A temporal consistent monolithic approach to fluid-structure interaction enabling single field predictors

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Introduction

Motivation and goals

- Possibility to choose time integration scheme in structure and fluid field differently and tailored to the needs of the respective field
- Interpolation of interface traction in presence of different temporal discretizations in structure and fluid field in order to avoid possible stability problems [1]
- Enable field specific predictors in order to reduce computational costs

Problem Definition

Domain of interest

- Structural domain $\Omega^{\mathcal{S}}$ (governed by elastodynamics)
- Fluid-ALE domain $\Omega^{\mathcal{F}}$ (governed by Navier-Stokes equations)
- Fluid-structure interface Γ_{FSI}

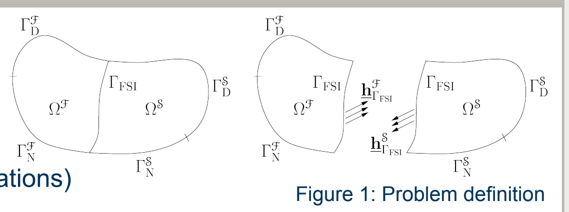

Figure 1: Problem definition

Coupling conditions at fluid-structure interface

- Weak enforcement of kinematic coupling condition by Lagrange multiplier field

$$\left(\delta\boldsymbol{\lambda}, \dot{\mathbf{d}}^{\mathcal{S}}_{\Gamma_{FSI}} - \dot{\mathbf{d}}^{\mathcal{G}}_{\Gamma_{FSI}}\right)_{\Gamma_{FSI}} = 0 \quad \text{in } \Gamma_{FSI} \times (0,T)$$

- Identify Lagrange multiplier field as interface traction $\boldsymbol{\lambda} = \mathbf{h}^{\mathcal{S}}_{\Gamma_{FSI}} = -\mathbf{h}^{\mathcal{F}}_{\Gamma_{FSI}}$

Discretization

Spatial discretization of structure and fluid field

- Mixed/hybrid finite elements for structure field
- Stabilized finite elements for fluid field
- Distinguish between interface DOFs (subscript Γ) and interior DOFs (subscript I)

Spatial disretization of Lagrange multiplier field

- Dual Mortar method for Lagrange multiplier field [2]
- Distinguish between *master* and *slave* side due to Mortar method

Temporal discretization

- Temporal discretization with *fully implicit, single-step, single-stage* time integration schemes with dynamic equilibrium at generalized mid-point t^m
- Different time integration schemes in structure and fluid field → $t^{\mathcal{S},m} \neq t^{\mathcal{F},m}$
- Single field predictors might result in a gap $\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p}$ at the interface that has to be accounted for in the discrete kinematic coupling conditions.

Discrete coupling conditions

- Kinematic continuity (accounting for possible predictors)

$$\mathcal{C}_{SF}\Delta\mathbf{d}^{\mathcal{S},n+1}_{\Gamma,i+1} + \delta_{i0}\,\mathcal{C}_{SF}\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p} = \tau\,\mathcal{C}_{FS}\Delta\mathbf{u}^{\mathcal{F},n+1}_{\Gamma,i+1} + \delta_{i0}\,\Delta t\,\mathcal{C}_{FS}\mathbf{u}^{\mathcal{F},n}_{\Gamma}$$

- Dynamic coupling conditions have to respect possible different time integrators in structure and fluid field. Hence, the Neumann-like interface traction has to be incorporated into the balances of linear momentum at the respective generalized mid-point $t^{\mathcal{S},m}$ or $t^{\mathcal{F},m}$ by interpolation (Fig. 2):

$$\mathbf{r}^{\mathcal{S},m}_{\boldsymbol{\lambda},i} = -\mathcal{C}^{T}_{SF}\left(a\boldsymbol{\lambda}^n + (1-a)\boldsymbol{\lambda}^{n+1}_i\right)$$

$$\mathbf{r}^{\mathcal{F},m}_{\boldsymbol{\lambda},i} = \mathcal{C}^{T}_{FS}\left(b\boldsymbol{\lambda}^n + (1-b)\boldsymbol{\lambda}^{n+1}_i\right)$$

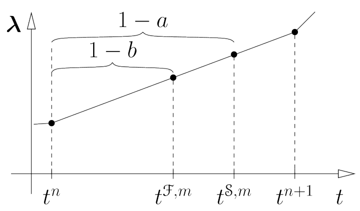

Figure 2: Temporal interpolation

Nonlinear FSI residual

$$\mathbf{r}^{FSI} = \begin{bmatrix} \mathbf{r}^{\mathcal{S}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{S}}_{\Gamma} \\ \mathbf{r}^{\mathcal{F}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{F}}_{\Gamma} \\ \mathbf{r}^{coupl} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{r}^{\mathcal{S}}_{\boldsymbol{\lambda},\Gamma} \\ \mathbf{0} \\ \mathbf{r}^{\mathcal{F}}_{\boldsymbol{\lambda},\Gamma} \\ \mathbf{0} \end{bmatrix} = \mathbf{0}$$

Linearization for Newton-Krylov solver

- Linearization with respect to unknown Lagrange multipliers

$$\frac{\partial \mathbf{r}^{\mathcal{S},m}_{\boldsymbol{\lambda},i}}{\partial \boldsymbol{\lambda}^{n+1}_i} = -(1-a)\,\mathcal{C}_{SF}, \quad \frac{\partial \mathbf{r}^{\mathcal{F},m}_{\boldsymbol{\lambda},i}}{\partial \boldsymbol{\lambda}^{n+1}_i} = (1-b)\,\mathcal{C}_{FS}$$

- Monolithic system of linearized equations

$$\begin{bmatrix} \frac{\partial \mathbf{r}^{\mathcal{S}}}{\partial \mathbf{x}^{\mathcal{S}}} & \mathbf{0} & \frac{\partial \mathbf{r}^{\mathcal{S}}}{\partial \boldsymbol{\lambda}} \\ \mathbf{0} & \frac{\partial \mathbf{r}^{\mathcal{F}}}{\partial \mathbf{x}^{\mathcal{F}}} & \frac{\partial \mathbf{r}^{\mathcal{F}}}{\partial \boldsymbol{\lambda}} \\ \frac{\partial \mathbf{r}^{coupl}}{\partial \mathbf{d}^{\mathcal{S}}_{\Gamma}} & \frac{\partial \mathbf{r}^{coupl}}{\partial \mathbf{u}^{\mathcal{F}}_{\Gamma}} & \mathbf{0} \end{bmatrix}^{n+1}_{i} \begin{bmatrix} \Delta\mathbf{x}^{\mathcal{S}} \\ \Delta\mathbf{x}^{\mathcal{F}} \\ \Delta\boldsymbol{\lambda} \end{bmatrix}^{n+1}_{i+1} = -\begin{bmatrix} \mathbf{r}^{\mathcal{S}} \\ \mathbf{r}^{\mathcal{F}} \\ \mathbf{r}^{coupl} \end{bmatrix}^{n+1}_{i}$$

Monolithic System of Equations

Linear System of Equations

- We exemplarily choose the structure field as master field → structure-governed interface motion → Mortar coupling operators: $\mathcal{C}_{SF} = \mathcal{M}, \ \mathcal{C}_{FS} = \mathcal{D}$

$$\begin{bmatrix} \mathcal{S}_{\mathrm{II}} & \mathcal{S}_{\mathrm{I}\Gamma} & & & & & \\ \mathcal{S}_{\Gamma\mathrm{I}} & \mathcal{S}_{\Gamma\Gamma} & & & & & -(1-a)\mathcal{M}^T \\ & & \mathcal{F}_{\mathrm{II}} & \mathcal{F}_{\mathrm{I}\Gamma} & \mathcal{F}^{\mathcal{G}}_{\mathrm{II}} & \mathcal{F}^{\mathcal{G}}_{\mathrm{I}\Gamma} & \\ & & \mathcal{F}_{\Gamma\mathrm{I}} & \mathcal{F}_{\Gamma\Gamma} & \mathcal{F}^{\mathcal{G}}_{\Gamma\mathrm{I}} & \mathcal{F}^{\mathcal{G}}_{\Gamma\Gamma} & (1-b)\mathcal{D}^T \\ & & & & \mathcal{A}_{\mathrm{II}} & \mathcal{A}_{\mathrm{I}\Gamma} & \\ & -\mathcal{M} & & \tau\mathcal{D} & & & \\ & \mathcal{M} & & & & -\mathcal{D} & \end{bmatrix}^{n+1}_{i} \begin{bmatrix} \Delta\mathbf{d}^{\mathcal{S}}_{\mathrm{I}} \\ \Delta\mathbf{d}^{\mathcal{S}}_{\Gamma} \\ \Delta\mathbf{u}^{\mathcal{F}}_{\mathrm{I}} \\ \Delta\mathbf{u}^{\mathcal{F}}_{\Gamma} \\ \Delta\mathbf{d}^{\mathcal{G}}_{\mathrm{I}} \\ \Delta\mathbf{d}^{\mathcal{G}}_{\Gamma} \\ \boldsymbol{\lambda} \end{bmatrix}^{n+1}_{i+1} =$$

$$= \begin{bmatrix} \mathbf{r}^{\mathcal{S}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{S}}_{\Gamma} \\ \mathbf{r}^{\mathcal{F}}_{\mathrm{I}} \\ \mathbf{r}^{\mathcal{F}}_{\Gamma} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}^{n+1}_{i} + \begin{bmatrix} \mathbf{0} \\ -a\mathcal{M}^T\boldsymbol{\lambda}^n \\ \mathbf{0} \\ b\mathcal{D}^T\boldsymbol{\lambda}^n \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \delta_{i0} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \Delta t\,\mathcal{D}\,\mathbf{u}^{\mathcal{F},n}_{\Gamma} - \mathcal{M}\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p} \\ \mathcal{M}\Delta\mathbf{d}^{\mathcal{S}}_{\Gamma,p} \end{bmatrix}$$

Condensation of Lagrange multipliers

- Use balance of linear momentum of slave interface DOFs for condensation
- Dual Mortar method leads to diagonal form of Mortar matrix $\mathcal{D}$ → Computationally cheap condensation of Lagrange multipliers and slave interface DOFs

Numerical Examples

Pseudo 1D FSI example with analytical solution

- Temporal convergence study with different time integrators in structure and fluid field (Fig. 4)
- Overall order of accuracy depends on single field accuracy → second order accuracy only if all time integrators are second order accurate

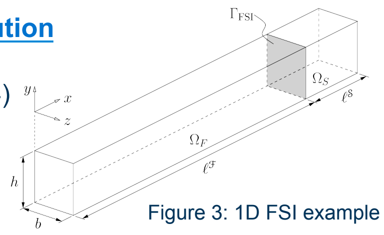

Figure 3: 1D FSI example

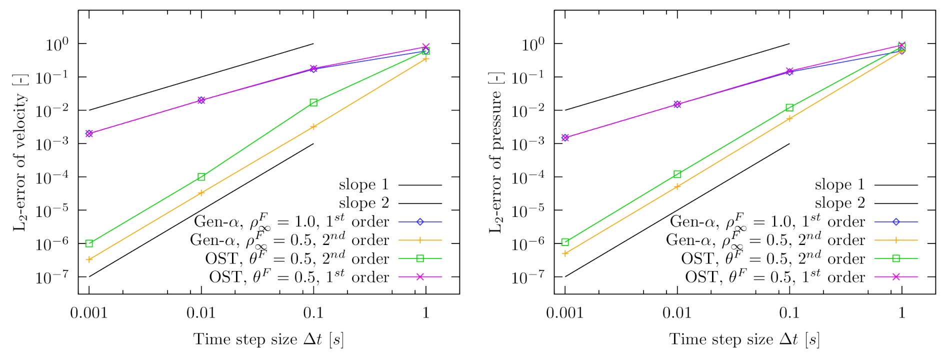

Figure 4: Temporal convergence of L2-error in velocity and pressure field

2D leaky driven cavity with flexible bottom

- Number of linear iterations reflects computational costs
- Reference solution without predictor (ConstDis)
- Reduction of number of linear iterations by 10% on average by employing simple predictors in structure field like constant velocity assumption (ConstVel) or constant acceleration assumption (ConstAcc) (see Fig. 5)

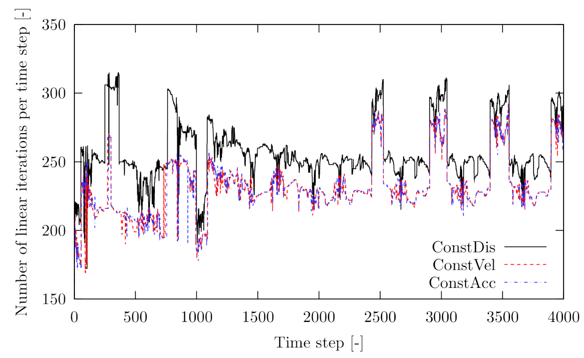

Figure 5: Saving of linear iterations as measure of computational costs

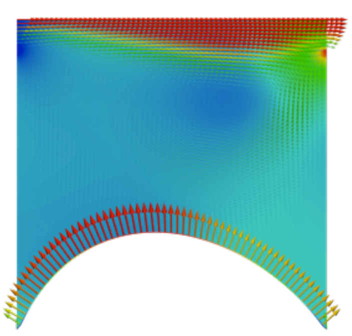

Figure 6: Velocity, pressure and Lagrange multiplier field

References

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