Efficient Uncertainty Quantification in Patient Specific Assessment of AAA Rupture Risk

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Introduction

AAA rupture risk prediction using FEM

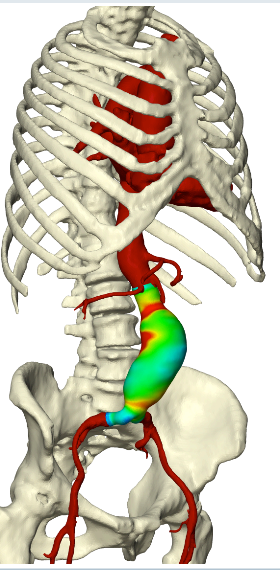
- Computational rupture risk indicators are superior to the diameter criterion [1]
- Most “patient-specific“ models use population averaged model parameters

Existing uncertainties

- Computational geometries (e.g. wall thickness, stress free configuration)
- Boundary conditions (e.g. intra luminal pressure)
- Physical parameters (e.g. constitutive parameters)

Towards more reliable rupture risk prediction

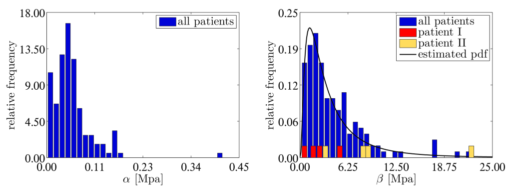
- In absence of truly patient-specific parameters: Include uncertainties in the FEM analysis
- As a first step in this direction uncertain constitutive parameters are considered



Stochastic Constitutive Law

Experimental research

- Tensile tests reveal significant inter- and intra-patient variations
- Random field approach to model fluctuations in the parameter β



Material behavior

- Stochastic extension of Raghavan and Vorp‘s hyperelastic constitutive model for aneurysmatic arterial wall [2]

$$\Psi(I_1, J, \boldsymbol{x}, \boldsymbol{\xi}) = \alpha(\bar{I}_1 - 3) + \beta(\boldsymbol{x}, \boldsymbol{\xi})(\bar{I}_1 - 3)^2 + \frac{\kappa}{\eta^2}(\eta \ln J + J^{-\eta} - 1)$$

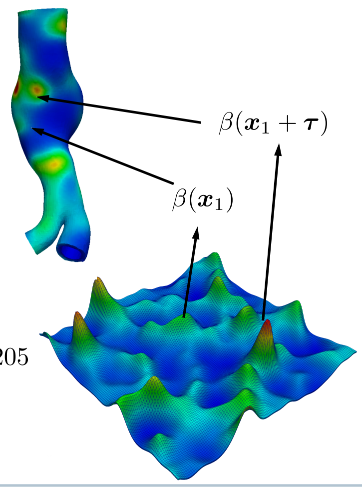
- The parameter $\beta(\boldsymbol{x}, \boldsymbol{\xi})$ is modeled as a three dimensional random field [4]:

Marginal probability density:

$$p_\beta(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}; \quad \mu = 1.0857 \;\; \sigma = 0.9205$$

Autocorrelation function:

$$R_{\mathrm{NG}}(\boldsymbol{\tau}) = \sigma_\beta^2 e^{-(\frac{|\boldsymbol{\tau}|}{d})^2}$$



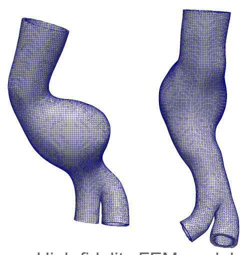
Propagation of Uncertainties

Monte Carlo

- Estimate the distribution of the quantity of interest $\pi_y(y)$ directly using:

$$\pi_y(y) \approx \frac{1}{N_{\mathrm{SAM}}} \sum_{i=1}^{N_{\mathrm{SAM}}} \delta_{y^{(i)}}(y(\boldsymbol{\xi}))$$

- Minimal implementational overhead
- Extremely expensive, verification only



High fidelity FEM models

Incorporation of approximate models [3]

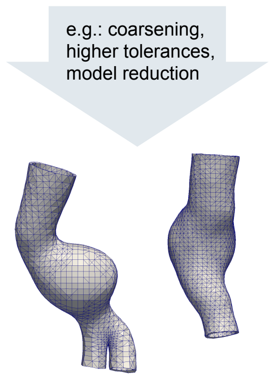
- Do sampling on cheap approximate model
- Establish a probabilistic link between high fidelity and approximate model with Bayesian regression

$$\pi_y(y) = \int p(y|x)\pi_x(x)dx$$

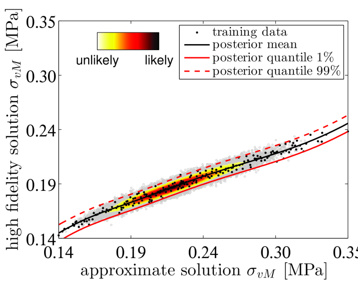
$$p(y|x) \approx p(y|x, \boldsymbol{\theta}, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y - f(x,\boldsymbol{\theta}))^2}{2\sigma^2}\right\}$$

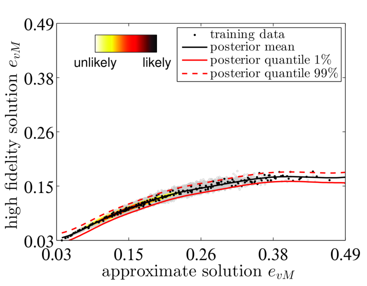
- Bayesian regression model $f(x, \boldsymbol{\theta})$
- Determination of posterior of the model parameters using Bayes‘ rule and advanced SMC scheme with few selected training samples of high fidelity model

$$\pi(\boldsymbol{\theta}) = \frac{p((x_{1:n}, y_{1:n})|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(x_{1:n}, y_{1:n})} \propto p((x_{1:n}j, y_{1:n})|\boldsymbol{\theta})p(\boldsymbol{\theta})$$



Approximate FEM models

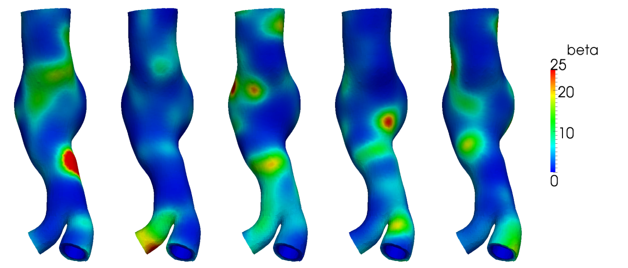




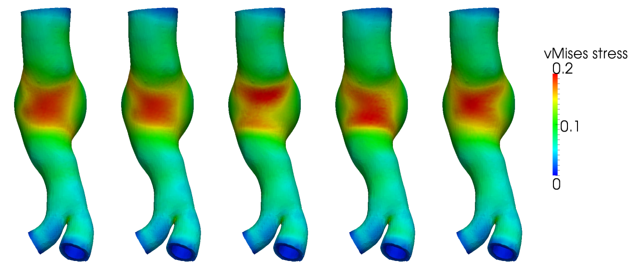
Results

Impact on stress

- Overall spatial pattern remains very similar
- Von Mises stress only mildly depends on β
- Stress state is decoupled from the strain state.
- Static equilibrium in deformed/imaged configuration.
- Prestress determines stress state to a large extend
- AAA is approximately statically determinate



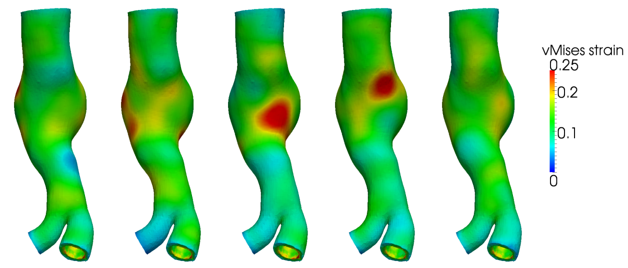
Different realizations of spatial distribution of $\beta(x,\xi)$ [MPa]



Resulting von Mises stress σ_{vM} [MPa]

Impact on strains

- Significant impact on spatial strain pattern
- Very high COV
- Implications for strain or strain energy based damage models [5]



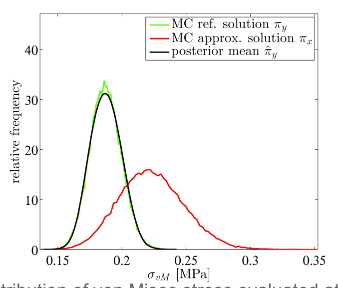
Resulting von Mises strain e_{vM}

Accuracy

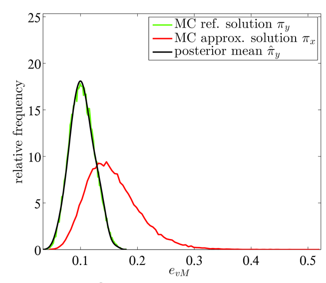
- Posterior mean approximation:

$$\hat{\pi}_y(y) = \mathbb{E}_{\boldsymbol{\theta},\sigma}[\pi_y(y)] = \int p(y|x, \boldsymbol{\theta}, \sigma^{-2})\pi_x(x)\pi_{\boldsymbol{\theta},\sigma^{-2}}(\boldsymbol{\theta}, \boldsymbol{\sigma^{-2}})d\boldsymbol{\theta}d\sigma^{-2}dx$$

allows accurate prediction of MC reference solution

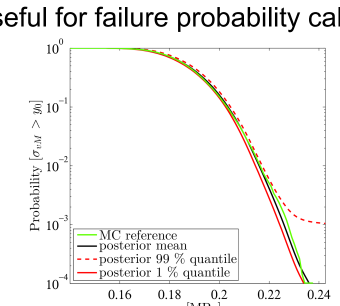


Distribution of von Mises stress evaluated at the center of the dorsal surface of the aneurysm sac

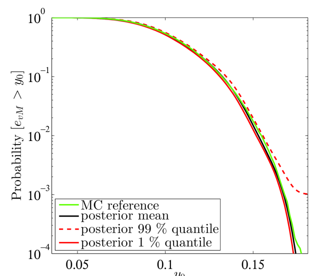


Distribution of von Mises strain evaluated at the center of the ventral surface of the aneurysm sac

- Confidence intervals based on quantiles of the posterior
- Useful for failure probability calculations



Failure probability at the center of the dorsal surface of the aneurysm sac for different failure thresholds



Failure probability at the center of the ventral surface of the aneurysm sac for different failure thresholds

Efficiency

- Tremendous reduction in computational costs
- Up to factor 40 cheaper than direct MC on high fidelity model
- Additional potential through numerical continuation schemes

Conclusion and Outlook

- Population mean values are not good enough for patient-specific assessment of AAA rupture risk
- Strains exhibits large variations whereas stresses are only mildly affected
- Advanced UQ methods cut down the cost to acceptable level
- Include more sources of uncertainty and apply method to a larger patient cohort

References

[1] Maier A, Gee MW, Reeps C, Pongratz J, Eckstein HH, Wall WA. A comparison of diameter, wall stress, and rupture potential index for abdominal aortic aneurysm rupture risk rrediction. *Annals of Biomedical Engineering* May 2010; 38(10):3124–3134.

[2] Raghavan M, Vorp D. Toward a biomechanical tool to evaluate rupture potential of abdominal aortic aneurysm: identification of a finite strain constitutive model and evaluation of its applicability. *Journal of Biomechanics* 2000; 33(4):475–482.

[3] Koutsourelakis P. Accurate uncertainty quantification using inaccurate models. *SIAM Journal of Scientific Computing* 2009; 31(5): 3274–3300.

[4] Shields M, Deodatis G. A simple and efficient methodology to approximate a general non-Gaussian stationary stochastic process by a translation process. *Probabilistic Engineering Mechanics* 2011.

[5] Marini G, Maier A, Reeps C, Eckstein HH, Wall WA, Gee MW. A continuum description of the damage process in the arterial wall of abdominal aortic aneurysms. *International Journal for Numerical Methods in Biomedical Engineering* Sep 2011; 28(1):87–99.