Semi-smooth Newton methods for finite strain plasticity

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Introduction

Computational plasticity at finite strains

- Highly nonlinear kinematics and material behavior
- Well-established local return-mappings schemes in the literature, e.g. [1,2]
- Difficult and only variationally inconsistent extension to general hyperelasticity
- Lack of stability at large time steps

Semi-smooth Newton methods for plasticity

- Global solution strategy in contrast to classical return-mapping schemes
- First approaches in [3] and further elaborated in [4,5] for small strain kinematics

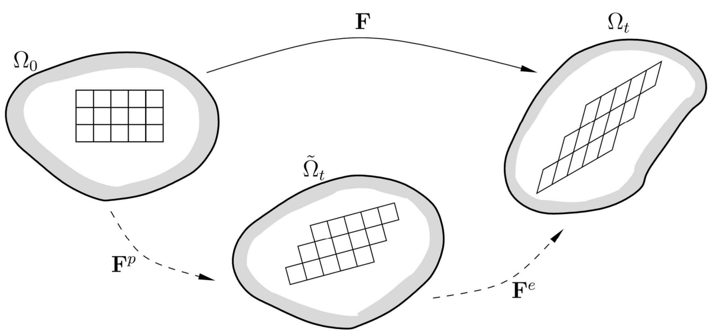
Aim of this work

- Application of a semi-smooth Newton method to finite strain plasticity
- Incorporation of general hyperelasticity
- Improved stability compared to classical return-mapping methods
- Combination with other semi-smooth strategies, e.g. for (frictional) contact [6]

Mathematical description of finite strain plasticity

- Multiplicative split of the deformation gradient to define a plastic, locally stress-free intermediate configuration:

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p$$



- General hyperelasticity in terms of Mandel stresses:

$$\boldsymbol{\Sigma} = 2\mathbf{C}^e \frac{\partial \psi^e}{\partial \mathbf{C}^e}$$

- Von Mises yield function with linear isotropic and kinematic hardening:

$$f^{pl} = \|\boldsymbol{\Sigma} - H^k \boldsymbol{\alpha}^k\| - \sqrt{\frac{2}{3}}(\sigma_0 + H^i \alpha^i) = \|\boldsymbol{\eta}\| - Y^{pl}$$

- Evolution equations for internal variables:

$$\dot{\mathbf{F}}^p \mathbf{F}^{p-1} = \gamma \frac{\boldsymbol{\eta}}{\|\boldsymbol{\eta}\|} \;,\; \dot{\alpha}^i = \sqrt{\frac{2}{3}}\gamma \;,\; \dot{\boldsymbol{\alpha}}^k = -\gamma \frac{\boldsymbol{\eta}}{\|\boldsymbol{\eta}\|}$$

- Complementarity and consistency condition for the plastic flow:

$$f^{pl} \leq 0 \quad \gamma \geq 0 \quad f^{pl}\gamma = 0 \quad \dot{f}^{pl}\gamma = 0$$

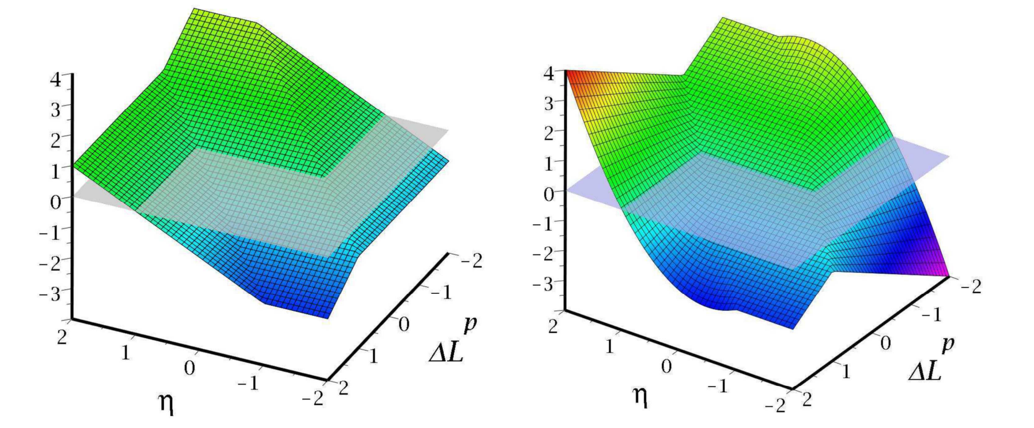
Reformulation as semi-smooth nonlinear system

- Introduction of the discrete plastic flow within one time step at each quadrature point as additional primary variables:

$$\Delta \mathbf{L}^p = \int_{t_n}^{t_{n+1}} \gamma \frac{\boldsymbol{\eta}}{\|\boldsymbol{\eta}\|} \mathrm{d}t$$

- Evolution equations and inequality constraints are reformulated in terms of a nonlinear complementarity (NCP) function:

$$\mathbf{C}^{pl,s} = \max\left(Y^{pl}, \boldsymbol{\eta} + c^{pl}\Delta\mathbf{L}^p\right)^s \left(\boldsymbol{\eta} - \min\left(1, \frac{Y^{pl}}{\|\boldsymbol{\eta} + c^{pl}\Delta\mathbf{L}^p\|}\right)\left(\boldsymbol{\eta} + c^{pl}\Delta\mathbf{L}^p\right)\right) = \mathbf{0}$$



(a) $s = 0$ (b) $s = 1$

- Piece-wise linear NCP-function, i.e. $s = 0$, is under further conditions (time integration, complemenetarity parameter) identical to return mapping methods
- Stabilization parameter $0 < s \lesssim 2$ allows for significantly larger time steps (up to factor 25) without changing the numerical solution
- Except isotropy, no restrictions to the hyperelastic material law have to be made

Semi-smooth Newton method

Consistent linearization

- Application of a variant of Newton's method based on a generalized definition of the derivatives in the semi-smooth NCP function [7]
- Linearization of the internal and external forces and the NCP function yields a block matrix system at each quadrature point:

$$\begin{bmatrix} \frac{\partial \mathbf{f}_{int}}{\partial \mathbf{d}} & \frac{\partial \mathbf{f}_{int}}{\partial (\Delta \mathbf{L}^p)} \\ \frac{\partial \mathbf{C}^{pl}}{\partial \mathbf{d}} & \frac{\partial \mathbf{C}^{pl}}{\partial (\Delta \mathbf{L}^p)} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{d} \\ \Delta(\Delta \mathbf{L}^p) \end{bmatrix} = \begin{bmatrix} -\mathbf{f}_{int} - \mathbf{f}_{ext} \\ -\mathbf{C}^{pl} \end{bmatrix}$$

Static condensation

- Discontinuous interpolation of inner variables at each quadrature point enables static condensation of the plastic flow increment $\Delta(\Delta\mathbf{L}^p)$ at Gauss point level:

$$\Delta(\Delta \mathbf{L}^p) = -\left(\frac{\partial \mathbf{C}^{pl}}{\partial (\Delta \mathbf{L}^p)}\right)^{-1} \left(\mathbf{C}^{pl} + \frac{\partial \mathbf{C}^{pl}}{\partial \mathbf{d}} \Delta \mathbf{d}\right)$$

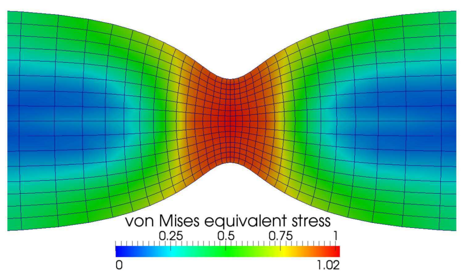
$$\left[\frac{\partial \mathbf{f}_{int}}{\partial \mathbf{d}} - \frac{\partial \mathbf{f}_{int}}{\partial (\Delta \mathbf{L}^p)} \left(\frac{\partial \mathbf{C}^{pl}}{\partial (\Delta \mathbf{L}^p)}\right)^{-1} \frac{\partial \mathbf{C}^{pl}}{\partial \mathbf{d}}\right] \Delta \mathbf{d} = \left[-\mathbf{f}_{int} - \mathbf{f}_{ext} + \frac{\partial \mathbf{f}_{int}}{\partial (\Delta \mathbf{L}^p)} \left(\frac{\partial \mathbf{C}^{pl}}{\partial (\Delta \mathbf{L}^p)}\right)^{-1} \mathbf{C}^{pl}\right]$$

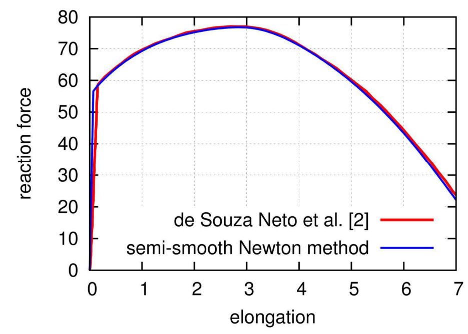
→ The remaining system only consists of the displacement degrees of freedom

Numerical results

Necking of a cylindrical bar

- Popular benchmark problem for finite strain plasticity in the literature, e.g. [1,2]
- Simulation with 960 F-bar elements to avoid volumetric locking

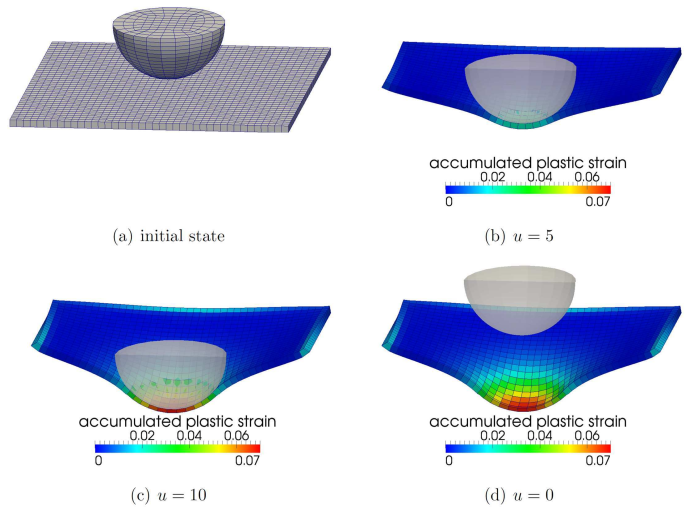




→ Excellent agreement with results from the literature

Cupping of a metal sheet

- A rigid hemisphere is pushed into a quadratic metal sheet
- Setting the stabilization parameter $s = 1$ permits to simulate the intrusion in only 25 equal time steps
- A semi-smooth Newton method is used for both plasticity and contact [6]



(a) initial state (b) $u = 5$ (c) $u = 10$ (d) $u = 0$

Conclusions

- Semi-smooth Newton method for finite strain plasticity results in a more robust formulation compared to classical return mapping schemes
- General isotropic hyperelasticity is naturally incorporated
- Unified treatment of plasticity and contact in a general framework of semi-smooth Newton methods

Outlook

- Further analysis of the interaction between plasticity and contact
- Application of the stabilization to frictional contact

References

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