An ALE-formulation for dual mortar finite deformation contact problems with wear

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Introduction

Wear due to frictional contact

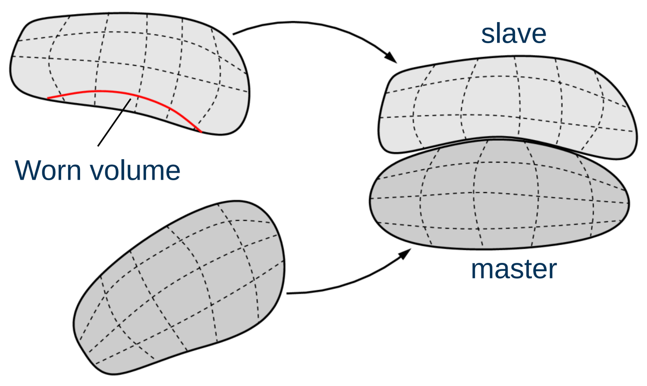
- Complex phenomena characterized by loss of material at contact surfaces
- One of the main causes for component damage and subsequent failure of machines and devices [1]
- High relevance in biomechanical applications, e.g. for joint prostheses [2]

Aim of this work

- Prediction of structural failure due to wear
- Consideration of different wear types with changing underlying physical effects, such as abrasive-, adhesive-, corrosive- and fretting- wear

Structure-ALE approach

Aspects of continuum mechanics

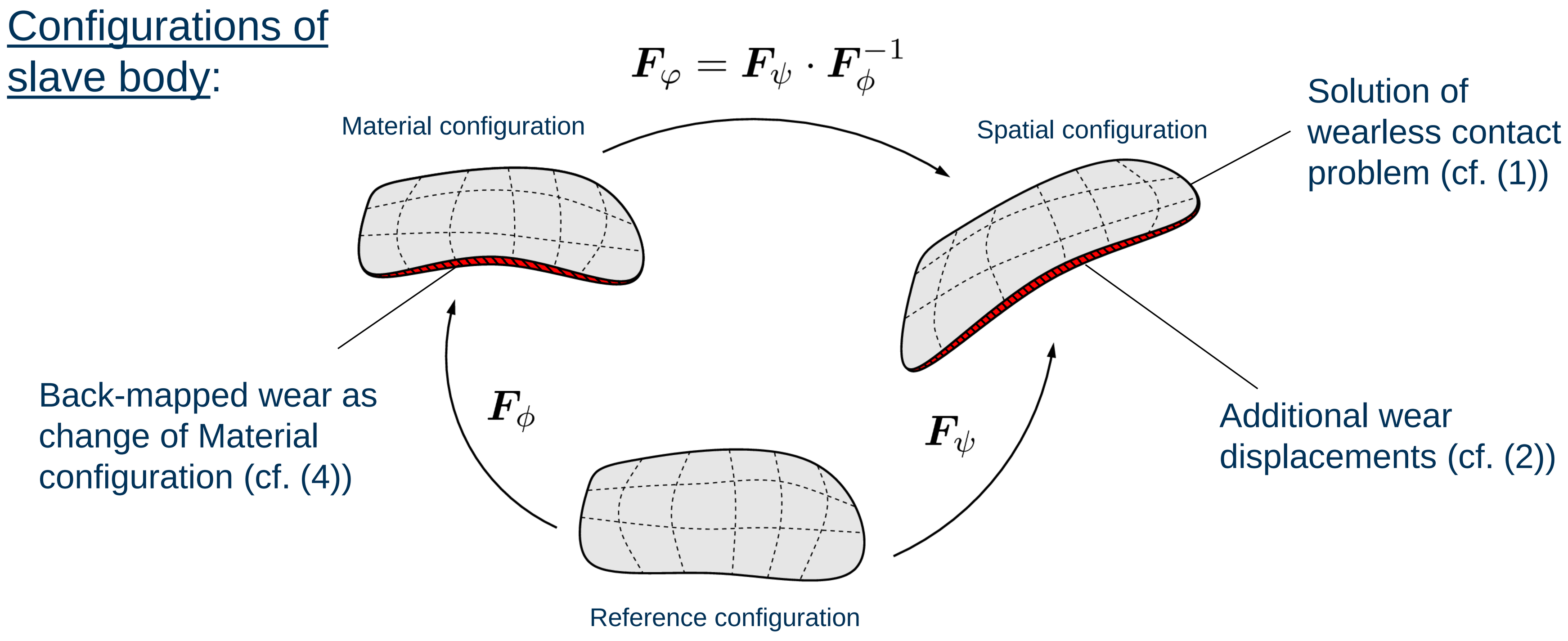


- Shape evolution due to wear is directly modeled
- Time dependent *reference configuration* which is called *material configuration*
- Consideration of wear only on mortar-slave surface (one-body wear)

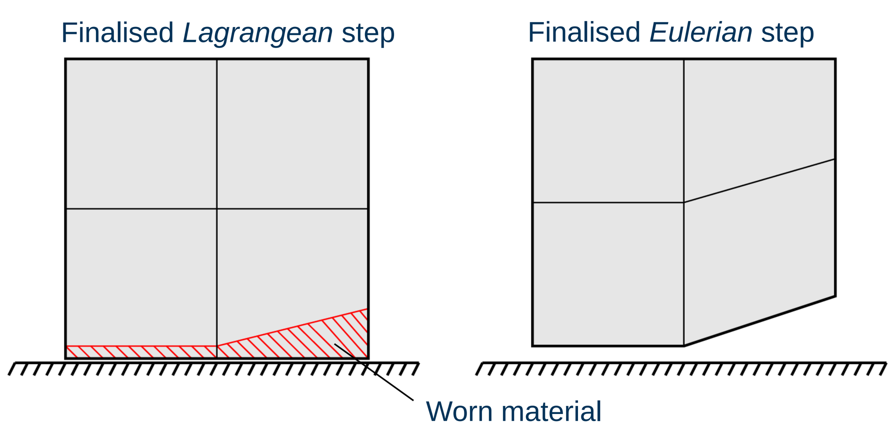
Staggered structure-ALE algorithm

1. Calculate standard frictional mortar contact with holding the material displacements constant (*Lagrangean* step)
2. Determine wear in spatial configuration by employing nodal quantities from (1)
3. Solve *Eulerian* step in spatial configuration with calculated wear as Dirichlet boundary condition
4. Map resulting spatial nodal positions to material configuration (Advection map)

Configurations of slave body:



Arbitrary Lagrangean-Eulerian formulation - ALE



Employing fractional step method [4] as decoupled solution procedure:

1. Pure *Lagrangean* step
2. Pure *Eulerian* step

- ALE-formulation is used as an auxiliary structure
- Purely Dirichlet-based *Eulerian* step to relocate the positions of nodes within the slave-body
- At least one surface of the body has to be fixed for the *Eulerian* step

→ Preventing degeneration of nodes when accumulated wear displacements are larger than element thickness [5]

Advection map

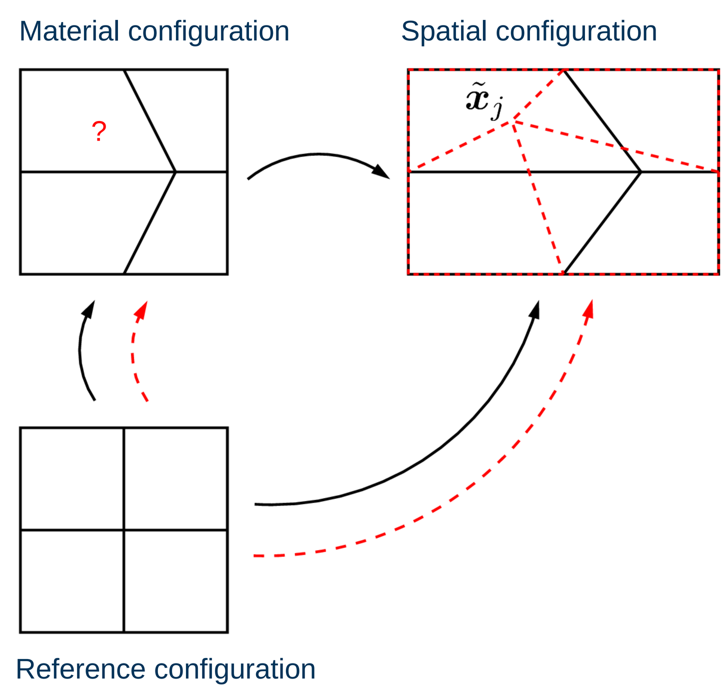
- The last step in the structure-ALE algorithm is well-known as Advection Map problem
- Based on employing the isoparametric properties of the Finite Element Method

1. Find the parameter space coordinate $\tilde{\boldsymbol{\xi}}_j$ by solving

$$\sum_{i=1}^{n_{ele}} N_i(\tilde{\boldsymbol{\xi}}_j)\boldsymbol{x}_i - \tilde{\boldsymbol{x}}_j = \boldsymbol{0}$$

2. Calculate material displacements with

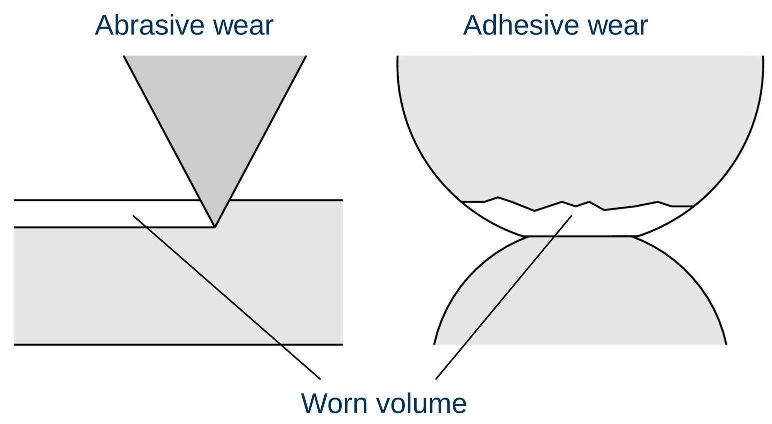
$$\tilde{\boldsymbol{d}}_{\phi j} = \boldsymbol{N}(\tilde{\boldsymbol{\xi}}_j) \cdot (\boldsymbol{X}^{ele} - \boldsymbol{\mathcal{X}}^{ele})$$



Archard's wear law

- General wear description for numerous types of wear, e.g. abrasive and adhesive wear [1]
- Definition of the worn volume as product of normal force P, sliding length S and the dimensionless wear coefficent K divided by the hardness H of the softer material: $V = K\frac{PS}{H}$
- Redefining the wear as pseudo displacement for employing it in the FEM-context

$$w = k_w |p_n| \cdot ||\boldsymbol{v}_{\tau,rel}|| \Delta t$$



- Including the wear expression as additional gap in the mortar framework yields the modified Karush-Kuhn-Tucker conditions [3]

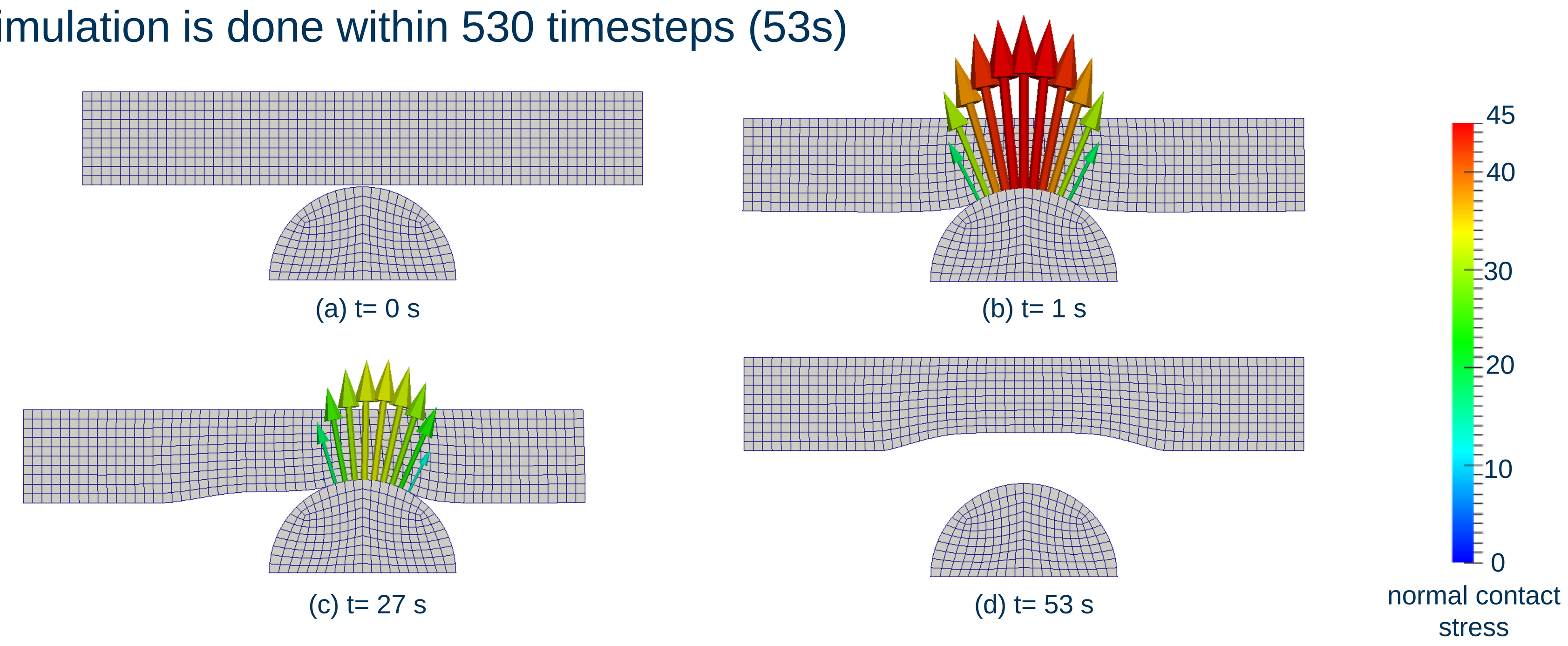
$$g(\boldsymbol{X}^{(1)},t) + w(\boldsymbol{X}^{(1)},t) \geq 0 \qquad p_n \leq 0 \qquad p_n\left(g(\boldsymbol{X}^{(1)},t) + w(\boldsymbol{X}^{(1)},t)\right) = 0$$

- The resulting discrete weighted wear increment and the unweighted wear increment are defined as $\Delta\tilde{w}_j = \int_{\gamma_c^{(1)}} \Phi_j(\lambda_n^h ||\boldsymbol{u}_{\tau,rel}^h||) d\gamma$ and $\Delta w_j = \frac{\Delta\tilde{w}_j}{\boldsymbol{D}_{jj}}$
- Removal of the weighting with the well-known mortar matrix $\boldsymbol{D}$ [5] is necessary because the weighted wear is rather a volumetric quantity than an applicable displacement

Results

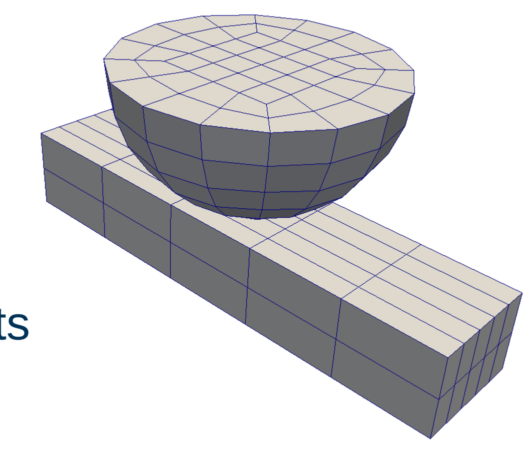
Oscillating 2D block on cylinder

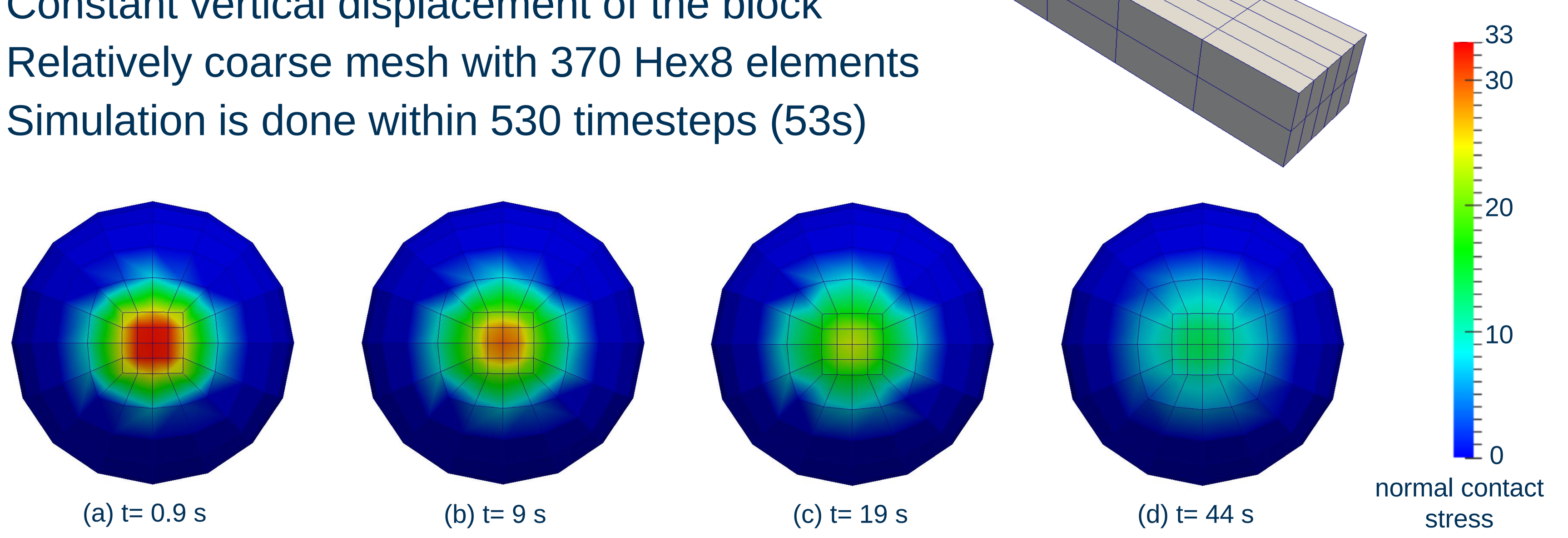
- Elastic beam is pressed onto a rigid cylinder [5]
- 5 Sinusoidal horizontal slidings
- Visualised normal contact tractions decreasing due to ongoing wear removal
- Simulation is done within 530 timesteps (53s)



Oscillating 3D block on sphere

- Rigid block is pressed onto an elastic sphere
- 5 Sinusoidal horizontal slidings
- Constant vertical displacement of the block
- Relatively coarse mesh with 370 Hex8 elements
- Simulation is done within 530 timesteps (53s)





Conclusions

- Wear modeled as stress-free displacements resulting in macroscopic structural effects
- Included wear variationally consistent as additional gap in constraint condition
- Embedded wear theory into the existing dual mortar framework
- Prevented element degeneration with *Eulerian* phase

Outlook

- Calculation of wear on both interacting bodies
- Consideration of wear on critical geometries, e.g. edges and corners
- Developing interaction of wear and thermomechanical effects

References

[1] V. L. Popov, Contact Mechanics and Friction, Physical Principles and Applications, *Springer*, 2010.
[2] F. Jourdan, Numerical wear modeling in dynamics and large strains: Application to knee joint prostheses, Wear, 261: 283-292, 2006.
[3] N. Strömberg, An augmented Lagrangian method for fretting problems, European Journal of Mechanics – A/Solids, 16:573-593, 1997.
[4] A. Huerta, F. Casadei, New ALE applications in non-linear fast transient solid dynamics, Engineering Computations, 11: 317-345, 1994.
[5] M. Gitterle, A dual mortar formulation for finite deformation frictional contact problems including wear and thermal coupling, PhD thesis, Technische Universität München, 2012