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PROCUREMENT PLANNING IN OIL REFINING INDUSTRIES CONSIDERING BLENDING OPERATIONS

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Abstract
This paper addresses procurement planning in oil refining, which has until now only had limited attention in the literature. We introduce a mixed integer nonlinear programming (MINLP) model and develop a novel two-stage solution approach, which aims at computational efficiency while addressing the problems due to discrepancies between a non-linear and a linearized formulation. The proposed model covers realistic settings by allowing the blending of crude oil in storage tanks, by modeling storage tanks and relevant processing units individually, and by handling more crude oil types and quality parameters than in previous literature. The developed approach is tested using historical data from Statoil A/S as well as through a comprehensive numerical analysis. The approach generates a feasible procurement plan within acceptable computation time, is able to quickly adjust an existing plan to take advantage of individual procurement opportunities, and can be used within a rolling time horizon scheme.

Key words: Procurement planning • oil refining industry • mixed integer non-linear programming • solution approach • crude oil scheduling • decision support

1. Introduction

At oil refineries, crude oil is processed and refined into petroleum products such as gasoline, kerosene and diesel oil. Due to increased competition and low refining margins oil refining activities need to be operated efficiently. Furthermore, the oil refining industry is one of the most complex chemical industries, with many different processes and chemical reactions, and the industry is regulated by strict environmental regulations.

Challenges in oil refining operations range from strategic to operational, and from purchasing raw materials to distribution and sales. There is great economic potential in enterprise-wide optimization; however, a lack of comprehensive optimization models and computational tools are one of the major issues that must be addressed (Shah et al. 2011; Grossmann, 2012). In this paper, we are presenting a step to this direction by presenting a decision support model for procurement planning.

Planning the procurement of crude oils is strongly linked to crude oil scheduling, as it has to be assured that the procured crude oils can be processed by the refinery. However, there are a number of marked differences between the crude oil purchasing problem and the crude oil scheduling problem. The main and most obvious difference is the procurement decision that is at the center of the crude oil purchasing problem, whereas in crude oil scheduling, all procurement decisions are assumed to be given. Crude oil purchasing is an extremely important step in refinery operations, since it directly impacts finished goods quality and quantity and can result in large economic benefit. A second key difference is that procurement planning and crude oil scheduling have significantly different planning horizons. Crude oil scheduling problems are usually treated with a time horizon of 7 – 10 days, whereas procurement planning is usually carried out with a planning horizon of up to three months. Accordingly, the planning granularity is days or longer periods in procurement planning and hours in crude oil scheduling.

In the next section, the procurement planning problem is defined. An overview of the existing literature related to procurement planning in the oil refining industries is given in Section 3. In Section 4 we introduce a MINLP model for the problem presented in Section 2. A corresponding two-stage solution approach for the planning model is presented in Section 5. In Section 6 we illustrate the dynamic application of the proposed approach by using real life examples. In Section 7 we perform a comprehensive numerical analysis, in which we test the quality of the proposed solution method. Finally, a summary of the main findings is given in Section 8.

2. Procurement planning in the oil refining industry

The main input to oil refineries is crude oil, including condensate. Refineries are generally designed to process a wide range of crude oil types into finished goods, such as gasoline, diesel oil and jet fuel. Refineries have the flexibility to shift between crude oils and process various crude blends to adjust to market conditions. Crude oils can be transported from the petroleum fields to the refinery in various ways. The most common modes of transportation are pipeline and marine transportation.

The trading unit of a refinery is responsible for all purchasing decisions that relate to raw material supply. Some refineries get most of their supply through long-term contracts, whereas others buy crude on the spot market. Either way, the timing of purchase is always important. The crude commodity market is very dynamic; prices fluctuate constantly. Factors such as a sudden increase in demand, refinery outages, and supply cutbacks, significantly affect the market prices. Procurement planners aim at procuring crude oil with high refining margins, which is defined as the difference between purchasing price of crude oil (including shipment costs) and value of the refined petroleum products. Crude oil arrives at the refinery according to reached agreements, and is allocated to crude oil storage tanks. The storage tanks have floating roofs in order to minimize evaporation losses. Because of these floating roofs, each tank always requires a minimum crude oil level to avoid damage to the roof when the tank goes empty.

In order to make sure that the procurement decisions are feasible and optimal to the refinery, it is necessary to consider the purchasing decisions, the arrival of purchased shipments, the flow of material into storage tanks, the tank connections, and the feed into crude distillation units (CDUs). Figure 1 illustrates the typical procurement process.

Crude oil is usually classified based on three key components: sulfur, specific gravity content and a total acid number (TAN). The specific gravity is the industry's measure of density and it gives an

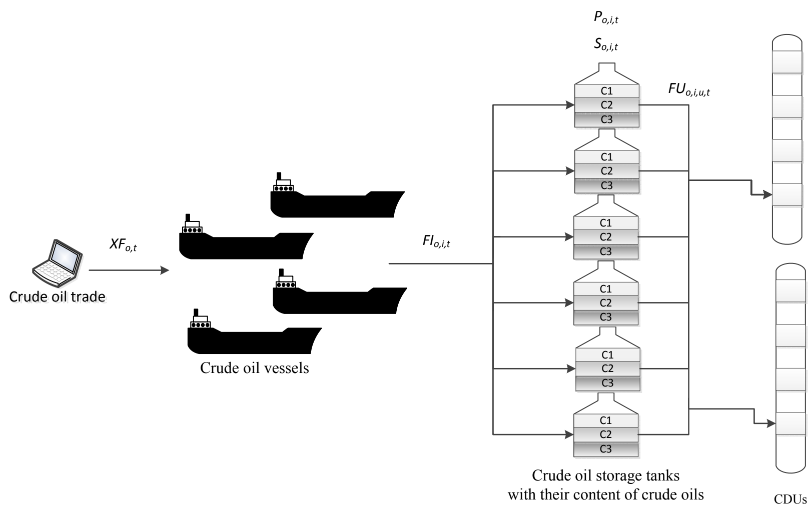

Figure 1. A typical procurement process.

indication whether the crude is heavy or light. Similarly, the sulfur amount in crude oil indicates if a crude is referred to as sweet our sour. The TAN represents a composite of acids present in the oil. Tracking these key components is important, since they affect the downstream processing, including product yields and product profitability.

It is therefore crucial for refineries to have a clear overview of what kind of crude oil or crude oil blend is stored in each storage tank. In practice, refineries often blend similar crude oils to preserve qualities and to make it easier to respect the processing limits of the CDUs in the further processing of the crude oils or their blends. Planning support, such as the work presented in this paper, should therefore also be able to account for product quality in relation to the blending of crude oils – both in the storage tanks and in subsequent processing.

Oil refineries operate 24 hours a day. A shut down is extremely costly; it results in major material losses and extreme cleaning and security activities. A procurement planner must make sure that there is always enough supply to the refinery to avoid shutdowns. At the same time, the supply should not exceed the storage capacities of the refinery and the quality of the supply has to be feasible for the downstream processing units.

Quite often, due to unexpected events, advantageous market opportunities emerge in the crude oil commodity market, where a shipment of some type of crude oil can be bought for less than the regular market price. This kind of situation can for example happen if there are breakdowns at other refineries that then have to quickly get rid of incoming shipments of crude oil. If a refinery is offered such a deal, the economic benefits of taking in such a shipment can be enormous, as will be illustrated in Section 6 of this paper. However, figuring out whether the crude oil is processible by the refinery, and making sure it fits the refinery's current feedstock profile and procurement plan is often very difficult. As the procurement decisions are currently made manually, the planner tends to rely on prior experience and therefore cannot always assess the impact of taking in a procurement opportunity. This means that the refinery often misses out on these special market opportunities.

All of the above-mentioned challenges stress the need for advanced decision support for procurement planners. In this paper, we present a MINLP model and solution approach which can be incorporated in a general decision support tool for procurement planning. Furthermore, the model can also indicate whether a specific shipment can be used at the refinery or not, and if that shipment needs to be compensated for by some other purchase in order for it to be processible by the refinery. The planner will see how procuring this specific shipment of crude oil will change the prior plan, including other future purchases, and how it will affect profitability.

3. Literature review

Oil refineries have used optimization techniques for a long time, ever since the introduction of linear programming (LP) (Symonds 1955; Manne 1956). However, LP models alone proved to be insufficient, since problems occurring in the petroleum refining industry very often lead to mixed integer and/or nonlinear problems.

The operations of a typical oil refinery start with the scheduling of crude oil deliveries. A typical crude oil scheduling problem deals with creating a detailed schedule for unloading arriving crude oil vessels, loading storage tanks and controlling crude blends and crude flow towards other processing units. It is a challenging problem and consequently substantial work in the literature has been devoted to address and solve this problem. Shah (1996) presented one of the earliest mathematical formulations for the crude oil scheduling problem, but does not consider blending different types of crudes. Lee et al. (1996) consider blending crude oils and propose a MIP model, in which all nonlinearities caused by the calculation of the key components of the crude oil blends are approximated by linear constraints. However, as was pointed out by Li et al. (2002), the main problem with this linearization is the so-called composition discrepancy. The reformulated linear constraints are not strict enough, as they do not force the concentration of the outlet flow from a storage tank to be equal to the concentration inside the tank. The composition discrepancy problem is serious, as it can lead to infeasibility and refinery shutdowns. Li et al. (2002) introduce an alternative solution approach to the crude oil scheduling problem, which decomposes the MINLP into a MIP and NLP, to avoid composition problems. Reddy et al. (2004) prove that the approach proposed by Li et al. (2002) does not always find a feasible solution, even if one exists. Furthermore, they propose a rolling-horizon solution algorithm for solving the MINLP model, in which they solve series of MIPs and obtain crude schedules that do not suffer from a composition discrepancy. The shortcomings of their solution approach are however the long solution times and the inability to give a feasible solution for industrial-sized problems. Li et al. (2007) further develop the work by Reddy et al. (2004) by ensuring crude oil feed quality in their model and a relaxation strategy to increase the speed of their solution approach. They demonstrate their improved solution with 24 industry-scale examples and the solutions give profits within 6% of an upper bound and solution times ranging from 28 minutes up to 5.5 hours.

Integration of different decision making levels in the refinery supply chain can be difficult due to the complexity of the operations. Some recent work has however been done on integrated planning and scheduling. For instance, Mendez et al. (2006) present a MILP-based method that addresses the simultaneous optimization of product blending and production unit scheduling. The product blending problem involves mixing the intermediate products (i.e. the output from the CDUs) with additives to produce certain oil products. By integrating the product blending problem with production scheduling problem, Mendez et al. (2006) aim at maximizing production profit while satisfying both the process and operations constraints, and the product demands and quality specifications. Erdirik-Dogan and Grossmann (2008) present a MILP model for the simultaneous planning and scheduling of single-stage multiproduct continuous plants with parallel units. While their model and solution approach show promising results, their planning framework is not specially designed for oil refining operations, and does not include the important blending constraints.

In this paper, we also focus on the integration of different decision problems, but our focus is on the procurement of crude oils, which requires integration with production scheduling to determine which crude oils can actually be processed profitably by the refinery and should hence be purchased. Little attention has been given to procurement planning in the petroleum refining industry. Pongsakdi et al. (2006) and Lakkhanawat and Bagajewicz (2008) address the issue of uncertainty and financial risk in refinery operations planning. A part of their problem is determining how much of each available crude oil one must purchase and decide on the anticipated production level given demand forecasts. The optimization model used in both papers is based on the network structure proposed by Pinto et al. (2000). The blending equations in the model are linearized by using bounds on the properties of each flow in the model; similar to the approach by Lee et al. (1996). Even though the model presented in Pongsakdi et al. (2006) and in Lakkhanawat and

Bagajewicz (2008) indicates how much of each available crude oil to purchase, it does not give sufficient support to procurement planners. It does not directly indicate what crudes to buy and when to buy, nor does it indicate whether a special procurement opportunity can be worth undertaking. It merely reports how much crude oil is needed for the refinery to be able to fulfill demand forecasts, and it does not take initial inventory into account. Moreover, the model presented in the papers can only handle three time periods and six crude oil types, which also indicates that the model is not suitable as decision support tool for procurement planners.

Julka et al. (2002a; 2002b) propose an agent-based framework for decision support and demonstrate its application through crude procurement modeling and simulation. Their framework does not directly indicate which crude oils should be procured; it only gives insight into how the business responds to changes in procurement policies by performing ‘what-if’ studies involved in the framework. Göthe-Lundgren et al. (2002) introduce a MIP model for oil refining planning and scheduling, and claim that their model encourages integration between procurement planning and production scheduling by letting planners test and analyze alternative procurement plans and production schedules. However, their planning model focuses on the production and operation decisions, and does not generate procurement plans or support procurement planners in other ways.

Most recent and most closely related to our work is the contribution by Zhang et al. (2012). They present a MINLP model for the integration of short-term crude oil blending and purchase planning, and are as such also addressing the importance of the interrelation between these planning problems, providing a significant contribution towards decision support for procurement planning. The focus of their work is on the analysis of operational flexibility; they characterize the capability of a refinery for handling delivery delay uncertainties and aim to quantify the relationship of profit maximization and flexibility maximization. However, their modeling approach does contain a number of simplifications. It considers only a single quality parameter (sulfur) and limits the number of crude oil types to the number of tanks. They do not model storage tanks or CDUs at the refinery in detail, but represent these units as aggregated logical units. This has the advantage that the resulting problem definition is easily treatable by standard solution methodologies, but it has the disadvantage that detailed control variables and operating rules, such as the inventory profile per storage tank and especially the quality parameters and feed rates towards each CDU (and their composition), cannot be included in their model. Furthermore, their approach segregates crude oil in storage tanks, making the model only applicable for refineries that have separate tanks for every crude oil they procure and process. These assumptions are however often not met in practice. Planning support, such as the work presented in this paper, should therefore also be able to account for product quality in relation to the blending of crude oils – both in the storage tanks and in subsequent processing.

Summarizing, the existing models in the literature do not provide the necessary decision support for procurement planning. Therefore we develop novel approaches for formulating and solving the procurement planning problem. The problem is modeled as a MINLP model and links to state-of-the-art methods for solving the crude oil scheduling problem. For typical planning horizons, the model indicates what shipments of crude oil should be procured; it keeps track of inventory compositions and levels, and gives insight into profitable crude oil blends that should be processed. Furthermore, the model can help planners to schedule individual procurement opportunities, in instances where distressed cargoes of crude oil can be bought at a low price.

The overall contribution of this paper is to address the procurement planning problem for oil refining industries, which has until now had limited attention in the literature. The paper makes three more specific contributions, which are (*i*) a formal representation of the problem where all relevant production units and operational rules are included (*ii*) the development of a solution approach able to solve realistic industrial-sized problems in acceptable (and relatively robust) solution times, and (*iii*) a demonstration of the practical applicability of the model and solution approach, using real life examples. Compared to previous work, we allow for the commonly seen practice of blending crude oils in the storage tanks, we model storage tanks and relevant processing units individually to be able to model CDU feed rates in detail and take relevant operating rules into account, and allow for the handling of more crude oil types and quality parameters.

4. Mathematical Model

Nomenclature

Sets:

$t \in T$	*set of time periods*
$o \in O$	*set of crude oil types*
$i \in I$	*set of crude storage tanks*
$u \in U$	*set of CDUs*
$(o,i) \in CI$	*set of pairs (o,i) such that tank i cannot hold crude oil type o*
$k \in K$	*set of key components (e.g. Sulfur, gravity, and TAN)*

Parameters:

RV_o	*Refining margin of crude oil o (\$/m^3)*
VF_o	*Fixed shipment size for crude oil type o (m^3)*
$VesselLim$	*The number of vessels allowed to unload crude oil during the same time period*
TL	*Maximum number of tanks used when unloading from arriving vessels*
CL	*Maximum number of CDUs a single storage tank can feed simultaneously*
UL	*Maximum number of storage tanks feeding a single CDU*
FI^-	*Minimum amount of crude oil flowing into tank per period (m^3)*
FI^+	*Maximum amount of crude oil flowing into tank per period (m^3)*
FU^-	*Minimum amount of crude oil flowing into CDU per period (m^3)*
FU^+	*Maximum amount of crude oil flowing into CDU at a time (m^3)*

$Smin_i$	*Minimum amount of inventory allowed in crude tank i (m^3)*
$Smax_i$	*Maximum amount of inventory allowed in crude tank i (m^3)*
Ssafety	*Safety stock limit (m^3)*
$Sinit_{o,i}$	*Initial inventory of crude oil o in crude tank i (m^3)*
$KC_{k,o}$	*Share of key component k in crude oil type o (%)*
${QL_{k,u}}^+$	*Maximum share of key component k in oil blend processed in CDU u*
${QL_{k,u}}^-$	*Minimum share of key component k in oil blend processed in CDU u*
$Pmin_{o,i}$	*Minimum share of crude oil o that should be stored in tank i (%)*
$Pmax_{o,i}$	*Maximum share of crude oil o that can be stored in tank i (%)*
Variables:	
$FI_{o,i,t}$	*Amount of crude oil o flowing into tank i during period t (m^3)*
$S_{o,i,t}$	*Amount of crude o in tank i at the end of period t (m^3)*
$Stot_{i,t}$	*Amount of inventory in tank i at the end of period t (m^3)*
$FU_{o,i,u,t}$	*Amount of crude oil o flowing from tank i to CDU u during period t (m^3)*
$FIU_{i,u,t}$	*Amount of crude oil blend flowing from tank i to CDU u during period t (m^3)*
$XF_{o,t}$	*Binary variable, equal to 1 if a shipment of crude oil o is procured with delivery date t, 0 otherwise*
$XT_{o,i,t}$	*Binary variable, equal to 1 if tank i is receiving oil o during period t, 0 otherwise*
$XC_{i,u,t}$	*Binary variable, equal to 1 if tank i is feeding CDU u during period t, 0 otherwise*
$P_{o,i,t}$	*Share of crude oil o in tank i at the end of period t (%)*

When dealing with long planning horizons, aggregation often becomes necessary. Whenever applying aggregation, we make assumptions about detailed activities which will simplify the aggregated problem. However, one must be careful in making such assumptions since it can lead to aggregation errors. The following assumptions are made in our modeling approach:

1. Each time period represents 72 operating hours. In reality, purchased cargoes of crude have a three day delivery time-window and therefore a time unit of three days is acceptable at the

procurement planning level. The detailed scheduling of arrival of these shipments must be done on a less aggregate level and is usually included in the crude oil scheduling problem.

2. Due to the aggregate level of the planning model, demurrage is neglected. Demurrage is a penalty cost for delays in unloading of the vessel, also called detention charge. It is assumed that the excess time taken to discharge cargoes and the associated costs are accounted for at scheduling level.
3. All crude oil is assumed to be available at any given time and quantity. If a specific crude oil becomes unavailable during the planning horizon, the planner can force the decision variable for that specific crude to be equal to zero, so the model does not include that crude oil in the procurement plan.
4. Standard shipment sizes of 100,000 m^3 are assumed for each crude oil type. This is valid since in reality, refineries usually receive full shipments of crude. In some exceptional cases, planners can negotiate the shipment size. However, in the proposed model that option is excluded.
5. It is assumed that all oil products produced can be sold. This is the case for many oil refineries; the production quantity is constrained by the refinery's capacity but not by demand. Therefore demand constraints are excluded from the model. Varying demand is, however, reflected in varying refining margins.

As mentioned above, refineries have complex operations and processes. Most refineries define some general operating rules and procedures that are meant to reduce operating complexity and to increase controllability. These rules and procedures need to be respected when modeling any refinery operations. The following operating rules are considered in the illustrative examples and numerical tests presented in this paper:

1. When a storage tank is receiving crude oil from a crude vessel, the tank cannot feed a CDU at the same time
2. All storage tanks need to be left idle for some specific amount of time after receiving crude oil for brine settling and removal.
3. Only a limited number of storage tanks can feed a CDU simultaneously.
4. Each crude oil shipment has to be allocated into a limited number of storage tanks during crude unloading
5. Capacity limitations per crude storage tank have to be respected as well as the minimum total safety stock for the refinery as a whole.
6. In order to preserve the quality of the crude oil and to avoid infeasible crude oil blends, it is common to specify what crude oil can be blended together. Therefore we include the according operating rules.

The mathematical model developed to solve the procurement planning problem partly builds upon Reddy et al.'s (2004) and Lee et al.'s (1996) modeling approaches for the crude oil scheduling problem. While they focus on scheduling the refinery based on predetermined incoming shipments of crude oil, our model extends that scope and focuses on planning for procurements, i.e. what crude oil should be procured and when to make the procurements. Furthermore, due to a higher level of aggregation, our model can handle the longer planning horizons used in procurement planning (typically 90 days). It should be noted that this level of aggregation also reflects the level of detail present in procurement plans. The modeled system is the same as is depicted in Figure 1. In comparison with previous work (notably Zhang et al., 2012), the scope of our model is more comprehensive, making it more applicable for realistic settings, and suitable to be used as a decision support tool. In the model, all physical storage tanks and CDUs are modeled, along with their configurations, capacities and operating rules. Since we allow the blending of crude oil in tanks and all tank connections and all CDU connections are modeled, we are able to include product qualities and flow rates in detail, but the problem becomes a mixed-integer, nonlinear planning model. We propose the following MINLP model for the procurement planning problem.

Objective function

The objective is to maximize the profit margin the refinery makes by purchasing and refining each volume unit of crude oil o.

Maximize

$$\sum_{o \in O} \sum_{i \in I} \sum_{u \in U} \sum_{t \in T} (FU_{o,i,ut} \cdot RV_o) \tag{1}$$

The objective function is subject to the following constraints:

Material Balance Constraints for Purchased Shipments

The amount of purchased crude oil o that will be delivered to the refinery during period t must be transferred into storage tanks during the same period. It can however be divided over various storage tanks. VF_o denotes the shipment size for each crude oil type, and as mentioned in the assumptions above it is fixed at 100,000 m^3.

$$XF_{o,t} \cdot VF_o = \sum_{i \in I} FI_{o,i,t} \qquad \forall o \in O, t \in T \tag{2}$$

The refinery allows only a limited number of vessels to unload at the dock during time period *t*, denoted by $VesselLim$. Each vessel carries only one type of crude oil o.

$$\sum_{o \in O} XF_{o,t} \leq VesselLim \qquad \forall t \in T \qquad (3)$$

Material Balance Constraint for Crude Storage Tanks

The refinery has operating constraints on crude oil transfer rates from vessels to the refinery's crude oil storage tanks during period *t*. This means that the total flow of crude oil from a vessel to a crude storage tank *i* must be within the operating limits FI^- and FI^+.

$$FI^- \cdot XT_{o,i,t} \leq FI_{o,i,t} \leq FI^+ \cdot XT_{o,i,t} \qquad \forall o \in O, i \in I, t \in T \qquad (4)$$

Inventory Balance Constraints

Inventory of crude *o* in crude storage tank *i* at the end of period *t* is equal to the inventory at the end of the previous period plus the amount of crude oil *o* flowing into the tank *i* during period *t*, minus the amount of crude oil *o* flowing out of tank *i* during period *t*.

$$S_{o,i,t} = S_{o,i,t-1} + FI_{o,i,t} - \sum_{u \in U} FU_{o,i,u,t} \qquad \forall o \in O, i \in I, t \in T \land t > 1 \qquad (5a)$$

The initial inventory in crude tanks for the first time period is denoted by $Sinit_{o,i}$.

$$S_{o,i,t} = Sinit_{o,i} + FI_{o,i,t} - \sum_{u \in U} FU_{o,i,u,t} \qquad \forall o \in O, i \in I, t = 1 \qquad (5b)$$

The amount of inventory in crude storage tank *i* at the end of period *t* equals the sum of inventories of individual crude oils *o*.

$$Stot_{i,t} = \sum_{o \in O} S_{o,i,t} \qquad \forall i \in I, t \in T \qquad (6)$$

The amount of inventory in crude storage tank *i* at the end of period *t* should be within minimum and maximum limits for that tank.

$$Smin_i \leq Stot_{i,t} \leq Smax_i \qquad \forall i \in I, t \in T \qquad (7)$$

The amount of total inventory at the refinery should always be equal to or greater than the safety stock limits that the refinery has defined.

$$\sum_{i \in I} Stot_{i,t} \geq Ssafety \qquad \forall t \in T \qquad (8)$$

Material Balance Constraint for Crude Distillation Units

The total flow of crude oil from tank *i* to CDU *u* during period *t* is equal to the sum of flows of individual crude oils *o* flowing from tank *i* to CDU *u* during period *t*.

$$FIU_{i,u,t} = \sum_{o \in O} FU_{o,i,u,t} \qquad \forall i \in I, u \in U, t \in T \qquad (9)$$

If tank *i* is feeding CDU *u* during period *t*, then the total flow must be within a pre-defined flow rate limit, according to the operational constraints at the refinery.

$$FU^{-} \cdot XC_{i\,u,t} \leq FIU_{i,u,t} \leq FU^{+} \cdot XC_{i,u,t} \qquad \forall i \in I, u \in U, t \in T \qquad (10)$$

The total feed to CDU *u* during *t* must also be within the processing limits of CDU *u*. This constraint makes sure that each CDU *u* always has enough feed so it never has to shut down, and it also makes sure that the feed to CDU *u* does not exceed the CDU's processing capacities.

$$FCDU^- \leq \sum_i FIU_{i,u,t} \leq FCDU^+ \qquad \forall u \in U, t \in T \qquad (11)$$

Tank Composition Constraints

The share of crude oil *o* in tank *i* at the end of period *t* is equal to the amount of crude oil *o* in tank *i* at the end of period *t* divided by the total inventory level in tank *i* at the end of period *t*.

$$Stot_{i,t} \cdot P_{o,i,t} = S_{o,i,t} \qquad \forall o \in O, i \in I, t \in T \qquad (12)$$

The next constraint ensures that when a tank feeds a CDU, the amount of individual crude oils delivered to that CDU must be in proportion to the crude composition in the tank.

$$FIU_{i,u,t} \cdot P_{o,i,t} = FU_{o,i,u,t} \qquad \forall o \in O, i \in I, u \in U, t \in T \qquad (13)$$

Storage Tank Constraints

The following constraint makes sure that the tank segregation rules hold, i.e. that tank i will never be charged with crude oil types that it is not allowed to hold.

$$FI_{o,i,t} = 0 \qquad \forall (o,i) \in CI, t \in T \qquad (14)$$

Another segregation rule defines the minimum and maximum concentration of each type of crude oil that each tank is allowed to storage.

$$Pmin_{o,i} \leq P_{o,i,t} \leq Pmax_{o,i} \qquad \forall o \in O, i \in I, t \in T \qquad (15)$$

In practice, a vessel will only unload crude to a limited number of storage tanks. Some refineries would like to minimize the number of tanks used when unloading crude, while other refineries constrain the number to a pre-defined number. For the sake of simplicity, we constrain the number of tanks used when unloading crude to TL.

$$\sum_{i \in I} XT_{o,i,t} \leq TL \qquad \forall t \in T, o \in O \qquad (16)$$

Similarly, operating rules at most refineries dictate that a storage tank can only charge a limited number of CDUs simultaneously.

$$\sum_{u \in U} XC_{i,u,t} \leq CL \qquad \forall i \in I, t \in T \qquad (17)$$

And vice versa; a single CDU can only receive crude from a limited number of storage tanks simultaneously.

$$\sum_{i \in I} XC_{i,u,t} \leq UL \qquad \forall u \in U, t \in T \qquad (18)$$

CDU Quality Constraint

As mentioned above, the concentration of key components in crude oil or crude oil blends affect the processing of these crudes. Therefore, all refineries have limits on the concentration of a key component, such as sulfur, for every CDU. These limits avoid processing problems in both CDUs and other downstream processing units. We ensure feed quality by using the known fraction of key component k in crude oil o ($KC_{k,o}$) and constrain the volume-weighted average of these qualities to be within the pre-defined quality limits (${QL_{k,u}}^-$ and ${QL_{k,u}}^+$).

Figure 2. The PRONODIS solution approach.

$$\left(\sum_{i\in I} FIU_{i,u,t}\right)\cdot QL_{k,u}^{-} \le \sum_{o\in O}\sum_{i\in I}\left(FU_{o,i,t,u}\cdot KC_{k,o}\right) \le \left(\sum_{i\in I} FIU_{i,u,t}\right)\cdot QL_{k,u}^{+} \qquad \forall u\in U, k\in K, t\in T \qquad (19)$$

5. The PRONODIS Solution Approach

The model presented in Section 4 is solvable for small problem instances. However, as is common for MINLP problems, the computing time for industry-sized problem instances becomes intolerable, especially for a detailed model like ours, including e.g. tank and feed rate compositions. Thus, we propose a solution approach that considerably reduces computing time. We will refer to this approach as the PROcurement-NO-DIScrepancies (PRONODIS) approach throughout this paper.

The main idea behind the algorithm is as follows: First, we solve a linearized MIP model, since the main reason for the long CPU time in the MINLP model is the nonlinear blending constraints. Then, we fix all purchasing decisions from the achieved solution and feed them into the original MINLP model. The process is illustrated in Figure 2. By doing this, the MINLP model has fewer decision variables, since $XF_{o,t}$ is no longer a decision variable but a parameter, and therefore the solution time for this two-stage approach is shorter than by only solving the initial MINLP model.

In order to change the original MINLP model into a MIP model we reformulate constraints (12), (13) and (15) to avoid all nonlinearity in the model. Inequality (15) is multiplied by $FIU_{i,u,t}$ and we get:

$$FIU_{i,u,t} \cdot Pmin_{o,i} \leq FIU_{i,u,t} \cdot P_{o,i,t} \leq FIU_{i,u,t} \cdot Pmax_{o,i} \qquad \forall o \in O, i \in I, u \in U, t \in T \qquad (20)$$

Then we insert the right-hand side of equation (13) into equation (20) and get:

$$FIU_{i,u,t} \cdot Pmin_{o,i} \leq FU_{o,i,u,t} \leq FIU_{i,u,t} \cdot Pmax_{o,i} \qquad \forall o \in O, i \in I, u \in U, t \in T \qquad (21)$$

Similarly, multiplying equation (15) with $Stot_{i,t}$ leads to:

$$Stot_{i,t} \cdot Pmin_{o,i} \leq Stot_{i,t} \cdot P_{o,i,t} \leq Stot_{i,t} \cdot Pmax_{o,i} \qquad \forall o \in O, i \in I, t \in T \qquad (22)$$

We insert the right-hand side of equation (12) into equation (22), and get:

$$Stot_{i,t} \cdot Pmin_{o,i} \leq S_{o,i,t} \leq Stot_{i,t} \cdot Pmax_{o,i} \qquad \forall o \in O, i \in I, t \in T \qquad (23)$$

Equations (12) and (13) are then replaced by equations (21) and (23) and the model becomes a MIP model instead of MINLP. This reformulation is similar to what many other researchers have used (e.g. Lee et al., 1996). Constraints (21) and (23) do not explicitly calculate the concentration of crude oils and thus we might experience composition discrepancy which can lead to infeasibility, as was explained in Section 3. Therefore we included the second stage in our solution approach. By fixing the procurement variables from the MIP model and feed them into the MINLP model, we can achieve a near-optimal solution that is free from composition discrepancy within acceptable CPU time.

The PRONODIS solution approach is a novel way of solving planning models within the oil refining industry. The existing solution approaches in the literature avoid solving full MINLP models, because of the long CPU times associated with nonlinear models. However, due to the level of aggregation of our problem, we are able to solve the MINLP model after having fixed the procurement decision variables. By solving the nonlinear model, we guarantee that we will not experience composition discrepancy. On the other hand, we cannot guarantee that the procurement decisions from the MIP always lead to feasible solutions when the other variables are re-optimized in the MINLP model. However, as we show in Section 6, we never experience infeasibility in any of our numerical tests.

6. Illustrative Examples

In order to illustrate how the PRONODIS model can be used for decision support and how it deals with composition discrepancies, several illustrative examples are presented. The modeled system in the illustrative examples represents the six storage tanks at a collaborating refinery (tanks T1 – T6), the two CDUs at this refinery (CDU1 and CDU2), their processing capacities, and a typical planning horizon of 90 days. For the illustrations in this section, it is assumed that the crude quality is only measured by sulfur content and that there are eight different types of crude oil available for the refinery (C1 – C8), which is usually the number of available crudes for the collaborating refinery.

The first illustrative example shows how the model creates a feasible procurement plan, followed by a second illustrative example that shows the need for the second stage in our solution approach. The third illustrative example shows how the model can indicate whether a specific procurement opportunity is worth taking. The fourth and final example illustrates how the model will most likely be used in reality, that is, to generate a continuously revised plan with a rolling planning horizon.

The modeling system GAMS (Brooke et al. 1998) was used to implement the optimization model. The number of computer solvers that solve both non-linear constraints and integer variable is still rather limited. We chose to use the widely used DICOPT solver, which is a MINLP solver available in GAMS. The DICOPT solver is based on mixed-integer programming (MIP) problem and a non-linear programming (NLP) heuristic that works reasonably well in many problems (Grossmann, 2002; 2012). The MINLP algorithm inside DICOPT thereby solves a series of MIP and NLP problems. These sub-problems can be solved using any NLP or MIP solver that runs under GAMS. The numerical experiments were performed on a laptop computer with Intel Core 2 Duo / 2.0 GHz PC platform.

6.1 Procurement Planning

The following example illustrates how the MIP/MNLP model generates a feasible procurement plan for the refinery. The input data for the example can be seen in Table 1.

The initial crude composition and initial volumes are based on a snapshot inventory position from the collaborating refinery. The initial crude oil blends consist of eight crude types, which differ in quality and profitability. All crude oil storage tanks are allowed to hold any type of crude blend, except for tank T6, which is reserved for sweet crude oil (containing less than 0.5% sulfur). CDU1 is capable of processing crude oils higher in sulfur percentage than CDU2; CDU1 has a quality limit of maximum sulfur content of 0.4%, whereas CDU2 has a limit at 0.15%. Total safety stock for the refinery is set at 56,000 m3, and the starting inventory for constraint (5b) is given in Table 1, as well as the capacities of each crude oil tank. It is assumed that no procurements have been planned from time t=0.

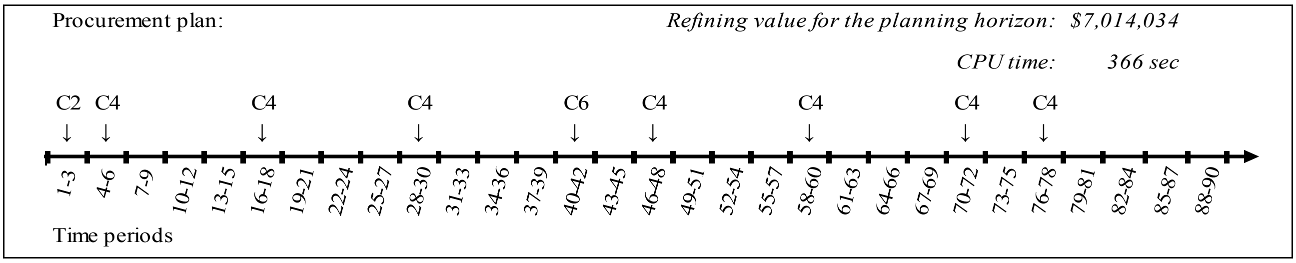

Figure 3. Results from the illustrative example.

Table 1 - Input data for illustrative example 1

Tank *i*	**Initial inventory of crude oil *o* [m^3]**								**Tank capacity [m^3]**	
	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**Min**	**Max**
T1	418	7,817	14,362	4,132	335	0	0	0	3,295	28,936
T2	0	0	9,525	0	96	0	14,735	0	3,205	28,857
T3	0	4,070	727	0	11	0	0	851	2,929	37,184
T4	153	356	4,474	0	102	9,700	98	0	6,622	68,148
T5	1,043	19,646	34,368	0	2,688	601	0	0	6,311	68,880
T6	0	0	7,071	0	0	0	26,179	6,495	8,435	79,673
Sulfur Content [%]	0.56	0.25	0.22	0.50	0.27	0.01	0.02	0.21	-	-
Refining Margin [$/m^3]	7.86	4.78	1.64	9.43	2.83	2.70	1.89	1.57	-	-

The optimization model creates a three-month intermediate procurement plan, choosing from these eight crude oils. The proposed plan can be seen in Figure 3.

The plan consists of 9 procurements of three different crude oils, and a total import volume of 900,000 m^3. This solution generated is free from composition discrepancy as it was generated with the PRONODIS solution approach. The computing time for solving this kind of problem is acceptable. It takes approximately 6 minutes to solve this specific example. In addition to proposing a procurement plan, the model reports various detailed outputs.

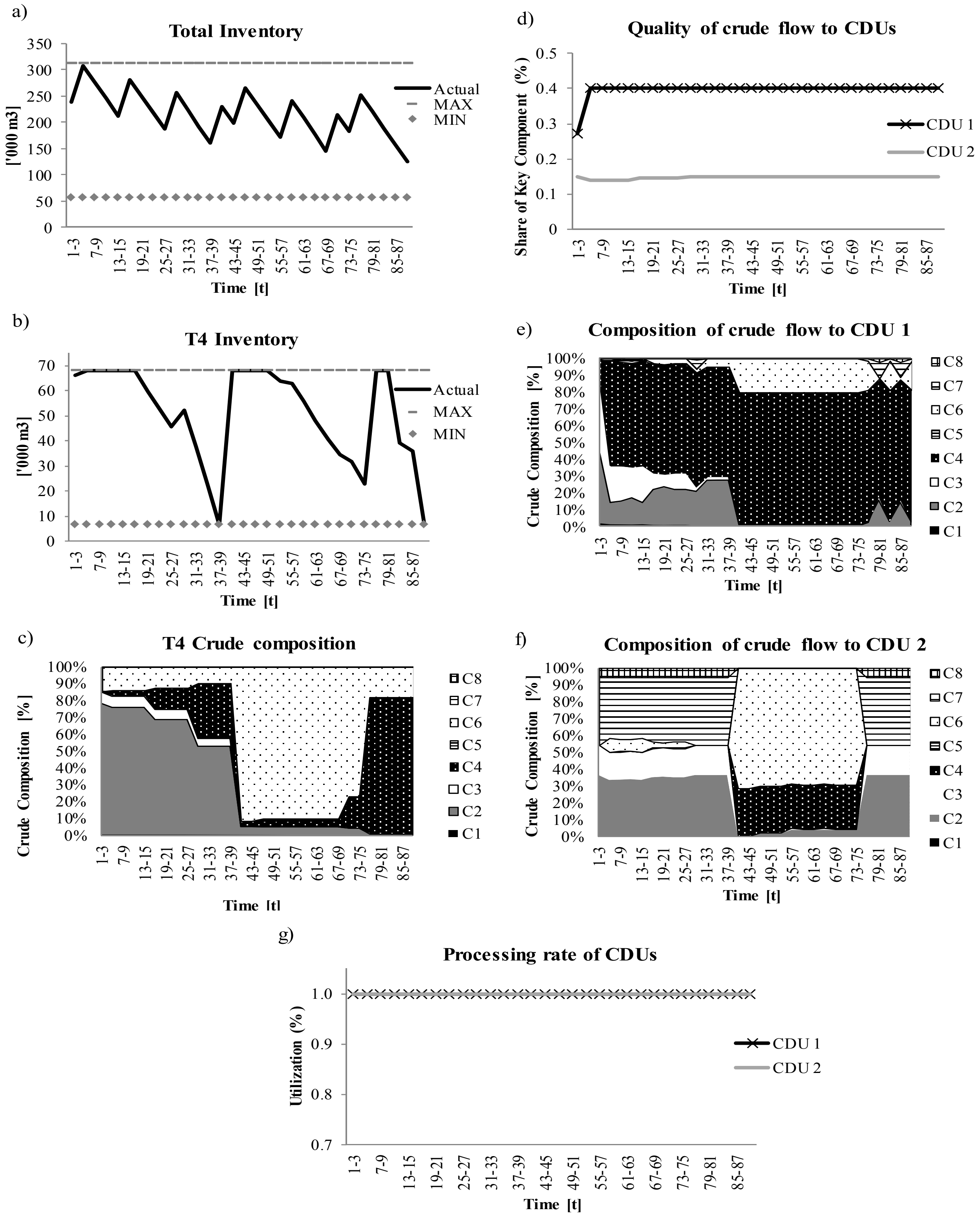

Figure 4. Various outputs of the model.

Figure 4 gives seven examples of analyses that can be created from the model's output variables. Figure 4(a) shows the total amount of inventory, with respect to maximum storage capacities and safety stock limits. The planned shipments from Figure 3 can be clearly seen in the graph. Figure 4(b) shows a similar analysis, but only for tank T4.

We can for instance see in this graph that a large share of the shipments scheduled to arrive in period t=40-42 and period t=76-78 are planned to be fed to tank T4. This is also clearly reflected in Figure 4(c), which shows the composition of the blend stored in tank T4; the crude composition changes drastically when it receives a large volume of one specific crude oil. During time period t=40-42, the tank receives a large volume of C6 and during time period t =76-78, a large volume of C4. Graphs similar to Figures 4(b) and 4(c) can be created for each tank, and are very useful for the production planners at the refinery, since they give a detailed overview of what is in stock at the refinery.

As has been stressed earlier, it is very important to make sure that the quality of the blended crude oils is sufficient and feasible for downstream processing units. Figure 4(d) shows the sulfur content of the crude oil blends fed to each CDU, and Figures 4(e) and 4(f) the specific compositions of the crude oil blend that is being fed into CDU1 and CDU2. Figures 4(e) and 4(f) demonstrate that the refinery can have substantial variation in the blends they process, while still achieving a stable feed to the CDUs (Figure 4(d)). These results highlight the relevance of modeling the compositions of the blends and quality parameters.

Finally, Figure 4(g) illustrates that procuring the right mix of crude oils, and blending them correctly in the storage tanks and CDU feeds allows the refinery to reach maximum utilization levels for both CDUs.

The detailed output depicted in Figure 4 shows how the proposed procurement plan will affect the production plans at the refinery. Furthermore, it is an important link between procurement planners and production planners. This detailed connection between procurement planning and production planning is not covered in previous procurement planning literature, although it is highly relevant as it gives detailed decision support for the planners.

6.2 Solution approach analysis

In order to explain the need for the second stage in our solution approach we compare the solutions for the first stage (MIP model) and the second stage (MINLP model) in the PRONODIS solution approach, using the data from the first illustrative example. The comparison confirms that the composition discrepancies are eliminated by running the second stage MINLP model.

Figure 5 shows the crude oil tanks that are feeding CDU 2 during time period *t*=70-72. The left part of the figure shows the solution from the first stage (MIP model) of the PRONODIS solution approach and the right part shows the second stage (MINLP model) solution. The composition discrepancy from the first stage solution is evident in the left half of Figure 5; the flow out of tank T4 and T5 has a different composition than the existing blends in the tanks. The flow of crude blend towards the CDU has 0.15% sulfur content, which is the maximum amount allowed. However, based on the actual blends in the tanks, the flow should have contained 0.19% sulfur. This solution would hence be infeasible.

The second stage in the PRONODIS solution approach includes fixing the procurement decision variables from the MIP model solution. The MINLP model then allocates the incoming crude oil shipments differently into storage tanks, and ensures that the flow out of all crude oil tanks is consistent with the tank content. This can be seen in the right part of Figure 5, where the composition of the feed towards CDU2 is identical to what is being stored in the tanks. The total flow of crude oil towards the CDU has 0.15% sulfur content and is feasible. This example illustrates the need for the second stage in our modeling approach. The second stage eliminates the composition discrepancies.

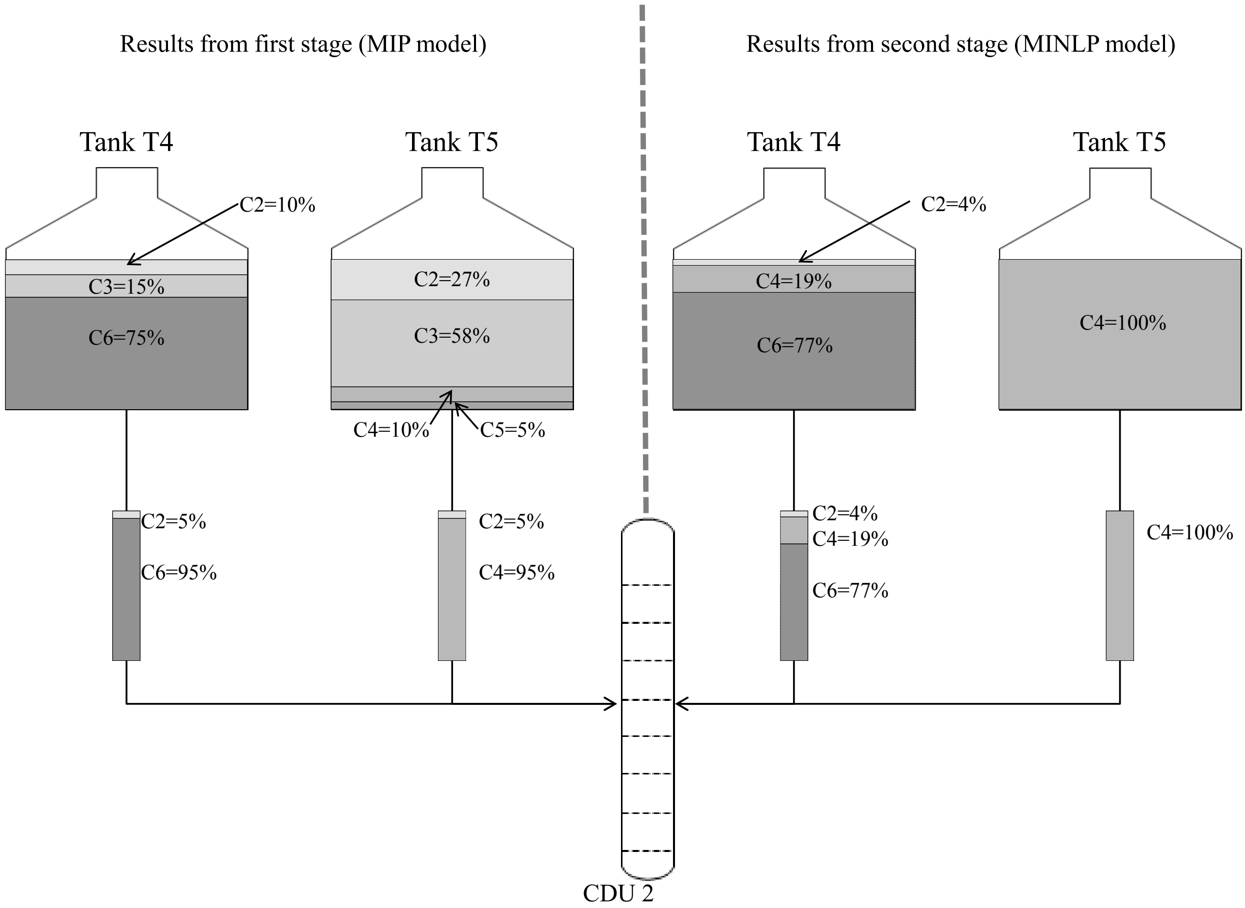

Figure 5. Comparison of the MIP and MINLP solutions.

6.3 Scheduling Procurement Opportunities

An important advantage of this model will be demonstrated in this section. Assume that the plan in Figure 6 has been created and decided upon. All shipments that are supposed to arrive within the first 30 days are fixed, since the tradition at the refinery is to sign all purchasing agreements within 30 days to crude oil arrival. The refinery is committed to buy these shipments, but all other shipments are still flexible. Now it might happen that an unexpected market opportunity arises; a distressed cargo of C1 becomes available at a discounted price. The shipment could be delivered in the time window *t*=37-39 and the quality of the crude is rather low, i.e. the sulfur percentage is high. It is not straightforward for the procurement planners to figure out if the refinery can use this shipment; they have to estimate how economical this will be for the refinery and how this will

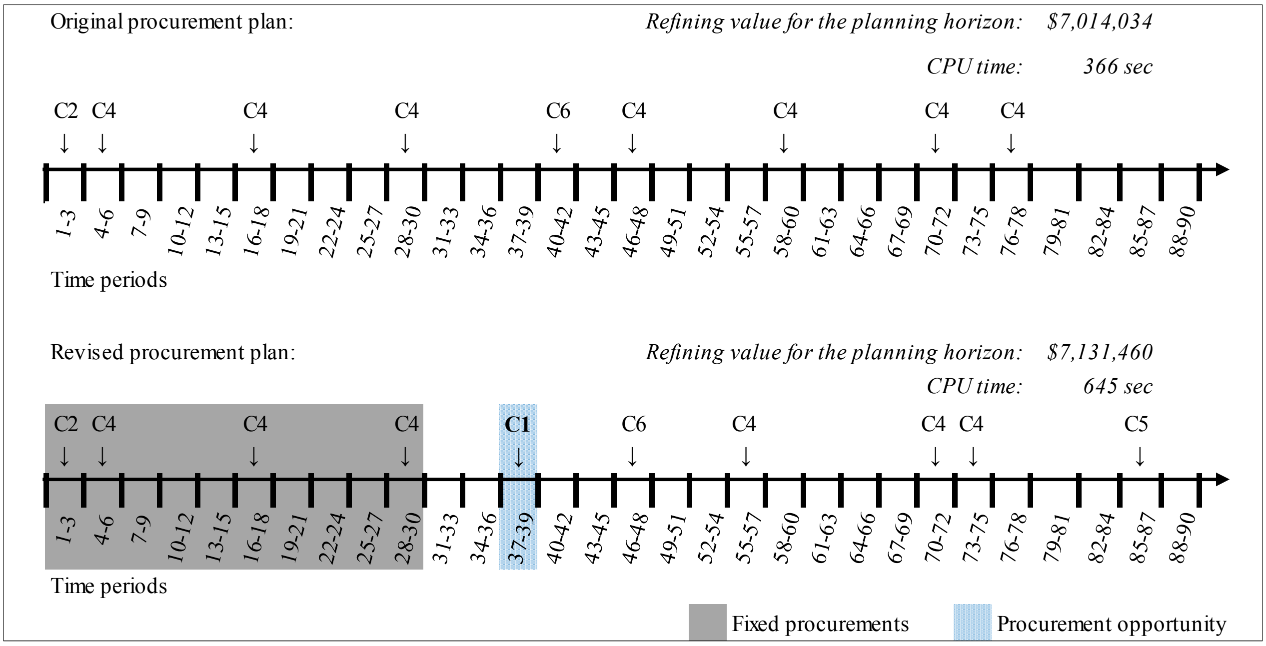

Figure 6. Results from illustrative example 2.

impact all future purchases (i.e. affect their current procurement plan). When this kind of offer becomes available, which happens quite often, the crude oil trader has to act rapidly in order to be able to take advantage of the opportunity.

The planner takes this new information and feeds it into the model, along with the current procurement plan and ends up with an updated plan. From the updated procurement plan, the planner can see whether it is economical to take advantage of this market opportunity and how it will affect future crude purchases. The updated procurement plan can be seen in Figure 6.

The new plan is more profitable, with a difference of approximately $117,500, indicating that it would be beneficial for the refinery to take in that distressed cargo of crude C1. The model updates the current procurement plan in 11 minutes, and comes up with a new feasible plan that includes this special procurement opportunity. Compared to current manual planning procedures, 11 minutes is a considerable time improvement. At present, the process of estimating whether a procurement opportunity is worth undertaking takes significant effort, often involving multiple organizational

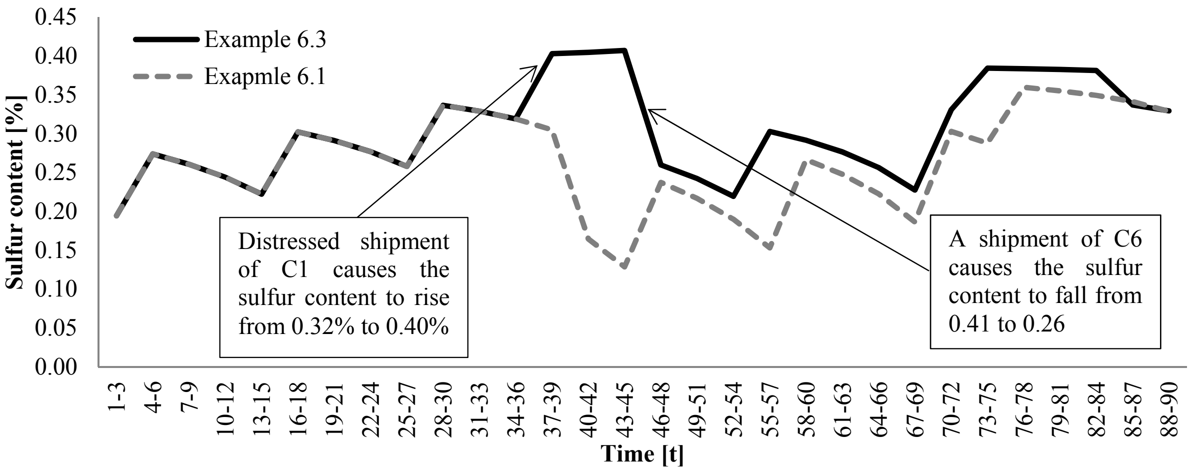

Figure 7. Sulfur content in total inventory per time period.

units, and cannot always be concluded on the same day. Very often, this means that the opportunity will have disappeared.

We see that the shipments that are scheduled to arrive after the first 30 days change. The updated plan has one less shipment of C4, and an additional shipment of C5 which is considerably lower in terms of sulfur percentage. This seems sensible since the distressed cargo has very high sulfur percentage that needs to be compensated for.

Figure 7 compares the sulfur content in the overall inventory at the refinery between the new solution, including the procurement opportunity, and the original procurement plan from illustrative example 6.1. We see that when the distressed cargo of C1 arrives at the refinery, the overall sulfur content rises, and remains higher for most of the planning horizon.

6.4 Creating a Rolling Procurement Plan

The following example will demonstrate how the model can also make procurement plans within a rolling time horizon. In reality, the procurement planner will update the procurement plans periodically, or whenever new information, that can influence the choice of procurements, becomes available. This new information usually concerns updated refining margins. The new data is fed to the model and a new procurement plan is generated. The model can thus be used dynamically and the outcome will be a rolling procurement plan.

To illustrate this rolling horizon planning procedure, we created the following example. We assume the same refinery configuration and input data as we did for the example in Section 6.1. We also assume that the planners receive updated refining margins every 10-12 days, as is the operating procedure for the collaborating refinery. When this happens, the current procurement plan is re-optimized and all procurements are subject to change except for those who are scheduled to arrive within the next 30 days. For the following illustration, we assume that no procurement opportunities become available. The changes in refining margins can be seen in Table 2 and are based on data from the collaborating refinery.

Table 2 - Updated refining margins

Crude oil *o*	***t*=0** [$/m^3]	***t*=10** [$/m^3]	***t*=22** [$/m^3]
C1	7.86	7.47	7.10
C2	4.78	4.54	4.31
C3	1.64	1.72	1.81
C4	9.43	8.96	8.51
C5	2.83	2.97	2.82
C6	2.70	2.84	2.98
C7	1.89	1.98	2.08
C8	1.57	1.65	1.65

The results from the dynamic application of the optimization model can be seen in Figure 8. The model is fairly robust to changes in refining margins. At *t*=10, the refining margin for C4 has decreased slightly, but it is still the most profitable crude. Therefore the model chooses to buy as much of that crude as possible. At *t*=22 the refining margin for C4 has decreased further, resulting in the model to choose other types of crude rather than trying to procure solely C4. The refining margin for C6 has risen, and since the crude is low in sulfur it is now an attractive option.

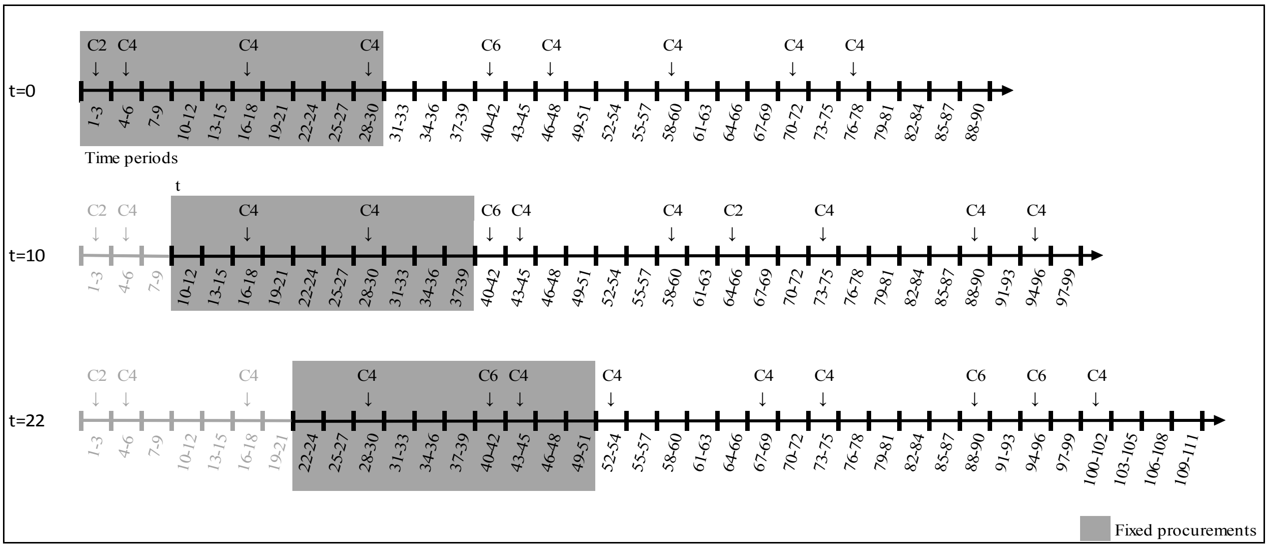

Figure 8. Results from illustrative example 3.

7. Numerical Analysis

In this section we will test the quality and robustness of the proposed model and solution approach. First, in order to gain understanding of the approximation error associated with the solution approach, we compare solving the full MINLP model to solving the model with the proposed MIP/MINLP solution approach. Secondly, we test the proposed model and solution approach with 180 different data sets, in order to see how robust our framework is with respect to changes in the problem instances.

7.1 Solution approach performance

Since the proposed PRONODIS solution approach involves linear approximation, it is important to analyze the effects the approximation will have on the solution. To estimate the approximation error associated with the PRONODIS approach versus solving the exact MINLP model, a numerical test is performed.

The full MINLP model is not solvable for industrial size cases and therefore we scale down in example size for this numerical analysis. We assume the actual refinery configuration, as described in Section 6, but we exclude CDU2 from our tests and assume the planning horizon to be 60 days. Furthermore, it is assumed that only two types of crude oils are available throughout the planning horizon, namely C1 and C2, and the quality of the crude oil is only measured by its sulfur content. C1 represents a crude oil grade that has a high refining margin, but is relatively high in sulfur (i.e. it needs to be mixed with C2 to be fit for production) and C2 represents the most commonly purchased crude oil at the refinery.

Table 3 - Input data for numerical test 1

Tank *i*	Initial inventory level of crude oil *o*		Tank capacity [m^3]	
	C1	C2	Min	Max
T1	Randomly generated for 20 instances		3.295	28.936
T2			3.205	28.857
T3			2.929	37.184
T4			6.622	68.148
T5			6.311	68.880
T6			8.435	79.673
Sulfur Content [%]	0,56	0,25	-	-
Refining Margin [$/$m^3$]	7,86	4,78	-	-
CDU1 sulfur content limit [%]	-	-	0,0	0,4

Starting inventory for each tank at time $t=t_0$ is randomly generated with a pseudo random number generator, restricting the random numbers to be between the minimum and maximum capacity levels of each tank. Table 3 summarizes the experimental design. It is assumed that no procurements have been planned from time $t=t_0$. The numerical test consists of 20 independent input data sets. The MINLP model and model that uses the PRONODIS solution approach are solved for all data sets and the solutions are compared to each other.

The results from the numerical test can be seen in Table 4. It took on average five hours to solve the data sets using solely the MINLP model, with computing time ranging from 47 seconds to almost

40 hours. Unpredictability in computing times is a well-known problem when dealing with MINLP problems. By using the PRONODIS solution approach, the computing time decreases drastically, and is also more predictable.

The average solution time for the PRONODIS approach model is 19 seconds and a feasible solution was found in every case. However, as could be expected, there is some change in the objective value when using the PRONODIS approach. The objective values are on average 1.1% lower.

However, it is clear that using the two-staged approach saves enormous amount of CPU time without much compromise of the quality of the solution. Furthermore, the PRONODIS model is capable of solving larger problem instances.

7.2 Problem Size Robustness

In order to study how the model and the solution approach react to changes in problem instances, we created the following numerical test, in which we will only look at the PRONODIS modeling approach. We assume the same refinery configuration as in the illustrative examples in Section 6.

We randomly generate the initial conditions for each data set, and always assume that no procurements have been made at time t=t_0. Starting inventory level per tank is randomly generated with a pseudo random number generator, restricting the random numbers to be between the minimum and maximum capacity levels of each tank. The crude oil included in each test is randomly chosen between the 12 crude oils allowed at the collaborating refinery. The composition of crude oil in each tank was also randomly chosen between 0 and 1 and then normalized so that the compositions would sum up to 100%.

Table 4 - Results from numerical test 1

Data set no.	**MINLP CPU [sec]**	**PRONODIS CPU [sec]**	**Relative difference in obj. function value**
1	87	11	0.5%
2	15,871	15	1.4%
3	259	46	2.8%
4	9,300	16	0.6%
5	11,305	21	0.1%
6	22,248	16	1.2%
7	141,544	12	0.0%
8	1,850	33	0.0%
9	32,811	10	3.4%
10	59	15	0.3%
11	2,130	21	3.5%
12	6,487	31	0.0%
13	3,887	24	0.2%
14	7,222	19	0.0%
15	168	16	2.6%
16	5,665	10	0.8%
17	278	14	2.0%
18	1,228	17	2.0%
19	198	27	0.3%
20	47	14	1.3%
Average	**13,132**	**19**	**1.1%**

Since CDU2 can only process light, sweet crude oils, we have to make sure that the starting crude oil inventory is feasible for that CDU. We ensure this by restricting tank T6 to only store sweet crudes in the beginning, i.e. no crude oil containing more than 0.5% is allowed in that tank at $t=t_0$.

We test the model with 180 different data sets. We created 20 independent data sets with four crude oil types, another 20 independent data sets considering six crude oil types and 20 considering eight crude oil types. Each data set was then tested with 1-3 key components (KC). A feasible solution was found for every single data set. The resulting CPU times are summarized in Figure 9.

The PRONODIS solution approach always generates a feasible solution. The CPU time increases as we include more crude oil types in the problem. The average computation time for a problem with

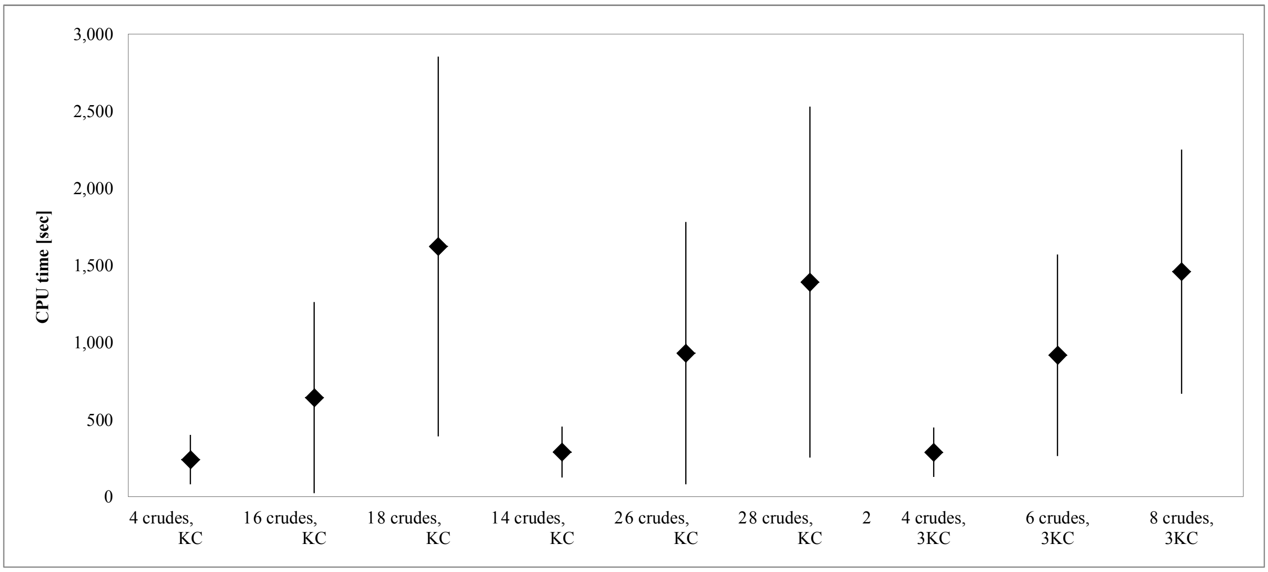

Figure 9. Results from illustrative example 3.

four crude oil types is 5 minutes, with a standard deviation of 3 minutes. The average computation time for a problem considering eight crude oil types is 25 minutes, with a standard deviation of 18 minutes. As is expected, the initial crude composition also has some impact on the solution time. Problems with starting inventories that contain a large proportion of low quality crude oil tend to have longer solution time. From Figure 9 we can see that changing the number of key components considered does not affect the CPU time significantly, even though we are adding more variables to the model. The data sets with one key component consider only sulfur, and sulfur is the most constraining key component. The other two key components considered are specific gravity and total acid number (TAN).

While our approach considers several additional aspects compared to previous literature (e.g. tank and feed rate compositions), our two-stage approach enables us to provide solutions significantly faster.

7. Conclusions and Future Research

This paper discusses the procurement planning problem in the oil refining industry, which has only recently seen contributions that support detailed decision support (Zhang et al., 2012). Compared to this previous literature, our model covers the following additional realistic factors: (*i*) blending crude oil in crude oil tanks is allowed, (*ii*) storage tanks, CDUs, and their feed rates are modeled individually, along with all the important operating rules that apply to these production units, and (*iii*) the model can handle more crude oil types and more quality parameters than are included in other procurement planning and crude oil scheduling literature, while still providing solutions in acceptable and robust solution times.

We developed a novel two-stage solution approach (termed PRONODIS) that gives solutions free of composition discrepancy, a problem that can appear due to linearization. The resulting framework helps procurement planners to plan, schedule and reschedule crude oil procurements. The approach also creates detailed inventory profiles for each crude storage tank per time period, and reports necessary information on feed quantity and quality being fed into each CDU.

The model was tested using historical data from a Statoil A/S refinery. The applicability of the approach was illustrated in three scenarios. A feasible, near-optimal, 90-day procurement plan was generated within 7 minutes of computing time, significantly shorter than the average computing times reported in previous literature despite including several additional factors, such as tank and feed rate compositions. Secondly, the approach is capable of scheduling individual procurement opportunities and generating procurement plans with a rolling time horizon. Finally, the quality of the PRONODIS solution approach was tested with a comprehensive numerical analysis. The developed solution approach obtains near-optimal solutions for industrial-sized problems within acceptable solution times.

The approach can be extended. Currently the costs of inventory are not considered, which potentially leads to higher inventory levels. On the other hand, minimizing inventory levels might result in an increased vulnerability to supply disruptions, as was discussed in Zhang et al. (2012). Other potential modifications to the approach include allowing variable shipment sizes. Also, the feasibility of the solution approach could be further studied. A way to guarantee feasibility would be to add a loop to the solution algorithm. A potential algorithm design could for instance be similar to the backtracking algorithm Li al. (2007) developed for the crude scheduling model. However, there was no necessity for this in the numerical tests we conducted.

The model presented in this paper is designed for decision support for procurement planners. For the model to be fully functional, it has to be implemented in a decision support tool that is accessible and useable for planners and refinery managers. This often also includes organizational changes, and is future work that the authors have already started.

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