cor} &= \Theta\_ - \mathbf{L}^\Theta U^{-1} (\mathbf{L}^\Gamma)^T \mathbf{W}^{-1} \alpha \textstyle\sum_i^{k+1} \Gamma^{(i)} \\ \mathbf{P}_{cor} &= \mathbf{L}^\Theta \mathbf{U}^{-1} \left(\mathbf{L}^\Theta\right)^T \end{aligned}$$

Results

The tools described above are now used to estimate a smooth fiber organization from 3D ex-vivo DT-MRI* data. For the ROUKF we use the a priori values Θ_ = 60° , std_0 =10.0, γ = 1.0, and **144 degrees-of freedom** for the surface angles distribution, on a **1.7M tetrahedra** mesh**.





Snapshot of the ventricular dynamic contraction simulation (at maximum active stress), superposed with original geometry (grey). The contraction with estimated fibers is closer to the case with DT-MRI than the standard 60° /-60° , but the stress field is smoother. The irregularities in the DT-MRI field induce non-physiological stress singularities in single elements

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[1] Wong, J., Kuhl, E.: Generating fibre orientation maps in human heart models using poisson interpolation. Computer Methods in Biomechanics and Biomedical Engineering (2012) 1{10 PMID: 23210529.

[2] Moireau, P., Chapelle, D.: Reduced-order Unscented Kalman Filtering with application to parameter identcation in large-dimensional systems. COCV 17 (2011) 380{405 doi:10.1051/cocv/2010006.

*DTMRI data Openly available on http://gforge.icm.jhu.edu/gf/project/dtmri data sets/ ; **Geometry created from in-vivo CT – imaging (courtesy of Klinikum Rechts der Isar)

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