Numerical integration of weak forms in Embedded Interface Methods

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Introduction

- Embedded interface introduces arbitrary polyhedral shaped volume cells
- Weak form integration over volume cells — stiffness matrix of cut elements
- Accurate and robust integration method over polyhedra is essential for EIM

Tessellation

- Decompose a polyhedron into a number of tetrahedra\(^{(1,2,3)}\)
- Integrate over each tet and sum it up

Direct divergence method

- Use divergence theorem to convert volume integral into surface integrals\(^{(2)}\)
- The divergence theorem
\[
\int_F \alpha d\Sigma = \int_G \beta d\Gamma
\]

- Applying it for a scalar integrand \(F\)
\[
\int_F d\Sigma = \int_G \beta d\Gamma
\]

- Two sets of Gauss points to evaluate the two integrals separately

Moment fitting method

- Construct quadratures for polyhedra by solving moment fitting equations\(^{(5,2)}\)
\[
\begin{bmatrix}
\sum \alpha_i (x_i, y_i, z_i) \\
\sum \alpha_i (x_i, y_i, z_i) \\
\sum \alpha_i (x_i, y_i, z_i)
\end{bmatrix}
= \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
\]

- Step 1: define base functions
- Step 2: integrate base functions and get L.H.S of (1) using divergence theorem

Results

- Robustness

- Accuracy of weak form integration

- Computational efficiency

References