Continuous turn over of Collagen

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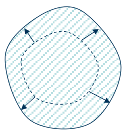
Introduction

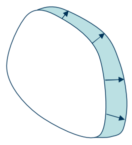
Growth

- Change in mass: open system thermodynamics

Volumetric growth
Swelling, arterial tissue, ...



Surface growth
Plating, wear, horns, shells, thrombus, ...



Remodeling

- Change of properties: reorientation of fiber directions, bones, ...

Aim

- Modeling the continuous turn over of collagen

Constraint mixture approach [1]

Material

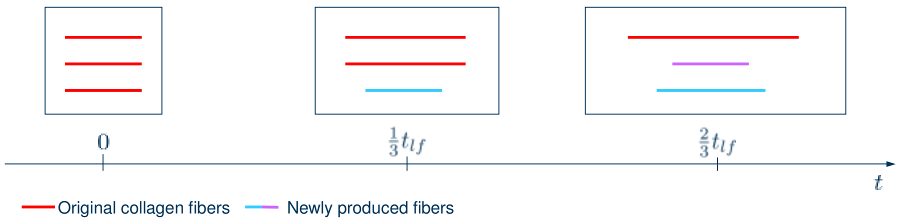
- Amorphous ground substance including elastin $\Psi^e = \frac{\mu}{2}(I_1 - 3) + \frac{\mu}{2\beta}(I_3^{-\beta} - 1)$
- Collagen fiber family $\Psi^l = \frac{k_1}{2k_2}(e^{k_2(\lambda_l^2-1)^2} - 1)$ with $\lambda_l^2 = \boldsymbol{a}_l^T \boldsymbol{C} \boldsymbol{a}_l$ oriented axial, circumferential and diagonally
- Volumetric strain energy $W^{vol} = \frac{\kappa}{2}(I_3 - 1)^2$
- Total strain energy function

$$W = \phi^e \Psi^e + \sum_{l=1}^{4} \phi^l \Psi^l + W^{vol} = W^e + \sum_{l=1}^{4} W^l + W^{vol}$$

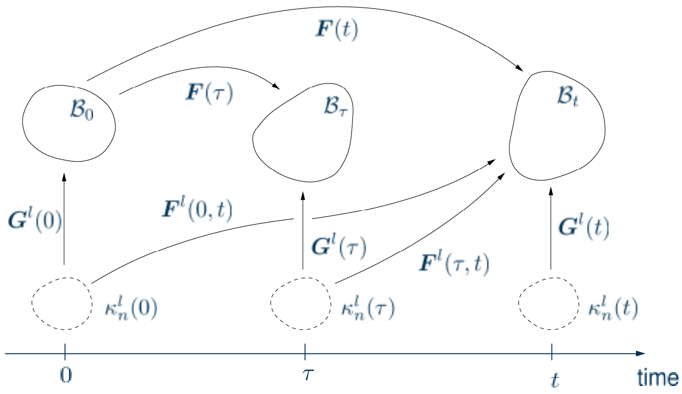
- All constituents are compatible in spatial configuration

Turn over of collagen

- Degradation of old collagen fibers
- Production of new collagen at a preferred prestretched state



- Scheme of important configurations



$\mathcal{B}_0$ Reference configuration
$\mathcal{B}_t$ Current configuration
$\boldsymbol{F}(t)$ Deformation gradient
$\kappa_n^l(\tau)$ Natural configuration of collagen fiber family l produced at time τ
$\boldsymbol{G}^l(t)$ Prestretch

Consequences for material modeling

- Mass production rate

$$m^l(\tau) = m_0^l \left[1 + K\left(\frac{\sigma^l(\tau)}{\sigma_h} - 1\right)\right]$$

- Variable mass fraction for collagen fiber families

$$W^l(t) = \int_{-\infty}^{t} \frac{m^l(\tau)}{\rho} q^l(\tau,t)\Psi^l\left(\lambda_l(\tau,t)\right) \mathrm{d}\tau$$

- Mass change in volumetric strain energy

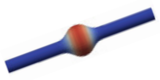
$$W^{vol} = \frac{\kappa}{2}\left(I_3 - \frac{M(t)}{M(0)}\right)^2$$

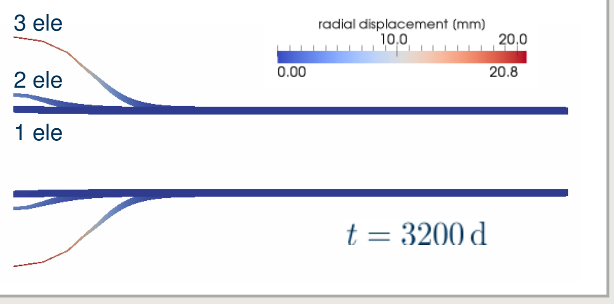
Exemplary formation of an aneurysm

- Geometry:
  - –Inner radius $R_i = 10.0\,\mathrm{mm}$
  - –Thickness $H = 2.0\,\mathrm{mm}$
  - –Length $L = 150.0\,\mathrm{mm}$
- Load:
  - –Inner pressure $P = 13.322\,\mathrm{kPa}$ (mean)
- Growth parameters:
  - –Time $t_{lf} = 118.0\,\mathrm{d},\ \Delta t = 1.0$
  - –Growth $\rho = 1.05 \cdot 10^{-3}\,\frac{\mathrm{g}}{\mathrm{mm}^3},\ K = 0.125$

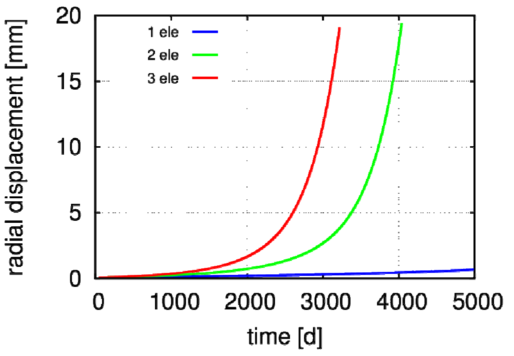
Axisymmetric AAA

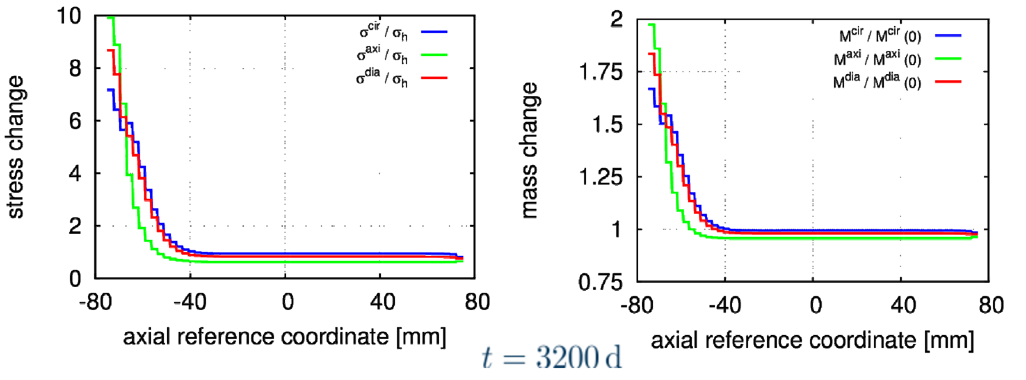
- Triggered by reduced stiffness of elastin at one end of the cylinder in 1, 2 and 3 element rings
- Aneurysms of different size develop





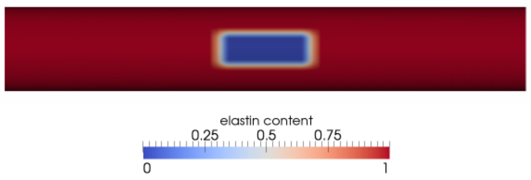
- Aneurysm develops abruptly
- Aneurysm formation local but not limited to initiation zone
- Largest stress/mass increase in axial fiber family, smallest in circumferential fibers

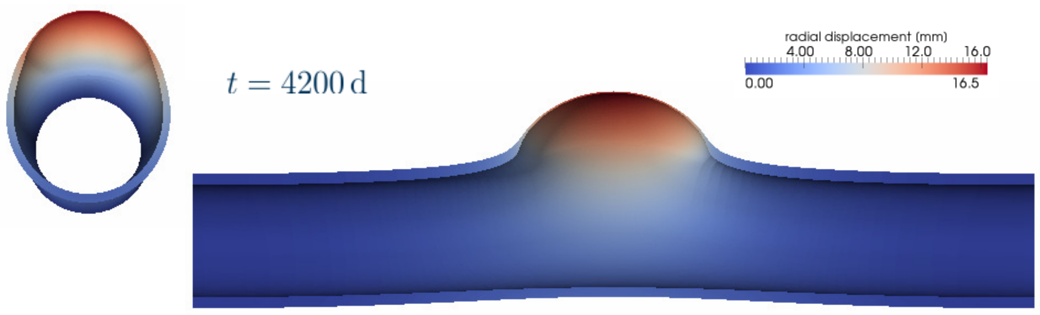




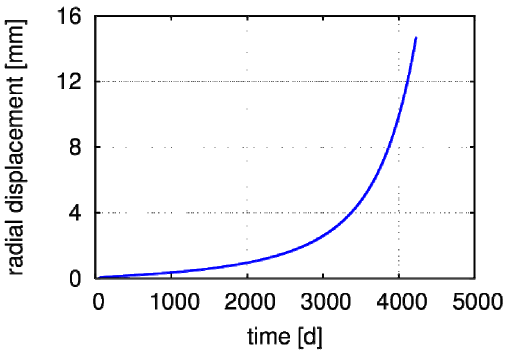
Asymmetric AAA

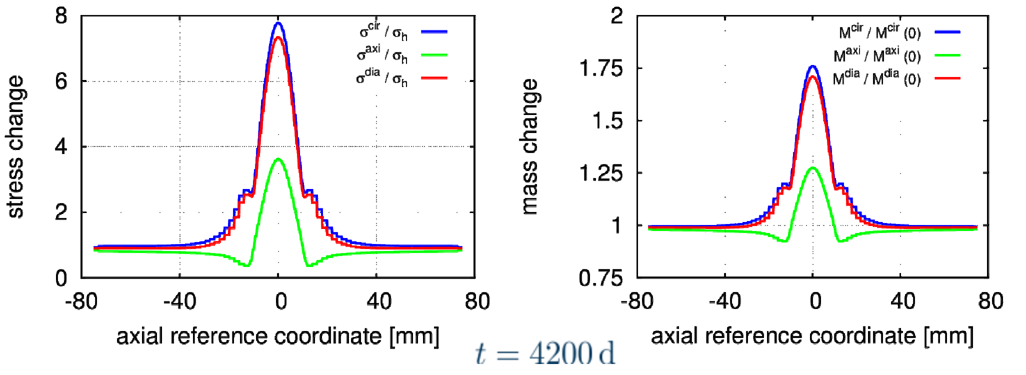
- Reduction of elastin stiffness in a small area to initiate the formation of a one sided aneurysm
- Enlargement mainly in one direction
- Opposing side concave
- Rest of the artery remains stable





- Time course similar to the axisymmetric 2 ele case
- Degradation of elastin more pronounced in axial direction → stress/mass change in axial direction smaller than before
- Reduction of stress/mass in axial direction at the boundary of the damage zone



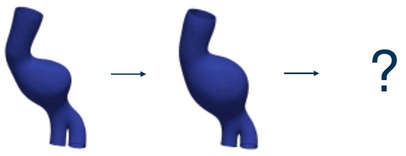


Conclusion

- Elastin loss initiates pathological collagen remodeling process
- Aneurysm formation can be reproduced qualitatively

Outlook

- Determination of growth parameters from clinical data
- Application to patient specific geometries
- Aneurysm growth prediction



References

[1] Humphrey JD, Rajagopal KR, A constrained mixture model for arterial adaptations to a sustained step change in blood flow, *Biomech. Model. Mechanobiol.*, 2: 109-126, 2003.