Face-oriented stabilized XFEM approach for 3D fluid-structure interaction

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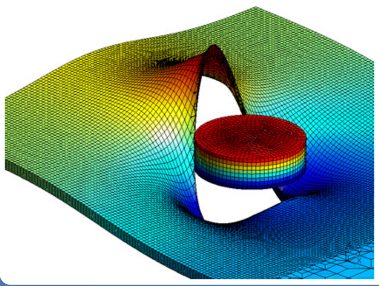
Motivation

Fig. 1: rotating wind turbine, contacting red blood cells, arriving U-Bahn train in Munich, biofilm streamer.

- The objective of this project is to derive a stabilized fixed-grid fluid discretization approach based on the eXtended Finite Element Method (XFEM) and to develop a robust fixed-grid fluid-structure interaction (FSI) scheme in 3D.
- Fixed-grid fluid-structure interaction methods are highly promising for a wide variety of industrial and biomedical applications since they allow for
  - large motions, rotations and deformations of flexible structures or even topological changes without remeshing,
  - contact of submersed structures (fluid-structure-contact interaction (FSCI) [3]).

Governing equations



- Fluid flow described in a fixed-grid Eulerian frame of reference using cut elements

$$\rho^f \frac{\partial \boldsymbol{u}}{\partial t} + \rho^f \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p - 2\mu \nabla \cdot \boldsymbol{\epsilon}(\boldsymbol{u}) = \rho^f \boldsymbol{b}^f \text{ in } \Omega^f(t)$$

$$\nabla \cdot \boldsymbol{u} = 0 \text{ in } \Omega^f(t)$$

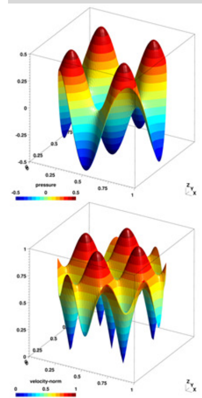
- Description of the structural motion in a Lagrangean framework

$$\rho^s \frac{d^2 \boldsymbol{d}}{dt^2} = \nabla \cdot (\boldsymbol{F} \cdot \boldsymbol{S}) + \rho^s \boldsymbol{g}^s \text{ in } \Omega^s(t)$$

- FSI-coupling conditions $[\![\boldsymbol{u}]\!] := \boldsymbol{u}^f - \boldsymbol{u}^s = \boldsymbol{0}$

$$[\![\boldsymbol{\sigma} \cdot \boldsymbol{n}]\!] := (\boldsymbol{\sigma}^f \cdot \boldsymbol{n}) - (\boldsymbol{\sigma}^s \cdot \boldsymbol{n}) = \boldsymbol{0} \text{ on } \Gamma_{FSI}(t)$$

Numerical Analysis and Results [1]



- Optimal spatial convergence in domain L^2-norms and interface $H^{1/2}$ and $H^{-1/2}$-norms for u and p in viscous and convection dominated cases.
- Good accuracy regarding the weak imposition of boundary conditions.
- Clear improvement of viscous and pressure fluxes at the interface compared to residual-based stabilized methods [6].
- Less sensitivity with respect to the interface position.
- Ghost penalty stabilization and face-oriented fluid stabilizations in the interface zone supersede any DOF-blocking strategy.
- Well conditioned system matrices in viscous and convection dominated cases.

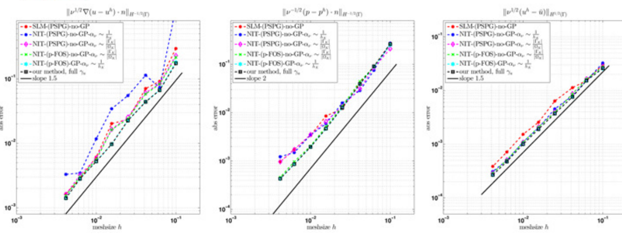


Fig. 3: Kim-Moin flow: viscous and pressure fluxes, enforcement of boundary condition in viscous dominated flow.

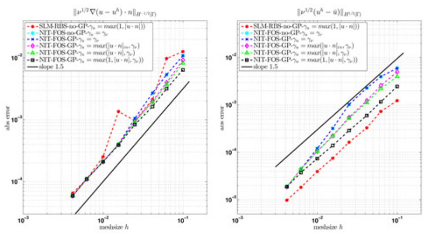
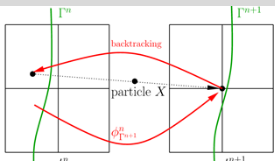


Fig. 4: Kim-Moin flow: viscous fluxes and enforcement of boundary condition in convection dominated flow.

Time integration involving moving domains

A Semi-Lagrangean method for time integration [5]

- Additional projected solution at time t^n w.r.t interface Γ^{n+1}
- Follow particle $\boldsymbol{X}$ in its motion from t^n to t^{n+1}
- Applicable to standard finite difference based time integrators.



1. Tracking $\boldsymbol{X}$ back in time t with $\boldsymbol{x}^* = \boldsymbol{x} - \int_{t^n}^{t^{n+1}} \boldsymbol{u}(\boldsymbol{x},t)\,\mathrm{d}t$

$$\Rightarrow \boldsymbol{x}^* = \boldsymbol{x} - \Delta t(\theta_{SL}\boldsymbol{u}^{n+1}(\boldsymbol{x}) + (1-\theta_{SL})\boldsymbol{u}^n(\boldsymbol{x}^*))$$

2. Solution at time t^n w.r.t. interface position Γ^{n+1}

$$\phi^n_{\Gamma^{n+1}}(\boldsymbol{x}) = \phi^n(\boldsymbol{x}^*) + \Delta t \left(\theta_{SL}\boldsymbol{u}^{n+1}(\boldsymbol{x})\frac{\partial \phi^{n+1}(\boldsymbol{x})}{\partial \boldsymbol{x}} + (1-\theta_{SL})\boldsymbol{u}^n(\boldsymbol{x}^*)\frac{\partial \phi^n(\boldsymbol{x}^*)}{\partial \boldsymbol{x}}\right)$$

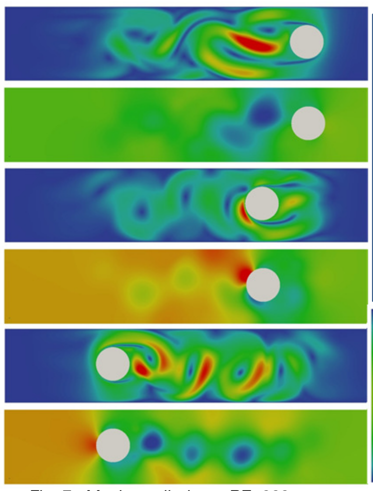


Fig. 7: Moving cylinder at RE=300, velocity norm and pressure at different t.

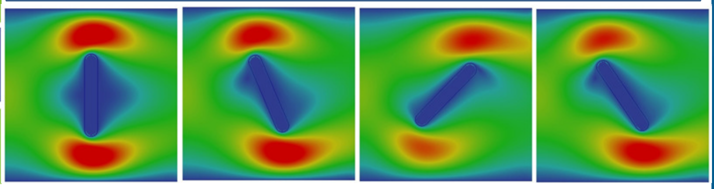


Fig. 8: Rotating beam with constant angular velocity: velocity norm at different t.

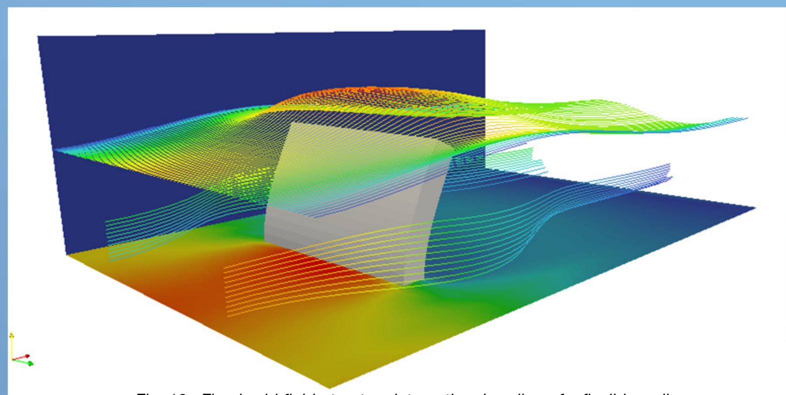


Fig. 10: Fixed-grid fluid-structure interaction: bending of a flexible wall.

Spatial discretization using cut elements

1. Volume-cell (3D polyhedra) representation of the physical fluid domain

$$\Omega^f = \bigcup_{K\in\mathcal{T}} \Omega^f_K = \bigcup_{K\in\mathcal{T}} \bigcup_{V\in\Omega^f_K} V \qquad \Omega^f_K = \{V \subseteq K \cap \Omega^f \text{ polyhedra} \mid V_i \cap V_j = \emptyset \text{ for } i \neq j\}$$

$$V^h = \left\{ v \in C^0(\Omega^f) : v|_V \in Q^l(K)\ \forall V \in \Omega^f_K, \forall K \in \mathcal{T} \right\}$$

2. Standard continuous approximations based on fluid volume-cells [1]
   - Build volume-cell connections C_i on $\mathrm{supp}(N^n)$ between elements via common facets
   - Number of connections C_i per node n determines the number of required degrees of freedom
   - Assign the respective degree of freedom (DOF) to the volume-cells
3. Integration of weak formulation based on the divergence theorem [2]
   - Heaviside enrichment for cut elements
   - Integration only on Ω^f

$$v|_V = \sum_{i=1}^{n_{en}} N^i \cdot \Psi^i|_V \cdot v^i \qquad \Psi^i = \begin{cases} 1 \text{ in } \Omega^f \\ 0 \text{ else} \end{cases}$$

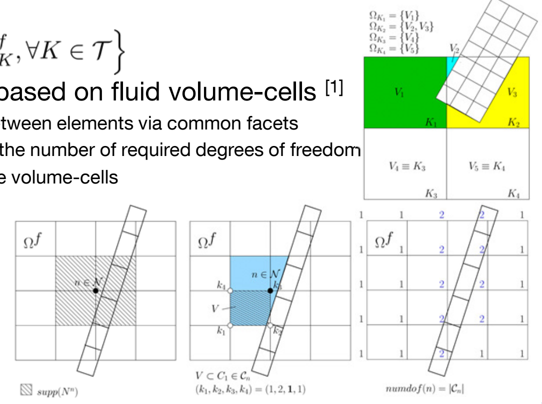


Fig. 2: Support for continuous approximations, volume-cell connections and dofset numbers, number of DOFs per node. [1]

Face-oriented stabilized fixed-grid fluid formulation

- Consistent Nitsche-type weak enforcement of interface conditions for viscous [4] and convection dominated flows

Find $(\boldsymbol{u}^h, p^h) \in V^h \times Q^h$ such that $\forall (\boldsymbol{v}^h, q^h) \in V^h \times Q^h$:

$$\left(\boldsymbol{v}^h, \rho\dot{\boldsymbol{u}}^h\right) + (\boldsymbol{v}^h, \rho\boldsymbol{u}^h \cdot \nabla \boldsymbol{u}^h) + (q^h, \nabla \cdot \boldsymbol{u}^h) - (\nabla \cdot \boldsymbol{v}^h, p^h) + (\boldsymbol{\epsilon}(\boldsymbol{v}^h), 2\mu\boldsymbol{\epsilon}(\boldsymbol{u}^h))$$
$$+ \langle \boldsymbol{v}^h, p^h \cdot \boldsymbol{n}\rangle_\Gamma - \langle \boldsymbol{v}^h, 2\mu\boldsymbol{\epsilon}(\boldsymbol{u}^h) \cdot \boldsymbol{n}\rangle_\Gamma$$
$$- \langle q^h \cdot \boldsymbol{n}, \boldsymbol{u}^h - \bar{\boldsymbol{u}}\rangle_\Gamma - \langle 2\mu\boldsymbol{\epsilon}(\boldsymbol{v}^h) \cdot \boldsymbol{n}, \boldsymbol{u}^h - \bar{\boldsymbol{u}}\rangle_\Gamma$$
$$+ \sum_{K\in G_h} \sum_{V\in\Omega^f_K} \langle \gamma_u \cdot \boldsymbol{v}^h, \rho(\boldsymbol{u}^h - \bar{\boldsymbol{u}})\rangle_{\Gamma_K \cap \bar{V}}$$
$$+ S_h\left[(\boldsymbol{v}^h, q^h), (\boldsymbol{u}^h, p^h)\right] = (\boldsymbol{v}^h, \rho\boldsymbol{b}) + \langle \boldsymbol{v}^h, \bar{\boldsymbol{h}}\rangle_{\partial\Omega^f_N}.$$

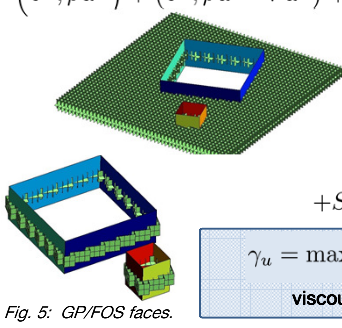


Fig. 5: GP/FOS faces.

$$\gamma_u = \max(\gamma_\nu, \gamma_{conv}, 1.0) = \max\left(\alpha_\nu \frac{\nu}{h_K}, |\boldsymbol{u}^h \cdot \boldsymbol{n}|, 1.0\right)$$

viscous part [4] + convective part [1] + penalty term [1]

- Face-oriented Ghost-penalty and fluid stabilizations (GP+FOS):

$$S_h\left[(\boldsymbol{v}^h, q^h), (\boldsymbol{u}^h, p^h)\right] = j_{GP}(\boldsymbol{v}^h, \boldsymbol{u}^h) + j_p(q^h, p^h) + j_{stream}(\boldsymbol{v}^h, \boldsymbol{u}^h)$$

viscous ghost penalty [4] + pressure stab. [4] + streamline stab. [1]

$$j_{GP}(\boldsymbol{v}^h, \boldsymbol{u}^h) := \sum_{f\in\mathcal{F}_G} \sum_{i=1}^{k} \int_{\substack{F\supseteq f \\ F\in\mathcal{F}_\mathcal{T}}} \alpha_{GP} \cdot \mu h_F^{2(i-1)+1} [\![D^i \boldsymbol{v}^h]\!] : [\![D^i \boldsymbol{u}^h]\!]\,\mathrm{ds}$$

$$j_p(q^h, p^h) := \sum_{f\in\mathcal{F}} \int_{\substack{F\supseteq f \\ F\in\mathcal{F}_\mathcal{T}}} \gamma_p \rho^{-1} \cdot [\![\nabla q^h]\!][\![\nabla p^h]\!]\,\mathrm{ds} \quad \text{with } \gamma_p = \alpha_p h_F^2 \cdot \left(1 + \frac{\nu}{h_F}\right)^{-1}$$

$$j_{stream}(\boldsymbol{v}^h, \boldsymbol{u}^h) := \sum_{f\in\mathcal{F}} \int_{\substack{F\supseteq f \\ F\in\mathcal{F}_\mathcal{T}}} \gamma_{stream}\rho \cdot |\boldsymbol{u}^h \cdot \boldsymbol{n}^F| [\![\nabla \boldsymbol{v}^h]\!] : [\![\nabla \boldsymbol{u}^h]\!]\,\mathrm{ds} \quad \text{with } \gamma_{stream} = \alpha_{stream} h_F^2$$

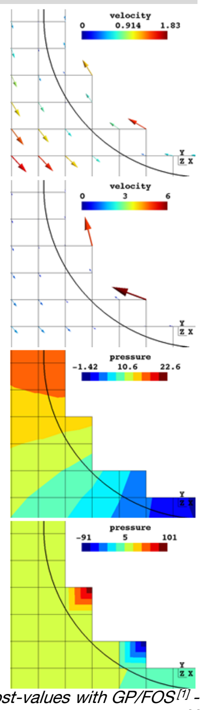
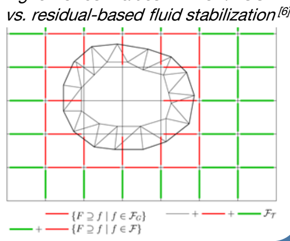


Fig. 6: Ghost-values with GP/FOS [1] - vs. residual-based fluid stabilization [6].



Partitioned Fluid-Structure Interaction

Iterative Dirichlet-Neumann Coupling (with Aitken relaxation)

- **Fluid-operator:** $\mathcal{F}^{n+1}_\Gamma(\boldsymbol{d}^{n+1}_\Gamma) = \boldsymbol{f}^{n+1}_\Gamma$
  solve fluid flow for $(\boldsymbol{u}_h, p_h) \Rightarrow \boldsymbol{f}^{n+1}_\Gamma$
  with prescribed **Dirichlet** velocity $\bar{\boldsymbol{u}} = \frac{\boldsymbol{d}^{n+1}_\Gamma - \boldsymbol{d}^n_\Gamma}{\Delta t}$
- **Structure-operator:** $\mathcal{S}^{-1,n+1}_\Gamma(\boldsymbol{f}^{n+1}_\Gamma) = \boldsymbol{d}^{n+1}_\Gamma$
  solve structure for $\boldsymbol{d}_h \Rightarrow \boldsymbol{d}^{n+1}_\Gamma$
  with prescribed **Neumann** interface forces $\boldsymbol{f}^{n+1}_\Gamma$
- Update using Aitken relaxation:

$$\boldsymbol{d}^{n+1}_{\Gamma,i+1} = \boldsymbol{d}^{n+1}_{\Gamma,i} + \omega_i(\mathcal{S}^{-1,n+1}_\Gamma(\mathcal{F}^{n+1}_\Gamma(\boldsymbol{d}^{n+1}_{\Gamma,i})) - \boldsymbol{d}^{n+1}_{\Gamma,i})$$

- Iterative coupling until convergence for each t^n.

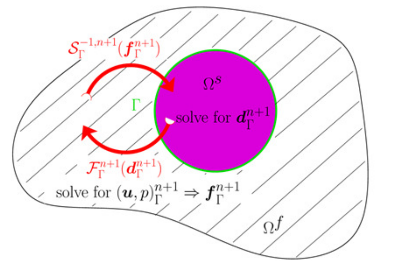


Fig. 9: Partitionened Dirichlet-Neumann coupling for fluid-structure interaction.

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