

# Intermediate product selection and blending in the food processing industry\*

Onur A. Kilic<sup>†1</sup>, Renzo Akkerman<sup>2</sup>, Dirk Pieter van Donk<sup>3</sup>, and Martin Grunow<sup>4</sup>

<sup>1</sup>Department of Management, Hacettepe University, Turkey

<sup>2</sup>Department of Management Engineering, Technical University of Denmark, Denmark

<sup>3</sup>Department of Operations, University of Groningen, The Netherlands

<sup>4</sup>Department of Production and Supply Chain Management, Technical University of Munich, Germany

## Abstract

This study addresses a capacitated intermediate product selection and blending problem typical for two-stage production systems in the food processing industry. The problem involves the selection of a set of intermediates and end product recipes characterizing how those selected intermediates are blended into end products to minimise the total operational costs under production and storage capacity limitations. A comprehensive mixed integer linear model is developed for the problem. The model is applied on a data set collected from a real-life case. The trade-offs between capacity limitations and operational costs are analysed, and the effects of different types of cost parameters and capacity limitations on the selection of intermediates and end product recipes are investigated.

**Keywords:** Production planning; Scheduling; Food processing; Capacity limitations; Intermediate Storage; Intermediate product;

## 1 Introduction

The food processing industry is characterised by divergent product structures where a relatively small number of (agricultural) raw materials are used to produce a large variety of often customer specific end products (see e.g. Akkerman and Van Donk, 2009). Due to the large variety of end

---

\*This article was published as: Kilic, O.A., Akkerman, R., van Donk, D.P., Grunow M. (2013); Intermediate product selection and blending in the food processing industry; *International Journal of Production Research* 51 (1), 26-42. The original publication is available at <http://www.tandfonline.com/> (doi:10.1080/00207543.2011.640955)

<sup>†</sup>Corresponding author (onuralp@hacettepe.edu.tr).

products, it is often not possible or at least inefficient to produce and stock all end products. A common practice used to mitigate the effect of the product variety on the operational performance in food-processing systems is to produce some or all end products by blending them from a limited number of selected intermediate products (Van Donk, 2001; Soman et al., 2004; McIntosh et al., 2010). The basic notion of this practice follows the well known principle of postponement which is widely used amongst various industries (Van Hoek, 1999; Venkatesh and Swaminathan, 2004; Pil and Holweg, 2004; Skipworth and Harrison, 2004; Caux et al., 2006; Forza et al., 2008).

The concept of postponement is often defined on the production stage where intermediates are transformed to end products after demand is realised. Zinn and Bowersox (1988) defined potential stages of postponement as manufacturing, assembly, packaging and labelling. In this respect, the approach considered in the current study concerns the postponement at manufacturing and assembly stages, which are respectively referred to as processing and blending in process industry terminology.

There is a variety of studies in the postponement literature that aim at assisting managerial decision making by putting the concept of postponement into action. However, as compared to other industries, the food processing industry has not been very active in taking up postponement strategies (Van Hoek, 1999; Cholette, 2010). Also, the research efforts taken in this domain are rather concentrated on the postponement practices at the packaging and labelling stages (see e.g. Cholette, 2009, 2010; Wong et al., 2011), mainly due to the fact that delayed packaging is considered to be a natural level of postponement in food industry (Van Hoek, 1999).

There are, however, a number of factors which grant a potential advantage in practicing postponement strategies at the blending level in the food processing industry (McIntosh et al., 2010). For instance, in food production, processing operations are often coupled with extensive setups. Also, blending operations in food processing are not as substantial as their assembly counterparts in discrete manufacturing. These, in connection to the inherent divergence of product flows due to the product variety, provide a strong motivation towards processing and stocking only a moderate number of intermediates, and then blending them into the whole range of end products following realised demands. This approach reduces the frequency of processing runs in the intermediate product level in expense of additional blending operations in the end product level.

Nevertheless, postponement practices in processing and blending are strongly coupled with the processing operations as well as the intermediates used in those operations. In particular, it may be required to use standardised intermediates for blending multiple end products. These intermediates may not even be marketable themselves. Also, it may be necessary to employ more complex production processes (Venkatesh and Swaminathan, 2004). These, all together, may lead to significant increases in productions costs, and overcome the advantage of employing the postponement strategy. As a result, companies employing such postponement strategies need to face a decision problem involving the selection of a set of intermediates from a large set of potential intermediates usually designed by quality management experts, and end product recipes which prescribe

how those selected intermediates are blended into end products in order to minimise the total operational costs (Rutten, 1993; Akkerman et al., 2010).

The current study seeks to address the aforementioned decision problem. The problem relates to the well-known blending problems, where, given a set of products, the objective is to find a minimum cost mix satisfying a set of quality related attributes. Due to their practical relevance, a considerable amount of work has been done on industry-specific production planning problems involving blending components, such as feedlot optimization problems (see e.g. Glen, 1980; Taube-Netto, 1996), sausage blending problems (see e.g. Steuer, 1984), multi-period production planning problems (see e.g. Williams and Redwood, 1974; Rutten, 1993), and grade selection and blending problems (see e.g. Karmarkar and Rajaram, 2001; Akkerman et al., 2010). However, these studies assume unlimited production and/or storage capacities. The problem we consider in this paper stands apart from the aforementioned literature with regard to two main aspects. First, we capture whether blending of intermediates is required to produce end products by acknowledging the possibility of direct use of intermediates as end products. Secondly, we approach the blending problem by considering the costs and the capacity limitations related to both the production and the storage operations which also affect the selection of the intermediates and end product recipes.

The rest of the paper is organised as follows: In Section 2, we provide a detailed description of the production system under consideration. In Section 3, we review the related literature. In Section 4, we present the mathematical programming formulation of the problem. In Section 5, we demonstrate an application of the model for a real-life case. We conduct a numerical study to illustrate the effects of some operational settings on the optimal decisions. Finally, in Section 6, we summarise our work and suggest directions for future research.

## 2 Problem description

The production system under consideration involves two production stages: processing and blending. The processing stage involves the production of intermediates. In the blending stage, intermediates are blended into end products following end product recipes which specify the blending proportions of intermediates. This production environment is common particularly in food processing because food products can often be prepared in a generic form. For instance, in dairy processing, the main raw material fresh milk is processed into fat, protein concentrate, cream, whey, dry milk, and skim milk. These materials are then used in processing a variety of milk products such as condensed and evaporated milk, nutritional products, buttermilk, and milk powder (Nicholson et al., 2011). In flour manufacturing, different types of starchy food are milled into a variety of grains which are then blended into flour products targeted for bakeries and industrial manufacturers (Akkerman et al., 2010). Also, in wine production, after being processed, different wines, possibly from different grape origins, can be blended to produce a particular brand (Cholette, 2010).

The recipe of an end product may involve single or multiple intermediates. In the former

case, demands can directly be satisfied from intermediate stocks. In the latter case, however, intermediates are first blended to form end products which are then used to serve demands. Figure 1 illustrates a small example of such a system involving two selected intermediates and three end products where circles and rectangles represent materials and production operations respectively. Notice that two of the three end products in the example require blending operations, whereas the last one does not.

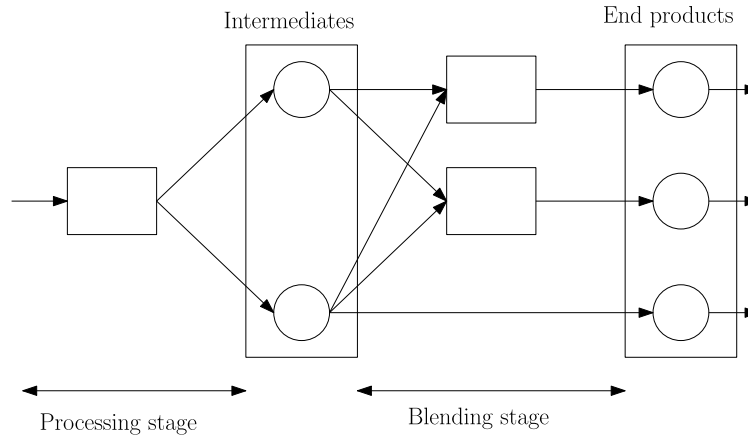


Figure 1: An example production system

The problem we address in this study involves the selection of (i) a set of intermediates to be stocked from a given set of potential intermediates, and (ii) end product recipes which specify how those intermediates are blended into end products. The selection of intermediates and end product recipes is associated with a set of cost factors and constraints. The total operational cost is composed of material procurement costs, processing costs, storage costs associated with selected intermediates; and blending costs associated with end products. There are two basic constraint sets. First, the compositions of end products, which are defined by their product recipes, must comply with a specified set of quality requirements in order to guarantee the conformity of end products. The composition of an end product characterises all types of attributes associated with it. Here, we refer to those attributes as quality parameters. For instance, in milk processing, fat, protein, and dry-matter concentration; in flour production, water absorption ability, dough extensibility, deoxynivalenon level, and bread volume; and in wine production, acid and tannin levels, and flavour intensity could be of importance. The quality requirement regarding a particular quality parameter states that the relevant parameter must be within a given range. Second, available production and storage capacities must be sufficient to put the selected intermediates and end product recipes to use. That is, given a set of selected intermediates and end product recipes, it must be possible to produce the necessary amount of intermediates and to blend them into end products to satisfy the demand, and the storage facilities must be sufficient to stock production lots.

The processing stage is characterised by processing and setup times/costs associated with each

intermediate. In order to avoid high setup costs and down times, long processing runs and/or a limited number of intermediates are preferred. Production operations are scheduled following the common cycle scheduling policy (Hanssmann, 1962). This approach is widely used in industry due to its simplicity and adaptability and has been proven to produce optimal or near-optimal schedules in many practical situations especially when products are similar in terms of their cost structure and setup times are relatively short (Jones and Inman, 1989). In a common cycle schedule, one lot of each product is produced in each production cycle and the cycle time is identical for each product (in our case selected intermediate). If the usage rates of selected intermediates were known in advance, then the optimal cycle time could easily be determined following the common cycle policy. However, in our case, the usage rates depend on the decisions regarding the set of selected intermediates and end product recipes. Hence, rather than optimizing the cycle time we aim at finding the optimal set of selected intermediates and end product recipes for a given cycle time. Due to the perishable nature of food products, cycle times are rather short in food processing industry. Furthermore, cycle times are usually not just arbitrary intervals but integer multiples of an applicable time period such as a shift or a day. Thus, in case it is needed, the model can be solved for a limited set of applicable cycle lengths.

As discussed previously, the product variety at the end product level often makes it impossible to store all end products. Because of this reason, blending operations run on a daily basis following end product demand. The blending stage usually involves very standardised operations. Hence we assume a constant blending rate for all end products. The setup operations in this stage are minor and are assumed to be negligible.

The selected intermediates are stored between the two production stages in a number of storage units (e.g. silos or tanks) which are identical in terms of their volume. The limitations on the storage capacities are rather restrictive in the food processing industry since only a single type of intermediate can be stored in a storage unit (Akkerman et al., 2007). The customer preferences and demands change gradually over time, and consequently, selected intermediates and end product recipes are usually revised to correct for those periodically. We assume that demand is stable within those revision intervals.

### **3 Related literature and positioning**

The first example of the blending problem is the famous diet problem of Stigler (1945) where a minimum cost diet is determined subject to a set of dietary allowances. Following the line of this problem a large body of literature has emerged addressing blending problems particularly in the petrochemical industry and the agricultural industry. Most of this work has been concentrated on stand-alone blending problems which usually concern the determination of a minimum cost blend or a recipe while respecting a set of quality related constraints. However, in processing systems, the production and the storage operations are tightly coupled with product recipes and demands

which together determine the consumption rates of the ingredients to be used in processing the blends. Crama et al. (2001) classify blending problems into three basic categories based on the degree of the integration of the blending problem with production and storage operations: (i) design problems where the blending operations are considered in isolation, (ii) long- or medium-term planning problems where the blending operations are integrated in the long- or medium-term (master) planning, and (iii) short term planning and scheduling problems where the blending problem is a part of everyday operations. The problem under consideration in this study falls into the category of medium-term planning problems. Here we briefly review some of the work in this domain.

Glen (1980) develops a method for the beef cattle feedlot operations to determine the rations to feed animals. His method gradually changes the rations over time in order to obtain a specified liveweight at the minimum cost. Steuer (1984) studies sausage blending problems which concern the optimization of meat blends to produce sausages under a set of quality constraints. Taube-Netto (1996) presents an integrated planning model for poultry production which encompasses, among other aspects, the formulation of feed to be used over the planning horizon. In the aforementioned examples, the processing and blending operations of the feedstuffs are not integrated into the overall production planning problem.

Williams and Redwood (1974) propose a multi-period blending model for a company that refines and blends different types of raw oils to produce a number of brand oils. Their model decides upon the purchasing and production quantities for each time period considering the price fluctuations of raw oils. Rutten (1993) develops a hierarchical approach for the operational planning of a dairy firm. He considers the planning problem at the operational planning level and decomposes it into smaller problems each of which can be solved in reasonable computational times. However, these studies do not consider the economies of scale resulting from the setup costs/times.

Karmarkar and Rajaram (2001) study the joint production and blending problem. They propose a general mixed integer non-linear program (MINLP) and a Lagrangean heuristic to solve the problem. Their work is substantial since they jointly optimise the lot sizes and end product recipes. However, they consider only a single quality parameter and use a cost function to penalise the nonconformity of end product. Furthermore, they assume uncapacitated production and storage.

Our study is closely related to the work of Akkerman et al. (2010) where a flour manufacturing system is considered. They study a system where a limited number of grains are milled and blended into various types of flour products. They propose a mixed integer linear program (MILP) to determine the recipes of flour products minimizing total milling and blending costs. Their approach also accounts for the option of using selected intermediates directly as end products. They do not explicitly consider the production and storage capacities. However, they approximate these limitations by using an upper bound on the number of intermediates to be selected. They mention that it is logical to limit the number of intermediates since the opposite would require large setup times and a huge storage capacity. In this study, we build on the model provided by

Akkerman et al. (2010) and extend their study by explicitly incorporating the capacity limitations and costs on production and storage operations.

## 4 Model formulation

In this section, we present a mathematical model for the intermediate selection and blending problem. We first provide the notation used in the rest of the paper. Then we outline the objective function and the constraints characterizing the problem.

### 4.1 Notation

Consider a food processing system producing a set of end products  $J$ . These end products can be produced by using a set of intermediates  $I$ . Intermediates and end products are characterised by their compositions in terms of a set of ingredients  $K$ . We refer to the proportions of those ingredients as quality parameters. The quality parameters of intermediates are known whereas they are defined on minimum and maximum levels for end products. The end product recipes should comply with those bounds.

We are given the quality specifications

$$\begin{aligned} q_{ik} &= \text{quality parameter } k \in K \text{ of intermediate } i \in I \text{ (\%)} \\ q_{jk}^{\min} &= \text{minimum quality parameter } k \in K \text{ of end product } j \in J \text{ (\%)} \\ q_{jk}^{\max} &= \text{maximum quality parameter } k \in K \text{ of end product } j \in J \text{ (\%)} \end{aligned}$$

demand and process characteristics

$$\begin{aligned} d_j &= \text{demand rate of end product } j \in J \text{ (tons/day)} \\ s_i &= \text{setup time of intermediate } i \in I \text{ (days)} \\ p_i &= \text{processing rate of intermediate } i \in I \text{ (tons/day)} \\ p^b &= \text{blending rate of end products (tons/day)} \\ N &= \text{number of available storage units} \\ V &= \text{capacity of each storage unit (tons)} \\ \pi &= \text{cycle time (days)} \end{aligned}$$

and cost parameters

$$\begin{aligned} a_i &= \text{setup cost of intermediate } i \in I \text{ (Euros)} \\ c_i &= \text{processing (and material) cost of intermediate } i \in I \text{ (Euros/ton)} \\ c^b &= \text{blending cost of end products (Euros/ton)} \\ h_i &= \text{holding cost of intermediate } i \in I \text{ (Euros/ton day)}. \end{aligned}$$

In order to specify the basic intermediates to be used and corresponding end product recipes we define the variables

$x_{ij}$  = fraction of end product  $j \in J$  supplied by intermediate  $i \in I_j$

where  $I_j \subset I$  is the set of intermediates which can be used in producing end product  $j$ ,

$$y_i = \begin{cases} 1, & \text{if intermediate } i \in I \text{ is selected as a basic intermediate} \\ 0, & \text{otherwise} \end{cases}$$

and

$$v_{ij} = \begin{cases} 1, & \text{if intermediate } i \in I_j^* \text{ is used directly as end product } j \in J \\ 0, & \text{otherwise} \end{cases}$$

where  $I_j^* \subset I_j$  is the set of intermediates which comply with all quality specifications of end product  $j$ , i.e.

$$I_j^* = \{i \in I_j \mid q_{jk}^{\min} \leq q_{ik} \leq q_{jk}^{\max}, \forall k \in K\}.$$

For notational simplicity, we also introduce the expressions

$w_i$  = the consumption rate of intermediate  $i \in I$  (tons/day)

such that,

$$w_i = \sum_{j \in J} d_j x_{ij} \quad i \in I \quad (1)$$

and

$$z_j = \begin{cases} 1, & \text{if end product } j \in J \text{ is produced with blending operations} \\ 0, & \text{otherwise} \end{cases}$$

such that,

$$z_j = 1 - \sum_{i \in I_j} v_{ij} \quad \forall j \in J. \quad (2)$$

Notice that, the domain of  $z_j$  can be verified since  $v_{ij}$  equals 1 for at most one intermediate. This will further be clarified in the constraints.

## 4.2 Objective function

The objective is to minimise the daily total cost which is comprised of cost components associated with setup, processing and storage of intermediates; and blending of end products. Setup costs are relevant to those intermediates which are selected as basic intermediates. Since processing operations are carried out following a common cycle schedule, in each cycle a setup is initiated for every basic intermediate. Thus, cost incurred in a single cycle equals  $\sum_{i \in I} a_i y_i$ . To obtain the setup cost per day, the cost per cycle is divided by the cycle time. Processing costs involve the material and operational costs of processing operations, and they are incurred for all basic intermediates in proportion to their consumption rates. Hence, daily processing cost can be expressed as  $\sum_{i \in I} c_i w_i$ .



It is important to note that processing cost, as a combination of material and operational costs, is usually the largest cost component of the total costs in food process industries. Storage costs depend on the average inventory levels of intermediates. The processing of an intermediate, say intermediate  $i$ , starts when the inventory drops down to zero, and stops when reaches up to  $\pi w_i(1 - w_i/p_i)$ . Because both production and consumption rates are assumed to be constant, the average inventory equals half of the maximum inventory level. Thus,  $\sum_{i \in I} 0.5h_i\pi w_i(1 - w_i/p_i)$  gives the daily storage cost. Blending costs are incurred for end products which go through the blending operation in proportion to their demand rates. Hence, the daily blending cost equals  $c^b \sum_{j \in J} d_j z_j$ . The following expression, therefore, provides daily total costs.

$$\frac{1}{\pi} \sum_{i \in I} a_i y_i + \sum_{i \in I} c_i w_i + \sum_{i \in I} \frac{1}{2} h_i \pi w_i \left(1 - \frac{w_i}{p_i}\right) + c^b \sum_{j \in J} d_j z_j. \quad (3)$$

### 4.3 Constraints

The capacitated intermediate selection and blending problem involves constraints related to the conservation and quality requirements of end product recipes, and capacity limitations on processing, storage and blending operations. These constraints are articulated in this subsection.

**Recipe conservation constraints.** For each end product  $j$ , the percentages  $x_{ij}$  defining the contribution of each intermediate  $i$  into end product  $j$  must sum up to 1 in order to specify a complete recipe:

$$\sum_{i \in I_j} x_{ij} = 1 \quad \forall j \in J. \quad (4)$$

The decision on whether intermediate  $i$  is selected to be used in one or more end product recipes is indicated by the binary decision variable  $y_i$ . Hence, intermediate  $i$  cannot take place in any end product recipe as long as  $y_i$  equals 0:

$$x_{ij} \leq y_i \quad \forall i \in I_j, \forall j \in J. \quad (5)$$

If end product  $j$  is directly supplied as intermediate  $i$  then its contribution in the associated recipe (in percentage) must equal 1 (i.e. %100):

$$v_{ij} \leq x_{ij} \quad \forall i \in I_j^*, \forall j \in J. \quad (6)$$

Notice that Eq. (6) together with Eq. (4) guarantees that  $\sum_{i \in I_j^*} v_{ij} \in \{0, 1\}$ , and hence  $z_j \in \{0, 1\}$ .

**Quality constraints.** Quality constraints guarantee that recipes comply with the quality requirements of end products. That is, each quality specification  $k$  of end product  $j$ , as the weighted

average of the specifications of the intermediates take place in the corresponding recipe, must be between the pre-specified minimum and maximum quality parameters:

$$q_{jk}^{\min} \leq \sum_{i \in I_j} q_{ik} x_{ik} \leq q_{jk}^{\max} \quad \forall j \in J, \forall k \in K. \quad (7)$$

**Processing capacity constraints.** Processing capacity constraints state that there must be enough time for the setup and the production operations of the selected intermediates within the given cycle length. This can be guaranteed by

$$\sum_{i \in I} \left\{ s_i y_i + \pi \frac{w_i}{p_i} \right\} \leq \pi \quad (8)$$

where the terms in the summation stand for the total setup time and the total processing time associated with the selected intermediates respectively. Notice that,  $w_i$  equals 0 for those intermediates that are not selected (see Eq. (1) and Eq. (5)).

**Storage capacity constraints.** Storage capacity constraints limit intermediate inventory levels. More specifically, there are  $N$  storage silos available each with  $V$  tons of capacity. Because the form of storage is homogeneous, the number of storage units constitutes an upper bound on the number of intermediates. However, it is also possible to assign multiple storage units to a particular intermediate. Henceforth, the number of storage silos assigned to an intermediate bounds the maximum inventory level of that intermediate.

The maximum inventory level of an intermediate depends on the production mode as discussed in forming the objective function. Following the same reasoning, we can write the storage constraints as

$$\sum_{i \in I} \left\lceil \frac{\pi w_i \left(1 - \frac{w_i}{p_i}\right)}{V} \right\rceil \leq N. \quad (9)$$

**Blending capacity constraints.** Blending capacity constraints limit the extent of the daily blending operations. The daily blending rate is given as  $p^b$ . The total daily blending volume is the sum of the demands associated with those end products which undergo blending operations as indicated by the binary variable  $z_j$ . Hence, the daily blending capacity constraint is expressed as

$$\sum_{j \in J} d_j z_j \leq p^b. \quad (10)$$

The mathematical formulation provided so far involves non-linear expressions both in the objective function and constraints. In particular, both storage costs and constraints are non-linear (see Eq. (3) and Eq. (9)). These expressions are difficult to handle with general purpose mathematical programming solvers. In Appendix A, we provide a linearisation scheme for those expressions

which enables us to express the model as a MILP. Also, in Appendix B, we provide some upper bounds on the consumption rates of potential intermediates which can be used to strengthen the formulation.

## 5 Numerical study

We implement our approach on a data set collected from a medium-sized flour manufacturer that supplies flour products to bakeries and industrial manufacturers. The main processing operation in flour manufacturing is the milling process where the grains are ground between successive sets of mill stones or rollers to produce different types of intermediate flour products. These flour products can be used directly as end products to satisfy demands. Alternatively, they can be blended into end products following to a blending operation where they are dispersed within each other and homogenised. Consequently, the decision problem is to select those flour products to be stocked, and to determine the recipes of end products specifying how they are blended.

The production system under consideration involves 76 potential intermediates and 45 end products with 9 different quality parameters. The flour mill can process flour products with a capacity around 350 tons/day. This does not include the setup times which are around 30 minutes per changeover. The blender mixes flour products with an average capacity of 200 tons/day. There are 18 storage silos each of which can store around 50 tons of material. The total demand sums up to 220 tons/day. However, the demands vary substantially between different end products. We do not provide the cost figures here for the sake of confidentiality. It is important to note that, as it is in most process industries, the material procurement costs (which are expressed in processing costs in the formulation) are dominant with respect to other operational costs. However, while good purchasing is pivotal, minimizing the non-procurement costs is important to stay competitive since profit margins are rather small in the food processing industry.

We analyse the case in a constructive manner by using several, increasingly comprehensive, scenarios, thereby illustrating the effect of the different constraints and cost factors. In Scenario 1, we consider the blending problem in isolation, ignoring all types of capacity limitations. This scenario establishes a benchmark to compare the following scenarios with. In Scenario 2, we add the production capacity limitations and setups costs to the problem showing us how these affect the selection of intermediates and end product recipes. In Scenario 3, we integrate the storage costs into the problem, demonstrating the trade-off between setup costs and storage costs. In Scenario 4, we finally add the storage capacity limitation, thereby considering all relevant costs and capacity limitations. This scenario reflects the actual production environment addressed in this study. In Scenario 5, we again study the complete problem, but change the production setup of the case company to look at possible ways to increase efficiency.

In each scenario, we communicate the daily costs and capacity utilization levels of production and storage operations, and provide some basic information regarding the selection of intermediates

and end product recipes. More specifically, we report:

1. Cycle time (CT):  $\pi$
2. Total costs (ToC):  $1/\pi \sum_{i \in I} a_i y_i + \sum_{i \in I} c_i w_i + \sum_{i \in I} 1/2 h_i \pi w_i (1 - w_i/p_i) + c^b \sum_{j \in J} d_j z_j$
3. Processing costs (PrC):  $\sum_{i \in I} c_i w_i$
4. Setup costs (SeC):  $1/\pi \sum_{i \in I} a_i y_i$
5. Blending costs (BIC):  $c^b \sum_{j \in J} d_j z_j$
6. Storage costs (StC):  $\sum_{i \in I} 1/2 h_i \pi w_i (1 - w_i/p_i)$
7. Processing utilisation (PrU):  $1/\pi \sum_{i \in I} \{s_i y_i + \pi w_i/p_i\}$
8. Blending utilisation (BIU):  $\sum_{j \in J} d_j z_j/p^b$
9. Storage utilisation (StU):  $\sum_{i \in I} \lceil \pi w_i (1 - w_i/p_i)/V \rceil / N$
10. Number of selected intermediates (#SI):  $\sum_{i \in I} y_i$
11. Number of end products directly supplied from intermediate stocks (i.e. end products involving a single intermediate in their recipes) (#EPFS):  $|J| - \sum_{j \in J} z_j$

Notice that, we define the utilization levels as the ratio of the engaged capacity to the available capacity. As such, the storage utilization relates to the percentage of storage units in use, rather than utilization of each individual storage unit.

## 5.1 Scenario 1

We start our analysis with the blending problem in isolation. That is, we minimise the sum of processing and blending costs subject to the quality constraints of end products. Thus, we assume that the production and storage capacities are both infinite and we neglect the setup and storage costs. Notice that, the optimal solution of this problem is independent of the cycle time, and it sets a lower bound on the costs for the original problem. The results are given in Table 1.

Table 1: Optimal solution – Scenario 1

	Costs (Euros/day)					Utilization (%)			Recipes	
CT	ToC	PrC	SeC	BIC	StC	PrU	BIU	StU	#SI	#EPFS
1	31202	31129	-	73	-	-	-	-	30	32

We observe that the daily total cost is minimised at 31202. Only a very small portion of this cost originates from the blending operations. The model selects 30 out of 76 intermediates to be stocked and 32 out of 45 end products to be supplied from stock. One could expect that supplying all end products from stock yields a lower total cost by preventing any blending costs. Yet, the optimal solution suggests that 13 of 45 end products should undergo blending operations. This shows that the unit production costs of those end products which are not directly supplied from stock are smaller when they are blended from a number of intermediates despite the additional blending costs. In other words, the intermediates which comply with the quality requirements of those end products possess larger unit processing costs than the unit processing cost of the optimal blend plus the blending costs. Another result is that 32 end products are supplied from stock although only 30 intermediates are selected to be stocked. This means that some selected intermediates comply with the quality requirements of multiple end products. Notice that this scenario reflects the minimum attainable combination of processing and blending costs. In the following scenarios, we analyse how additional costs and capacity limitations add on this cost figure.

## 5.2 Scenario 2

In this scenario, we integrate production capacities (i.e. processing rates and setup times) and related costs (i.e. setup costs) into the problem considered in Scenario 1 while still neglecting the storage capacity limitation and storage costs. Note that the optimal solution of the problem is now dependent on the cycle time. In Table 2, we therefore report the optimal solutions of the problem for cycle times of 1 to 10 days.

Table 2: Optimal solution – Scenario 2

CT	Costs (Euros/day)					Utilization (%)			Recipes	
	ToC	PrC	SeC	BIC	StC	PrU	BIU	StU	#SI	#EPFS
1	31875	31215	531	129	-	0.88	0.32	-	11	21
2	31585	31178	290	117	-	0.77	0.29	-	12	21
3	31480	31165	224	91	-	0.74	0.23	-	14	24
4	31423	31160	174	89	-	0.71	0.22	-	15	25
5	31386	31151	146	89	-	0.71	0.22	-	16	25
6	31363	31153	124	86	-	0.70	0.21	-	16	25
7	31346	31153	107	86	-	0.69	0.21	-	16	25
8	31331	31141	102	88	-	0.70	0.22	-	17	24
9	31319	31142	90	87	-	0.69	0.22	-	17	25
10	31311	31132	92	87	-	0.72	0.22	-	19	25

The results show that both the cost figures and the selection of intermediates are quite different

from the ones in Scenario 1. We notice a sharp decrease in the number of selected intermediates compared to Scenario 1. This leads to higher processing and blending costs which, together with setup costs, significantly increase the daily total cost. We also observe that neither the production nor the blending capacity is binding for the system under consideration. For the cycle times considered, the utilisation in production and blending does not reach 100%.

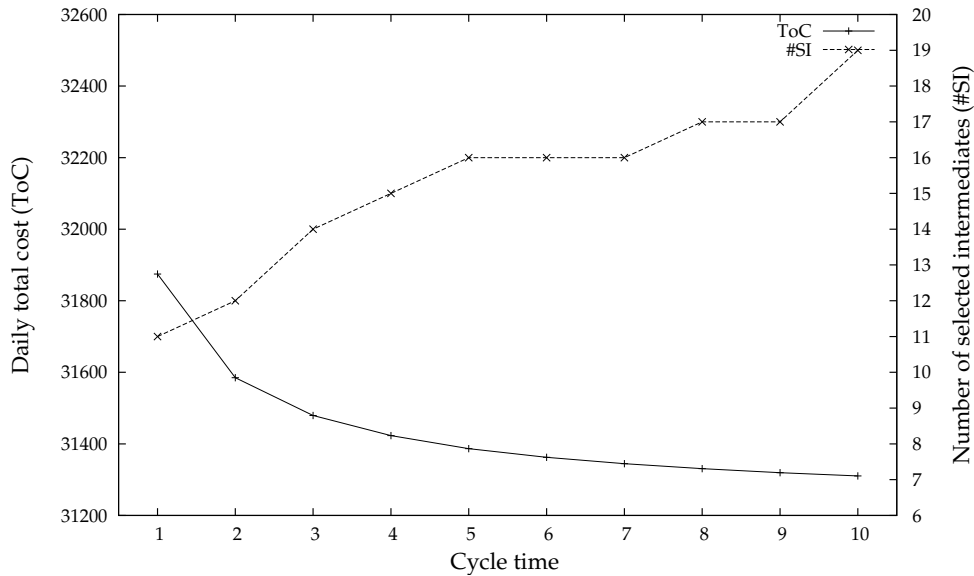


Figure 2: Daily total costs and the number of selected intermediates – Scenario 2

The trade-off between the daily total cost and the number of selected intermediates is one of the core issues considered in this study. In Figure 2, we illustrate this trade-off with respect to varying cycle times. As can be observed, the number of selected intermediates gradually increases with the cycle time. The optimal daily total cost, however, is decreasing on cycle time since we do not consider storage costs. A setup cost is incurred for each selected intermediate in every production cycle. Thus, the daily total setup cost is decreasing on the cycle time and increasing on the sum of the individual setup times of selected intermediates. In this sense, having a larger number of selected intermediates may lead to higher setup costs. The results clearly show that this effect is dominated by the cost reduction due to the increasing cycle time. On the other hand, having a larger number of intermediates may also result in lower processing and/or blending costs by bringing more options to supply end products. We can detect these effects in Table 2. The results show that increasing cycle time leads to a larger number of selected intermediates, and thus, reduce processing and blending costs.

### 5.3 Scenario 3

In this scenario, we integrate the storage costs into the problem considered in Scenario 2 while still neglecting the storage capacity limitation. In Table 3, we report the optimal solutions of the problem for cycle times of 1 to 10 days.

Table 3: Optimal solution – Scenario 3

CT	Costs (Euros/day)					Utilization (%)			Recipes	
	ToC	PrC	SeC	BIC	StC	PrU	BIU	StU	#SI	#EPFS
1	31892	31215	531	129	17	0.88	0.32	-	11	21
2	31623	31178	290	117	38	0.77	0.29	-	12	21
3	31539	31165	224	91	59	0.73	0.23	-	14	24
4	31503	31165	168	91	79	0.71	0.23	-	14	24
5	31485	31151	146	89	99	0.71	0.22	-	16	25
6	31482	31153	124	86	119	0.70	0.21	-	16	25
7	31485	31153	107	86	139	0.69	0.21	-	16	25
8	31490	31142	102	87	159	0.70	0.22	-	17	25
9	31498	31142	90	87	179	0.69	0.22	-	17	25
10	31509	31142	81	87	199	0.69	0.22	-	17	25

We observe that the cost figures, the utilisation rates, and the selection of intermediates are very similar to the ones reported in Scenario 2. Because the difference between these demonstrates the effects of storage costs, we can argue that the magnitude of storage cost is not sufficiently large to effect the decisions regarding the design of intermediates and end products. Nevertheless, we can clearly detect the trade-off between setup and holding costs, i.e. the daily setup cost is decreasing whereas the daily storage cost is increasing on cycle time. As a consequence, the optimal daily total cost is no longer decreasing on cycle time.

In Figure 3, we illustrate the daily total costs and the number of selected intermediates with respect to varying cycle times. We observe that the minimum daily total cost 31482 is achieved when the cycle time is 6 days. The optimal daily total cost is higher for cycle times shorter than 6 days due to larger setup costs, and for cycle times longer than 6 days due to larger storage costs. The effect of holding costs on the selection of intermediates is mostly visible for longer cycle times where the magnitude of storage costs is rather large. Consider the cycle time of 10 days. In Scenario 2, the model selects 19 intermediates which can supply 25 end products directly from stock. In Scenario 3, however, the model selects 17 intermediates which can also supply 25 end products directly from stock. The difference between those figures can be explained as follows. The total consumption rate of selected intermediates equals the total demand rate, and it is allocated

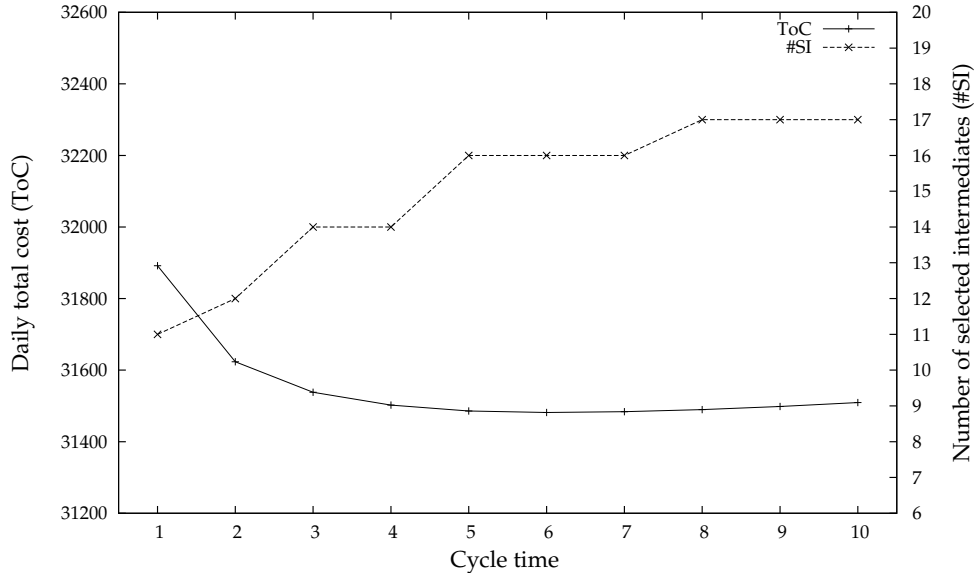


Figure 3: Daily total costs and the number of selected intermediates – Scenario 3

between the selected intermediates following the end product recipes. The average inventory level of a selected intermediate is increasing on intermediate’s production rate whereas it is concave on intermediate’s consumption rate. Thus, with other things held constant, the total holding cost would be lower when the production rates and holding costs of the selected intermediates are lower and/or the number of selected intermediates is smaller. The reduction in the number of intermediates also leads to a slight increase in the processing costs, demonstrating that more expensive raw materials are required to produce more flexible intermediates.

#### 5.4 Scenario 4

In this scenario, we integrate the storage capacity limitation into the problem considered in Scenario 3. Thus, we consider all types of capacity limitations and costs and investigate the actual real-life problem. In this particular case, there are 18 storage units available. In Table 4, we report the optimal solutions of the problem for cycle times of 1 to 6 days. We do not consider cycle times longer than 6 days because there is no feasible solution for those with the given storage capacity limitation.

The results show that the storage capacity limitation significantly affects the optimal cost structure and the selection of intermediates and end product recipes. Notice that, in general, a longer cycle time leads to higher inventory levels. Thus storage capacity limitation is more restrictive when the cycle time is longer. For cycle times longer than 2 days, the utilization of storage units reaches 100%, and the storage capacity becomes binding. As can be observed, for those cycle times, the storage capacity limitation leads to large differences in daily total costs



Table 4: Optimal solution – Scenario 4

CT	Costs (Euros/day)					Utilization (%)			Recipes	
	ToC	PrC	SeC	BIC	StC	PrU	BIU	StU	#SI	#EPFS
1	31892	31215	531	129	17	0.88	0.32	0.61	11	21
2	31623	31178	290	117	38	0.77	0.29	0.78	12	21
3	31539	31165	224	91	59	0.74	0.23	1.00	14	24
4	31577	31215	155	128	79	0.73	0.32	1.00	13	21
5	31891	31386	93	327	85	0.82	0.82	1.00	9	11
6	32451	31959	54	346	92	0.90	0.87	1.00	7	12

reported in Scenario 3 and Scenario 4. The magnitude of this difference gradually increases with the cycle time, and eventually, the storage capacity limitation results in an infeasible problem for cycle times longer than 6 days.

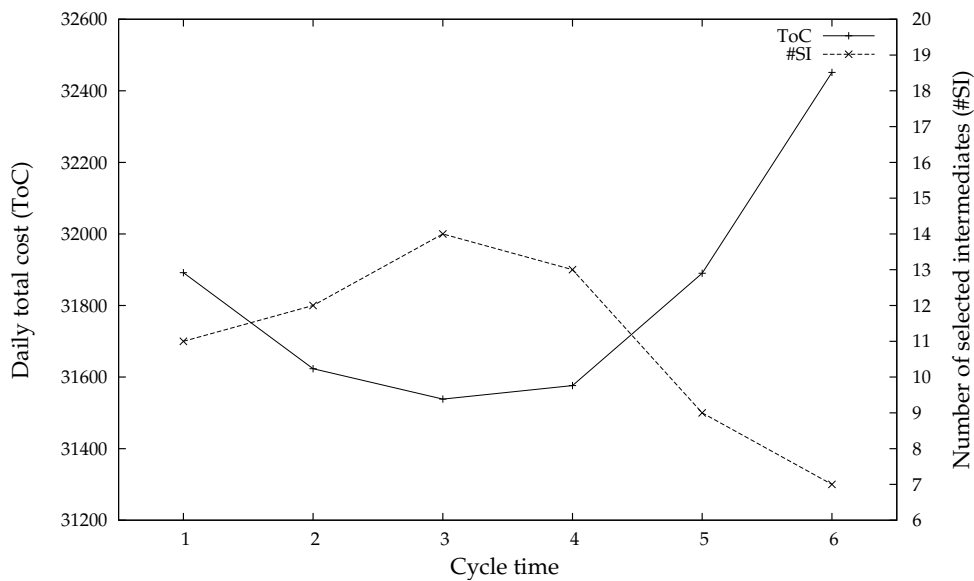


Figure 4: Daily total costs and the number of selected intermediates – Scenario 4

Figure 4 illustrates the daily total costs and the number of selected intermediates with respect to varying cycle times. We observe that daily total cost is minimised when the cycle time is 3 days, and it is increasing rapidly with the cycle time. We also see that the storage limitations significantly change the structure of the optimal set of selected intermediates and the end product recipes. In particular, for those cycle times where the storage capacity is binding, the model selects a smaller number of intermediates, preferably the ones with lower production rates, in order to reduce the stock levels. This, however, increases processing and blending costs because the set of

selected intermediates and end product recipes further move away from the optimal ones.

## 5.5 Scenario 5

In the previous scenarios, we have observed that the storage capacity limitation is the most critical one among other limitations for the particular example considered in this numerical study. Consequently, in this scenario, we focus on cost reductions that can be achieved by altering the storage capacity. We conduct our analysis as follows. First, for each cycle time, we find a critical storage capacity level which is large enough to provide the optimal daily total cost that can be achieved when there is no storage limitation. Since inventory levels gradually increase with cycle time, we expect critical storage capacity levels to be higher for longer cycle times. These levels can easily be found by solving the problem without storage capacity limitations, as in Scenario 3, and checking how many storage units are being used following the optimal solution. Notice that the storage capacity constraint is binding only when the storage capacity is below those critical levels, and if so then having extra storage capacity could reduce the daily total cost. Secondly, we solve the problem for all storage capacity levels up to critical ones so as to find the added value of expanding the storage capacity.

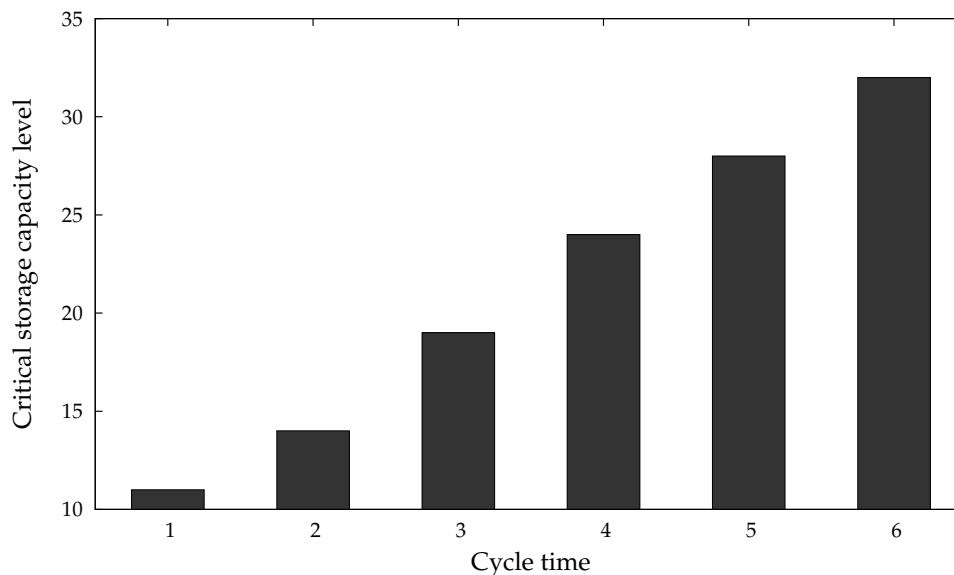


Figure 5: Critical storage capacity levels for cycle times of one to six days – Scenario 5

We know, from Scenario 3, that the optimal cycle time is 6 days for the problem without storage capacity limitations. This cycle time can be regarded as an upper bound for the optimal cycle time when storage capacity is limited. Thus we limit our analysis to cycle times from 1 to 6 days. In Figure 5, we report those critical resource levels. As expected, we observe that the critical storage capacity levels are increasing on cycle time. The critical storage capacity levels reflect the maximum number

of storage units that could possibly be needed when a given cycle time is employed. Up to this level, additional storage units lead to lower costs, but, there is no added value of expanding the storage capacity beyond the critical level as long as the same cycle time is employed. Nevertheless, from Scenario 3 we know that the daily total cost can still be improved by employing a longer cycle time (not more than 6 days).

In what follows, we solve the problem for cycle times of 1 to 6 days and for all feasible storage capacity levels up to the critical ones so as to find the added value of the extra storage capacity. In previous scenarios we have already demonstrated the effects of the storage capacity limitation on individual cost components, utilization rates, and the selection of intermediates and end product recipes. Thus, in this scenario we only communicate the optimal daily total costs. Figure 6 illustrates the minimum daily total costs that can be achieved with respect to the available number of storage units also including the cycle time linked to these solutions.

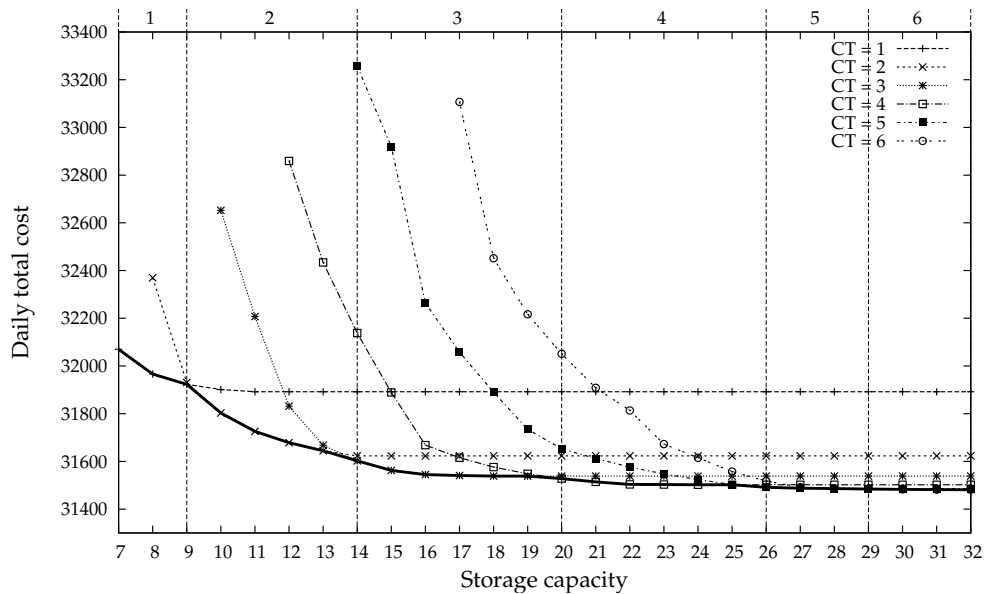


Figure 6: Optimal daily total costs – Scenario 5

We observe that different cycle times are optimal for different storage capacities. For example, when cycle time is 3 days, the critical storage capacity level equals 19 storage units. Further increasing the storage capacity does not lead to a reduction of cost as long as the cycle time remains the same. However, increasing the available number of storage units to 20, while also increasing the cycle time to 4 days open up the possibility to reduce costs, since the storage capacity of 20 units is less than the critical storage capacity level for the cycle time of 4 days. Hence, further increasing the storage capacity leads to a lower daily total cost until we reach a storage capacity of 25. After this, we would again need to increase the cycle time to enable further cost reductions. Notice that the cycle time that minimises the daily total cost tends to increase as the storage capacity gets

larger. Nevertheless, from Scenario 3, we know that the maximum cycle time that would be used equals 6 days. Hence, it would never be necessary to use more than 32 storage units as it is the critical storage level for the cycle time of 6 days.

We can also investigate the added value of extra storage capacity by looking at the minimum daily total cost for each storage capacity level over the cycle times. These costs are illustrated by the down-most bold line in Figure 6. We know that the daily total cost tends to decrease as the storage capacity expands up to 32 storage units and then levels off. However, we observe that this trend is not steady over the storage capacity. That is, the cost reduction due to an extra storage unit is not decreasing on the number of available storage units. This is mainly because when an additional storage unit increases the storage capacity to exceed a critical level, it brings up the possibility of reducing the cost also by altering the cycle time which is not possible otherwise.

In the actual storage setting of the case company, 18 storage units are used. We know now that expanding the storage capacity to 32 units will reduce the daily total cost but only if the cycle time is increased simultaneously. Minor, more realistic, increases in the storage capacity should also be considered in combination with changes in the cycle time. Adding one storage unit has almost no effect on costs whereas adding two or three storage units in combination with an increase of the cycle time to 4 would have a significant effect.

## 6 Conclusions

In this study, we addressed a capacitated intermediate product selection and blending problem confronted in the food processing industry. The problem involves the selection of a set of intermediates and end product recipes characterizing how those selected intermediates are blended into end products to minimise the total operational costs under capacity limitations. We developed a comprehensive MILP model for the problem. We applied the model to a data set collected from a real-life company and we analysed the problem under several scenarios to better understand the trade-offs between capacity limitations and costs. For the particular case considered in our numerical study, we observed that the production and the blending capacities are not binding for the case whereas the storage capacity is. Consequently, we investigated possible cost reductions that can be achieved by altering the storage capacity. We showed that the cost reduction due to an extra storage unit is not decreasing on the number of available storage units, mainly due to the use of different cycle times. This suggests that a careful investigation is required when deciding upon an expansion of the storage capacity.

In general, this study demonstrated important product-process interactions in the process industries, where the decisions on the selection of intermediates and the configuration of end product recipes are affected by the capacity limitations and the costs associated with production and storage operations. In this context, the problem addressed in this study can be regarded an extension of production lot sizing problems with integrated design decisions. The conventional production

lot sizing problems try to balance the trade-off between the fixed production setup costs and inventory holding costs. We observed that this trade-off is affected by integrated design decisions, since excessive setup and holding costs can be eliminated by reducing the number of intermediates, however, in expense of additional blending costs. We also analysed how capacity limitations affect the selection of intermediates and end product recipes. We saw that these limitations interact with each other, and based on their magnitude, one or more of those limitations could be binding at the same time. Especially the planning of the production operations within the capacitated situation (in this paper represented by the selection of a certain cycle time) had a significant effect on the selected intermediate products, the amount of storage units this requires, and the total costs. The model presented in this paper can be used to gain insight in the complex interactions between product design, process design, and operational planning. Also, an analysis as the one conducted in this paper, could be useful for evaluating alternative design and/or expansion decisions.

This research is particularly aimed at the food processing industry. Nevertheless, it encompasses characteristics, such as limitations on production and storage capacities, which are very common in many other processing systems. Therefore, the proposed model can also be adapted to other processing systems with some simple modifications. For instance, in some processing systems, production lots go through a series of quality checks before they are used. This could easily be covered by the proposed model by replacing the expression of maximum inventory levels  $\pi w_i(1 - w_i/p_i)$  with  $\pi w_i$  in the objective function and the storage capacity constraints. This would also simplify the linearization of storage capacity constraints. There are several directions we leave aside for further research. We analysed the problem assuming that a common cycle scheduling policy is employed. Although it is widely used in practice, it is known that under certain circumstances this policy may perform badly. Hence, the same problem can be considered under more sophisticated scheduling policies. We assumed that all storage units are identical, and they must be assigned to certain intermediates. Violating these assumptions require significant modifications in our approach. However, in some cases, storage units may possess different characteristics, and it may be possible to switch storage units between intermediates. Thus, it would be interesting to analyse the problem while relaxing these assumptions.

## A Piecewise linear approximation of storage capacities and costs

The mathematical formulation provided in Section 4 involves non-linearities in the objective function and constraints. The non-linearity arises due to the expression of the maximum inventory level of intermediates which is a quadratic polynomial. The expression appears in the objective function (see Eq. (3)) and the storage capacity constraints (see Eq. (9)). Here, we provide a piecewise linear approximation for this expression. Let us define

$$f_i(w_i) = \pi w_i \left(1 - \frac{w_i}{p_i}\right). \quad (11)$$

For any intermediate  $i$ , the consumption rate  $w_i$  is non-negative, and it cannot exceed the production rate  $p_i$  (see Eq. (8)). Hence, we analyse  $f_i(\cdot)$  in the domain  $[0, p_i]$ , where it is concave, equals 0 at  $w_i = 0$  and  $w_i = p_i$ , and reaches its maximum at  $w_i = p_i/2$  (see Figure A1).

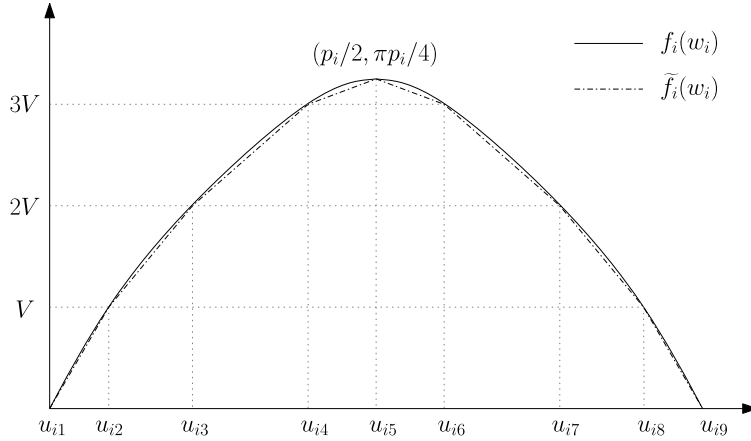


Figure 7: Approximation of  $f_i(\cdot)$

The proposed approximation scheme is based on setting breakpoints of the piecewise approximation to exact values of  $f_i(w_i)$  corresponding to integer multiples of the storage unit capacity  $V$ . In order to account for the exact maximum inventory level and extreme values of the domain of  $w_i$ , we also use breakpoints at  $w_i = 0$ ,  $w_i = p_i/2$ , and  $w_i = p_i$ . This approach guarantees the feasibility of any storage unit assignment while, in general, underestimating maximum inventory levels and hence storage costs.

The number of linear segments that must be used for intermediate  $i$  depends on the production rate  $p_i$ , the capacity of storage units  $V$ , and the length of the planning horizon  $\pi$ . We denote the set of linear segments for intermediate  $i$  by  $L_i$ . Each linear segment  $l \in L_i$  is bounded below and above by two breakpoints denoted by  $u_{il-1}$  and  $u_{il}$ . Notice that these values can easily be pre-computed. Figure A1 depicts a possible realization of  $f_i(\cdot)$  and the corresponding approximation function denoted by  $\tilde{f}_i(\cdot)$ .

The approximation scheme requires the usage a set of new variables and constraints. Let  $\alpha_{il}$  and  $\beta_{il}$  be the intercept and the slope of the  $l$ 'th linear segment of  $\tilde{f}_i$ . Now, we introduce variables:

$$\delta_{il} = \begin{cases} 1, & \text{if } u_{il-1} < w_i \leq u_{il} \\ 0, & \text{otherwise} \end{cases}$$

and

$$\mu_{il} = \begin{cases} w_i, & \text{if } u_{il-1} < w_i \leq u_{il} \\ 0, & \text{otherwise.} \end{cases}$$

It should be obvious that  $\delta_{il}$  and  $\mu_{il}$  represent the active segment and the corresponding consumption rate for intermediate  $i$ . In order to guarantee that these variables take appropriate values, we use the constraints:

$$u_{il-1}\delta_{il} < \mu_{il} \leq u_{il}\delta_{il} \quad \forall l \in L_i, \forall i \in I \quad (12)$$

and

$$\sum_{l \in L_i} \delta_{il} = 1 \quad \forall i \in I \quad (13)$$

$$\sum_{l \in L_i} \mu_{il} = w_i \quad \forall i \in I. \quad (14)$$

We can now re-write the objective function in Eq. (3) by replacing the exact expression of the maximum inventory levels given in Eq. (11) with the following approximate one:

$$\tilde{f}_i(w_i) = \sum_{l \in L_i} \alpha_{il}\delta_{il} + \beta_{il}\mu_{il}. \quad (15)$$

Next, we revise the storage capacity constraint. The number of storage tanks that must be assigned to intermediate  $i$  is uniquely defined for each linear segment  $l \in L_i$ . Let us denote these values by  $r_{il}$ , such that, if  $\delta_{il} = 1$ , then  $r_{il}$  storage tanks are assigned to intermediate  $i$ . We can now re-write the storage capacity constraint given in Eq. (9) as

$$\sum_{i \in I} \sum_{l \in L_i} r_{il}\delta_{il} \leq N. \quad (16)$$

## B Upper bounds on the consumption rates of intermediates

We used a piecewise linear approximation scheme in order to ensure the linearity of the mathematical formulation. However, this necessitated the use of a new set binary variables. In this section, we provide a simple method to reduce the number of those variables, and thus, to reduce the computation time. The method is based on the idea of finding upper bounds on the consumption rates of potential intermediates by using a slightly modified version of the blending sub-problem. These bounds are then used to cut-off some of the binary variables associated with the assignment of storage units to intermediates.

Let us consider the blending sub-problem:

$$\min \sum_{j \in J} \sum_{i \in I_j} c_i x_{ij} \quad (17)$$

$$\sum_{i \in I_j} x_{ij} = 1 \quad \forall j \in J \quad (18)$$

$$q_{jk}^{\min} \leq \sum_{i \in I_j} q_{ik} x_{ik} \leq q_{jk}^{\max} \quad \forall j \in J, \forall k \in K \quad (19)$$

The blending sub-problem provides the optimal end product recipes when production and storage capacities are unlimited and the only cost to be considered is the processing costs of intermediates. Hence, the constraints of the blending sub-problem define all feasible end product recipes. Now, let us replace the objective function by

$$\max w_i^{\max} = \sum_{j \in J} d_j x_{ij}. \quad (20)$$

It is clear that  $w_i^{\max}$  is an upper bound on any feasible  $w_i$  of intermediate  $i$  in the original formulation since it is the maximum possible consumption rate in the uncapacitated problem. The modified version of the blending sub-problem is a simple linear program and can easily be solved for each intermediate. Notice that it is possible to find stronger bounds by using more sophisticated sub-problems following the same approach. However, we experienced considerable reductions in the computation time even with this simple sub-problem.

Once upper bounds on the consumption rates are computed, for each intermediate  $i$ , we can replace the set of linear segments  $L_i$  with a smaller one  $\tilde{L}_i$  which can be defined as follows:

$$\tilde{L}_i = \{l \in L_i \mid u_{il} \leq w_i^{\max}\}. \quad (21)$$

## References

- Akkerman, R., D. Van Der Meer, and D. P. Van Donk (2010). Make to stock and mix to order: Choosing intermediate products in the food-processing industry. *International Journal of Production Research* 48(12), 3475–3492.
- Akkerman, R. and D. P. Van Donk (2009). Analyzing scheduling in the food-processing industry: structure and tasks. *Cognition, Technology & Work* 11(3), 215–226.
- Akkerman, R., D. P. Van Donk, and G. Gaalman (2007). Influence of capacity-and time-constrained intermediate storage in two-stage food production systems. *International Journal of Production Research* 45(13), 2955–2973.
- Caux, C., F. David, and H. Pierreval (2006). Implementation of delayed differentiation in batch process industries: a standardization problem. *International Journal of Production Research* 44(16), 3243–3255.
- Cholette, S. (2009). Mitigating demand uncertainty across a winerys sales channels through postponement. *International Journal of Production Research* 47(13), 3587–3609.
- Cholette, S. (2010). Postponement practices in the wine industry: Adoption and attitudes of california wineries. *Supply Chain Forum: an International Journal* 11(1), 4–15.



- Crama, Y., Y. Pochet, and Y. Wera (2001). A discussion of production planning approaches in the process industry. Technical report, Université catholique de Louvain.
- Forza, C., F. Salvador, and A. Trentin (2008). Form postponement effects on operational performance: A typological theory. *International Journal of Operations & Production Management* 28(11), 1067–1094.
- Glen, J. J. (1980). A mathematical programming approach to beef feedlot optimization. *Management Science* 26(5), 524–535.
- Hanssmann, F. (1962). *Operations Research in Production and Inventory Control*. New York City: John Wiley & Sons.
- Jones, P. C. and R. P. Inman (1989). When is the Economic Lot Scheduling Problem Easy? *IIE Transactions* 21(1), 11–20.
- Karmarkar, U. S. and K. Rajaram (2001). Grade selection and blending to optimize cost and quality. *Operations Research* 49(2), 271–280.
- McIntosh, R. I., J. Matthews, G. Mullineux, and A. J. Medland (2010). Late customisation: issues of mass customisation in the food industry. *International Journal of Production Research* 48(6), 1557–1574.
- Nicholson, C. F., M. I. Gomez, and O. H. Gao (2011). The costs of increased localization for a multiple-product food supply chain: Dairy in the united states. *Food Policy* 36(2), 300 – 310.
- Pil, F. K. and M. Holweg (2004). Linking product variety to order-fulfillment strategies. *Interfaces* 34(5), 394–403.
- Rutten, W. (1993). Hierarchical mathematical programming for operational planning in a process industry. *European Journal of Operational Research* 64(3), 363–369.
- Skipworth, H. and A. Harrison (2004). Implications of form postponement to manufacturing: a case study. *International Journal of Production Research* 42(10), 2063–2081.
- Soman, C. A., D. P. Van Donk, and G. Gaalman (2004). Combined make-to-order and make-to-stock in a food production system. *International Journal of Production Economics* 90(2), 223–235.
- Steuer, R. E. (1984). Sausage blending using multiple objective linear programming. *Management Science* 30(11), 1376–1384.
- Stigler, G. J. (1945). The cost of subsistence. *Journal of Farm Economics* 27, 303–14.

- Taube-Netto, M. (1996). Integrated planning for poultry production at Sadia. *Interfaces* 26(1), 38–53.
- Van Donk, D. P. (2001). Make to stock or make to order: The decoupling point in the food processing industries. *International Journal of Production Economics* 69(3), 297–306.
- Van Hoek, R. I. (1999). Postponement and the reconfiguration challenge for food supply chains. *Supply Chain Management: An International Journal* 4(1), 18–34.
- Venkatesh, S. and J. M. Swaminathan (2004). Managing product variety through postponement: Concept and applications. In T. Harrison, H. Lee, and J. Neale (Eds.), *The Practice of Supply Chain Management: Where Theory and Applications Converge*, pp. 139–155. Kluwer Publishers, Boston, MA.
- Williams, H. P. and A. C. Redwood (1974). A structured linear programming model in the food industry. *Operational Research Quarterly* 25(4), 517–527.
- Wong, H., A. Potter, and M. Naim (2011). Evaluation of postponement in the soluble coffee supply chain: A case study. *International Journal of Production Economics* 131(1), 355 – 364.
- Zinn, W. and D. J. Bowersox (1988). Planning physical distribution with the principle of postponement. *Journal of Business Logistics* 9(2), 117–136.